BOOTCAMP IN CREDIT RISK MODELLING

BY Peaks2tails

COURSE CONTENT

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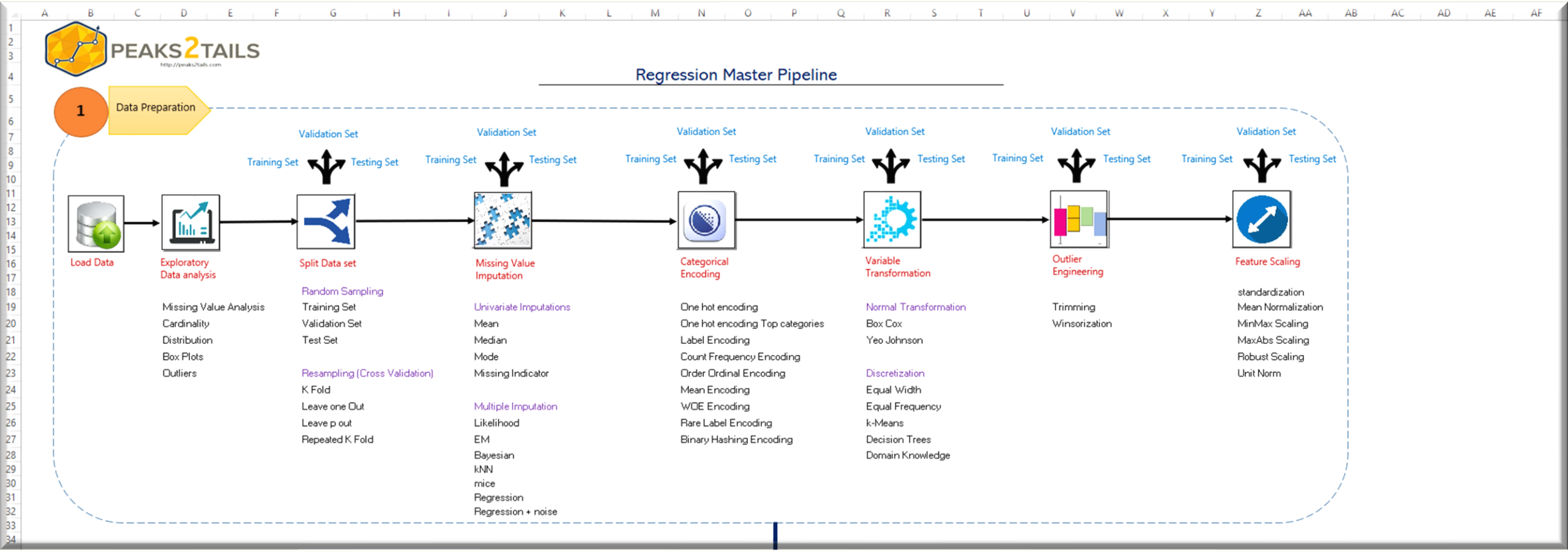
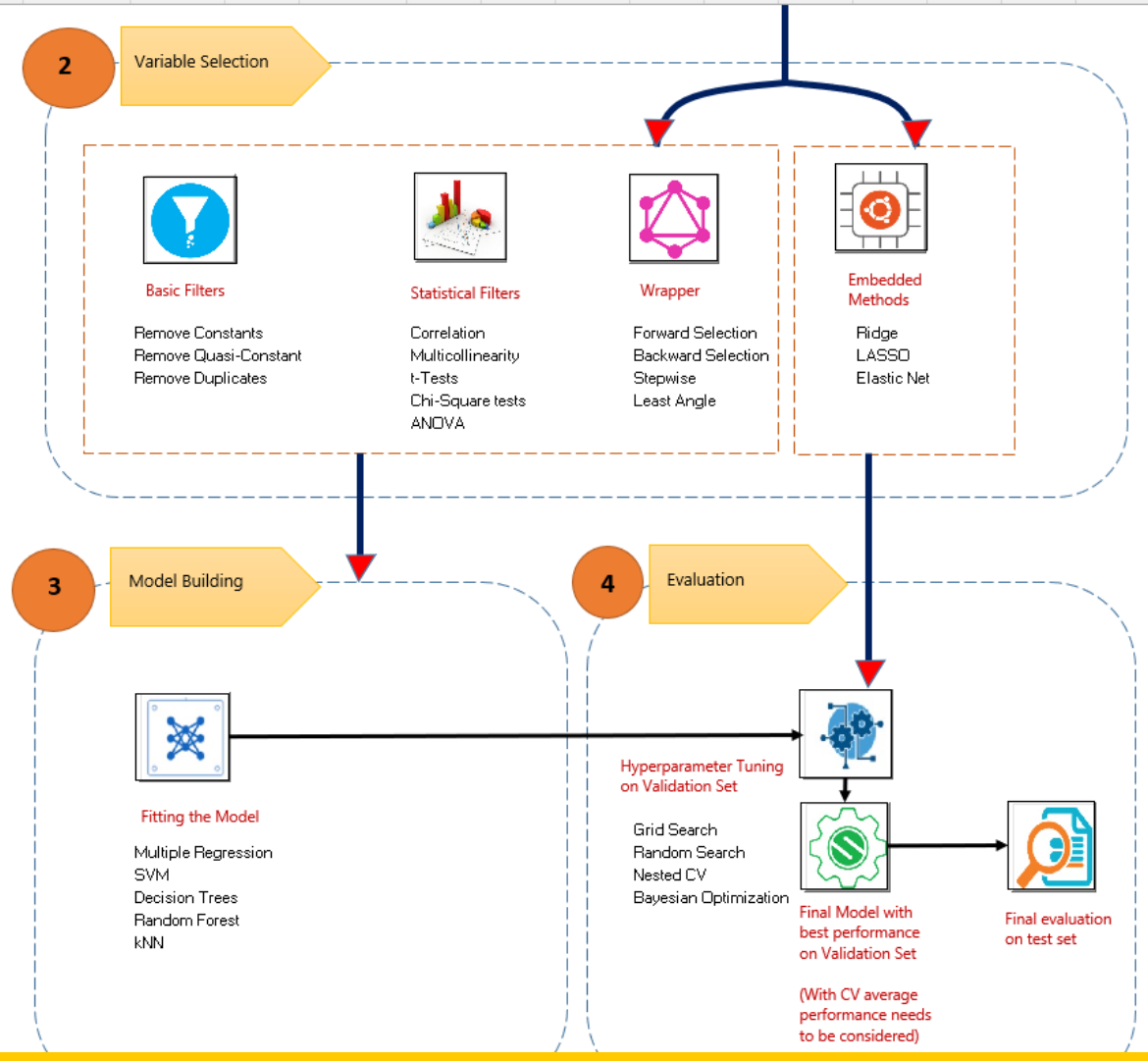
1.1 Regression Master Pipeline



1.2 Regression Master Pipeline



1.3 Exploratory Data Analysis

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Exploratory Data Analysis

1

Univariate Statistics

Variable Name	Measure	Missing Count	Missing Count %	Mean	Median	Mode	Min	Max	std dev	Cardinality
manufact	Nominal	0	0.000%	N/A	N/A	Dodge	N/A	N/A	N/A	30
model	Nominal	0	0.000%	N/A	N/A	Neon	N/A	N/A	N/A	156
sales	Scale	0	0.000%	52.99807643	29.45	N/A	0.11	540.561	68.029422	N/A
resale	Scale	36	22.930%	18.07297521	14.18	N/A	5.16	67.55	11.4533841	N/A
type	Ordinal	0	0.000%	N/A	N/A	Automobile	N/A	N/A	N/A	2
price	Scale	2	1.274%	27.39075484	22.799	N/A	9.235	85.5	14.3516532	N/A
engine_s	Scale	1	0.637%	3.060897436	3	N/A	1	8	1.04465297	N/A
horsepow	Scale	1	0.637%	185.9487179	177.5	N/A	55	450	56.7003209	N/A
wheelbas	Scale	1	0.637%	107.4871795	107	N/A	92.6	138.7	7.64130303	N/A
width	Scale	1	0.637%	71.15	70.55	N/A	62.6	79.9	3.45187186	N/A
length	Scale	1	0.637%	187.3435897	187.9	N/A	149.4	224.5	13.4317543	N/A
curb_wgt	Scale	2	1.274%	3.378025806	3.342	N/A	1.895	5.572	0.63050163	N/A
fuel_cap	Scale	1	0.637%	17.95192308	17.2	N/A	10.3	32	3.88792126	N/A
mpg	Scale	3	1.911%	23.84415584	24	N/A	15	45	4.28270556	N/A

Variable will be dropped

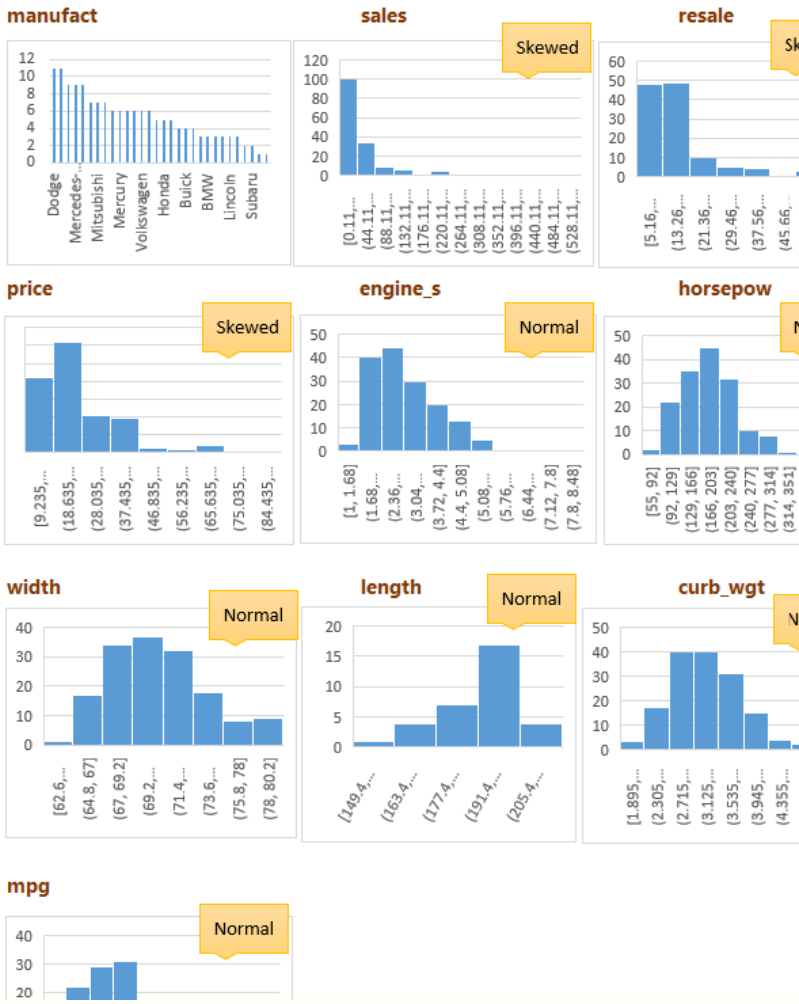
High *Cardinality*

3

Box Plot

2

Distribution



1.4 Regression Missing Value Imputation

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Regression Imputation

correlation	0.9522798
slope	0.7410197
intercept	-1.522177

*Regression is a multivariate imputation technique. In our case, it's obvious that **price** alone would explain **resale**. So we will use a simple regression in our case*

***Regression Imputation** - We can simply impute the fitted values of resale where they are missing. The drawback is it does not account for dispersion. (Not Recommended)*

***Regression Imputation with noise** - Normally we add random noise to the fitted value from error fitted distribution (normal) or empirical distribution (draw from observed residuals - bootstrapping). It may not work in this example, as errors appear heteroscedastic. We can use fGLS to estimate the conditional error means at various x = price*

$$Var(u\,|\boldsymbol{x}) = \sigma^2 \exp(\delta_0 + \delta_1 x_1 + \delta_2 x_1{}^2)$$

Dependent	***Regressor***			
resale	price	$\widehat{resale}$	residual	Missing
29.725	42	29.6006527	0.124347	
22.255	23.99	16.2548871	6.000113	
23.555	33.95	23.6354437	-0.08044	
39	62	44.4210476	-5.42105	
	26.99	18.4779463		0
28.675	33.4	23.2278829	5.447117	
12.475	21.975	14.7617323	-2.28673	
13.74	25.3	17.2256229	-3.48562	
13.36	27.885	19.141159	-5.78116	
22.525	39.895	28.0408061	-5.51581	
27.1	44.475	31.4346766	-4.33468	
25.725	39.665	27.8703716	-2.14537	
	46.225	32.7314611		0
10.31	18.89	12.4756863	-2.16569	
11.525	19.39	12.8461962	-1.3212	
36.225	45.705	32.3461309	3.878869	
9.125	13.96	8.822459	0.302541	
	18.89	12.4756863		0
13.725	22.245	14.9618076	-1.23681	
12.64	16.48	10.6898288	1.950171	
17.325	28.34	19.478323	-2.15332	
7.75	12.64	7.84431293	-0.09431	
12.545	19.045	12.5905444	-0.04554	

resale
29.6007
22.255
23.555
39
18.4779
28.675
12.475
13.74
13.36
22.525
27.1
25.725
32.7315
10.31
11.525
36.225
9.125
12.4757
13.725
12.64
17.325
7.75
12.545

-0.00313 0.29522 -4.339

g = log(residual^2)	price	price^2	$\hat{g}$	$\widehat{h}_i = \exp(\hat{g}_i)$
-4.169353653	42	1764	2.547229	12.77166717
3.583556586	23.99	575.5201	0.944838	2.572395498
-5.040394506	33.95	1152.603	2.081597	8.017259862
3.380578177	62	3844	1.950686	7.033510208
	26.99	728.4601	1.352495	3.867061505
3.390173001	33.4	1115.56	2.035	7.652250429
1.654247682	21.975	482.9006	0.639444	1.895426789
2.497293546	25.3	640.09	1.129767	3.094936666
3.509208354	27.885	777.5732	1.463217	4.319834939
3.415235626	39.895	1591.611	2.464584	11.75858498
2.933293999	44.475	1978.026	2.608973	13.58508801
1.526625547	39.665	1573.312	2.453875	11.63333764
	46.225	2136.751	2.629521	13.86712219
1.545474662	18.89	356.8321	0.122706	1.130552541
0.557075109	19.39	375.9721	0.210496	1.234290335
2.711087303	45.705	2088.947	2.625414	13.81029173
-2.391076917	13.96	194.8816	-0.82657	0.437548096
	18.89	356.8321	0.122706	1.130552541
0.425067086	22.245	494.84	0.681838	1.977508698
1.33583437	16.48	271.5904	-0.32236	0.724437981
1.534024421	28.34	803.1556	1.517586	4.561202728
-4.722273978	12.64	159.7696	-1.10652	0.330707551
-6.178134822	19.045	362.712	0.150088	1.161936984

1.5 KNN Missing Value Imputation

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kNN Imputation

kNN is a multivariate imputation technique. We will choose 'price' and 'horsepow' as the two features to perform inputation

corr	**0.9523**	**0.5079**	**0.74066077**	**-0.029**	**0.0967**	**0.0143**	**0.3545**	**0.3412**	**-0.433**
resale	price	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
29.725	42	3.5	210	114.6	71.4	196.6	3.85	18	22
22.255	23.99	1.8	150	102.6	68.2	178	2.998	16.4	27
23.555	33.95	2.8	200	108.7	76.1	192	3.561	18.5	22
39	62	4.2	310	113	74	198.2	3.902	23.7	21
	26.99	2.5	170	107.3	68.4	176	3.179	16.6	26
28.675	33.4	2.8	193	107.3	68.5	176	3.197	16.6	24
12.475	21.975	3.1	175	109	72.7	194.6	3.368	17.5	25
13.74	25.3	3.8	240	109	72.7	196.2	3.543	17.5	23
13.36	27.885	3.8	205	112.2	73.5	200	3.591	17.5	25
22.525	39.895	4.6	275	115.3	74.5	207.2	3.978	18.5	22
27.1	44.475	4.6	275	112.2	75	201	3.4514	18.5	22
25.725	39.665	4.6	275	108	75.5	200.6	3.843	19	22
	46.225	5.7	255	117.5	77	201.2	5.572	30	15
10.31	18.89	3.1	175	107.5	72.5	200.9	3.33	16.6	25
11.525	19.39	3.4	180	110.5	72.7	197.9	3.34	17	27
36.225	45.705	5.7	345	104.5	73.6	179.7	3.21	19.1	22
9.125	13.96	1.8	120	97.1	66.7	174.3	2.398	13.2	33
	18.89	3.4	180	110.5	73	200	3.389	17	27
13.725	22.245	2.7	200	113	74.4	209.1	3.452	17	26
12.64	16.48	2	132	108	71	186	2.911	16	27
17.325	28.34	3.5	253	113	74.4	207.7	3.564	17	23
7.75	12.64	2	132	105	74.4	174.4	2.567	12.5	29
12.545	19.045	2.5	163	103.7	69.1	190.2	2.879	15.9	24
10.185	20.23	2.5	168	108	71	186	3.058	16	24
15.51	21.315	3.9	175	109.6	78.8	192.6	4.245	32	15
	26.31	5.2	230	115.7	71.7	193.5	4.394	25	17

Available Cases

#	resale	engine_s	horsepow
1	29.725	3.5	210
2	22.255	1.8	150
3	23.555	2.8	200
4	39	4.2	310
5	28.675	2.8	193
6	12.475	3.1	175
7	13.74	3.8	240
8	13.36	3.8	205
9	22.525	4.6	275
10	27.1	4.6	275
11	25.725	4.6	275
12	10.31	3.1	175
13	11.525	3.4	180
14	36.225	5.7	345
15	9.125	1.8	120
16	13.725	2.7	200
17	12.64	2	132
18	17.325	3.5	253
19	7.75	2	132
20	12.545	2.5	163
21	10.185	2.5	168
22	15.51	3.9	175
23	12.025	2.4	150
24	7.425	2	110
25	12.76	3.8	190
26	10.055	3	155

Missing Cases

K = 3 .. K might be tunedin another held out set if needed

resale	engine_s	horsepow	*Neighbor1*	*Neighbor2*	*Neighbor3*
13.25833	2.5	170	44	44	21
20.26333	5.7	255	18	51	28
11.43	3.4	180	13	55	6
20.75833	5.2	230	49	28	28
8.608333	2	107	24	59	59
19.06833	3	240	7	28	51
52.74167	4	300	58	47	4
20.75833	4.7	230	49	28	28
23.81667	3	220	29	49	57
52.74167	5.4	300	58	47	4
16.77833	2.3	185	45	13	25
32.65	3.2	215	57	36	1
52.74167	5	302	47	58	4
32.65	3.2	215	57	36	1
13.25833	3.3	170	44	44	21
13.25833	3.3	170	44	44	21
33.56667	3.5	215	57	1	1
14.49167	2.4	150	23	23	64
16.985	3.5	253	18	51	7
13.25833	2.3	170	44	44	21
16.77833	2	185	45	13	25
13.03167	2.2	137	30	62	17
14.55667	2.5	165	20	21	39
13.25833	2.4	168	21	44	44
17.48	2.3	236	7	28	49
17.385	2.9	201	3	16	42

#	1	2	3
resale	29.725	22.255	23.555
engine_s	3.5	1.8	2.8
horsepow	210	150	200
distance			
	40.0125	20.0122	30.0015
	45.0537	105.072	55.0764
	30.0002	30.0426	20.009
	20.0721	80.0722	30.0958
	103.011	43.0005	93.0034
	30.0042	90.008	40.0005
	90.0014	150.016	100.007
	20.036	80.0525	30.0601
	10.0125	70.0103	20.001
	90.0201	150.043	100.034
	25.0288	35.0036	15.0083
	5.00899	65.0151	15.0053
	92.0122	152.034	102.024
	5.00899	65.0151	15.0053
	40.0005	20.0562	30.0042
	40.0005	20.0562	30.0042
	5	65.0222	15.0163
	60.0101	0.6	50.0016
	43	103.014	53.0046
	40.018	20.0062	30.0042
	25.045	35.0006	15.0213
	73.0116	13.0062	63.0029
	45.0111	15.0163	35.0013
	42.0144	18.01	32.0025
	26.0277	86.0015	36.0035
	9.01998	51.0119	1.00499

1.6 Cardinality Reduction

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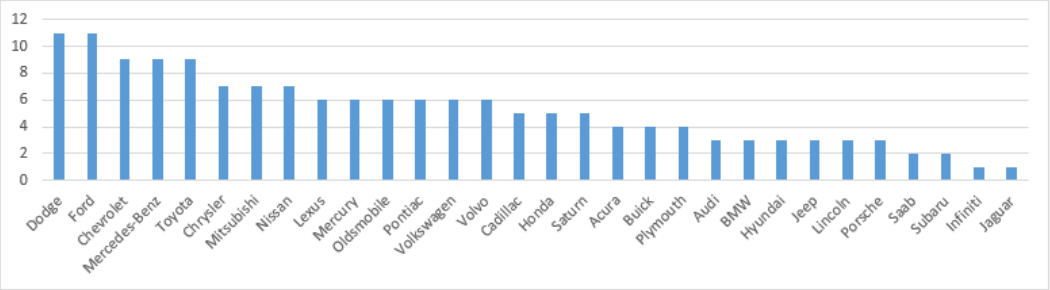
Reduce Cardinality

manufact	model
Acura	Integra
Acura	TL
Acura	CL
Acura	RL
Audi	A4
Audi	A6
Audi	A8
BMW	323i
BMW	328i
BMW	528i
Buick	Century
Buick	Regal
Buick	Park Avenue
Buick	LeSabre
Cadillac	DeVille
Cadillac	Seville
Cadillac	Eldorado
Cadillac	Catera
Cadillac	Escalade
Chevrolet	Cavalier
Chevrolet	Malibu
Chevrolet	Lumina
Chevrolet	Monte Carlo
Chevrolet	Camaro
Chevrolet	Corvette
Chevrolet	Prizm

Variable will be dropped

manufact



manufact	Frequency	%	Cum %
Dodge	11	7.01%	7.01%
Ford	11	7.01%	14.01%
Chevrolet	9	5.73%	19.75%
Mercedes-Benz	9	5.73%	25.48%
Toyota	9	5.73%	31.21%
Chrysler	7	4.46%	35.67%
Mitsubishi	7	4.46%	40.13%
Nissan	7	4.46%	44.59%
Lexus	6	3.82%	48.41%
Mercury	6	3.82%	52.23%
Oldsmobile	6	3.82%	56.05%
Pontiac	6	3.82%	59.87%
Volkswagen	6	3.82%	63.69%
Volvo	6	3.82%	67.52%

Possible Techniques to reduce cardinality are

- ***Target Mean encoding*** (Replace categories by the Target Mean in that category. Must be done on the training data set or CV scheme to prevent data leakage)

- ***Top Categories*** (Keep categories upto a threshold cumulative frequency and bucket the remaining rare labels under a category 'Others'. Works well when a few categories constitue majority of the cases.

- ***Frequency Encoding*** (replace categories by frequency. Works when the frequencies are different. Statistical interpretation of regression coefficient is difficult)

- ***Integer Encoding*** (Assigns integers 1,2,.. for each category. Statistical interpretation of regression coefficient is difficult)

In our example, we will first bucket the last 4 ccategories into a separate category 'Others'. Then choose ***Target mean encoding*

1.7 Categorical Encoding

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Categorical Encoding

Saab
Subaru
Infiniti
Jaguar

Others

In classification task, we do something similar called as WOE

type	One hot encoding
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Truck	1
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Automobile	0
Truck	1

manufact	Top Majority Encoding	Target sales	Target Mean Encoding
Acura	Acura	8.588	8.588
Audi	Audi	20.397	13.519
Audi	Audi	18.78	13.519
Audi	Audi	1.38	13.519
BMW	BMW	19.747	14.489
BMW	BMW	9.231	14.489
Buick	Buick	91.561	71.38933333
Buick	Buick	39.35	71.38933333
Buick	Buick	83.257	71.38933333
Cadillac	Cadillac	63.729	25.24825
Cadillac	Cadillac	15.943	25.24825
Cadillac	Cadillac	6.536	25.24825
Cadillac	Cadillac	14.785	25.24825
Chevrolet	Chevrolet	24.629	45.0926
Chevrolet	Chevrolet	42.593	45.0926
Chevrolet	Chevrolet	17.947	45.0926
Chevrolet	Chevrolet	32.299	45.0926
Chevrolet	Chevrolet	107.995	45.0926
Chrysler	Chrysler	31.148	25.63866667
Chrysler	Chrysler	32.306	25.63866667
Chrysler	Chrysler	13.462	25.63866667
Dodge	Dodge	76.034	75.29883333
Dodge	Dodge	4.734	75.29883333
Dodge	Dodge	71.186	75.29883333
Dodge	Dodge	16.767	75.29883333

1.8 Box Cox Transformation

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Variable Transformation

skewed. We can use Box Cox to convert it to normal. Although X need not be normal for regression, but once we convert it to m=normal, we can further convert it into Zscore. So beta coefficients can be compared for impact analysis.

Box Cox Transformation. λ is estimated using Maximum Likelihood.

#	manufact	sales	resale	type	price	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
4	8.588	8.588	29.6007	0	42	3.5	210	114.6	71.4	196.6	3.85	18	22
5	13.519	20.397	22.255	0	23.99	1.8	150	102.6	68.2	178	2.998	16.4	27
6	13.519	18.78	23.555	0	33.95	2.8	200	108.7	76.1	192	3.561	18.5	22
7	13.519	1.38	39	0	62	4.2	310	113	74	198.2	3.902	23.7	21
8	14.489	19.747	19.9473	0	26.99	2.5	170	107.3	68.4	176	3.179	16.6	26
9	14.489	9.231	28.675	0	33.4	2.8	193	107.3	68.5	176	3.197	16.6	24
11	71.3893	91.561	12.475	0	21.975	3.1	175	109	72.7	194.6	3.368	17.5	25
12	71.3893	39.35	13.74	0	25.3	3.8	240	109	72.7	196.2	3.543	17.5	23
14	71.3893	83.257	13.36	0	27.885	3.8	205	112.2	73.5	200	3.591	17.5	25
15	25.2483	63.729	22.525	0	39.895	4.6	275	115.3	74.5	207.2	3.978	18.5	22
16	25.2483	15.943	27.1	0	44.475	4.6	275	112.2	75	201	3.4514	18.5	22
17	25.2483	6.536	25.725	0	39.665	4.6	275	108	75.5	200.6	3.843	19	22
19	25.2483	14.785	33.4837	1	46.225	5.7	255	117.5	77	201.2	5.572	30	15
22	45.0926	24.629	10.31	0	18.89	3.1	175	107.5	72.5	200.9	3.33	16.6	25
23	45.0926	42.593	11.525	0	19.39	3.4	180	110.5	72.7	197.9	3.34	17	27
25	45.0926	17.947	36.225	0	45.705	5.7	345	104.5	73.6	179.7	3.21	19.1	22
26	45.0926	32.299	9.125	0	13.96	1.8	120	97.1	66.7	174.3	2.398	13.2	33
28	45.0926	107.995	12.369	0	18.89	3.4	180	110.5	73	200	3.389	17	27
31	25.6387	31.148	13.725	0	22.245	2.7	200	113	74.4	209.1	3.452	17	26
32	25.6387	32.306	12.64	0	16.48	2	132	108	71	186	2.911	16	27
33	25.6387	13.462	17.325	0	28.34	3.5	253	113	74.4	207.7	3.564	17	23
36	75.2988	76.034	7.75	0	12.64	2	132	105	74.4	174.4	2.567	12.5	29
37	75.2988	4.734	12.545	0	19.045	2.5	163	103.7	69.1	190.2	2.879	15.9	24

	mean	
	3.206309751	Sum LL
λ	SD	-158.964932
1E-06	1.395738065	

sales	sales_transformed	LL
8.588	2.150368192	-1.53854363
20.397	3.015392378	-1.26171709
18.78	2.932796774	-1.27156263
1.38	0.322083551	-3.38747454
19.747	2.98300603	-1.26516024
9.231	2.222569855	-1.50074538
91.561	4.517015618	-1.69329493
39.35	3.672502718	-1.30814382
83.257	4.421941986	-1.63164768
63.729	4.154648348	-1.48319017
15.943	2.769023695	-1.30144063
6.536	1.87732712	-1.7056776
14.785	2.693616781	-1.31982663
24.629	3.203929743	-1.25236334
42.593	3.751696958	-1.32870538
17.947	2.887427139	-1.27846089
32.299	3.475042308	-1.27089732
107.995	4.682095891	-1.81135829
31.148	3.43875595	-1.26622967
32.306	3.47525901	-1.27092723
13.462	2.599874281	-1.34675305
76.034	4.331189988	-1.57713101
4.734	1.55477172	-1.95242842

	mean	
	2.972218974	Sum LL
λ	SD	-123.885
0.035529	0.611572297	

resale	resale_transformed	LL
29.6007	3.600112027	-1.29217
22.255	3.280024122	-1.25376
23.555	3.343475325	-1.25719
39	3.912678167	-1.38043
19.9473	3.158032051	-1.25296
28.675	3.564297803	-1.28525
12.475	2.640329574	-1.33458
13.74	2.746156006	-1.30671
13.36	2.715388846	-1.31422
22.525	3.293491408	-1.25431
27.1	3.500715882	-1.27461
25.725	3.442223281	-1.26665
33.4837	3.73944373	-1.32531
10.31	2.432540838	-1.40603
11.525	2.553813041	-1.36164
36.225	3.828715397	-1.35179
9.125	2.3001796	-1.4631
12.369	2.630993053	-1.33731
13.725	2.744957158	-1.30699
12.64	2.654705277	-1.33046
17.325	3.00166596	-1.26311
7.75	2.124018892	-1.553
12.545	2.646450621	-1.33281

	m
	3.256
λ	
1E-06	0.468

price	price_tra
42	3.73
23.99	3.17
33.95	3.52
62	4.12
26.99	3.29
33.4	3.50
21.975	3.08
25.3	3.23
27.885	3.32
39.895	3.68
44.475	3.79
39.665	3.68
46.225	3.83
18.89	2.93
19.39	2.96
45.705	3.82
13.96	2.63
18.89	2.93
22.245	3.10
16.48	2.80
28.34	3.34
12.64	2.53
19.045	2.94

1.9 Outlier Engineering

PEAKS2TAILS
http://peaks2tails.com

Univariate Imputation on Training Set

Obs #34 is deleted

Median Imputation because of skewed dist. Training Set has no missing data, but this median to be imputed on validation / test set.

Mean Imputation

Mean Imputation

#	manufact	model	sales	resale	type	price	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
						25.3						3.4514		23.356
4	Acura	RL	8.588	29.725	Automobile	42	3.5	210	114.6	71.4	196.6	3.85	18	22
5	Audi	A4	20.397	22.255	Automobile	23.99	1.8	150	102.6	68.2	178	2.998	16.4	27
6	Audi	A6	18.78	23.555	Automobile	33.95	2.8	200	108.7	76.1	192	3.561	18.5	22
7	Audi	A8	1.38	39	Automobile	62	4.2	310	113	74	198.2	3.902	23.7	21
8	BMW	323i	19.747		Automobile	26.99	2.5	170	107.3	68.4	176	3.179	16.6	26
9	BMW	328i	9.231	28.675	Automobile	33.4	2.8	193	107.3	68.5	176	3.197	16.6	24
11	Buick	Century	91.561	12.475	Automobile	21.975	3.1	175	109	72.7	194.6	3.368	17.5	25
12	Buick	Regal	39.35	13.74	Automobile	25.3	3.8	240	109	72.7	196.2	3.543	17.5	23
14	Buick	LeSabre	83.257	13.36	Automobile	27.885	3.8	205	112.2	73.5	200	3.591	17.5	25
15	Cadillac	DeVille	63.729	22.525	Automobile	39.895	4.6	275	115.3	74.5	207.2	3.978	18.5	22
16	Cadillac	Seville	15.943	27.1	Automobile	44.475	4.6	275	112.2	75	201	3.4514	18.5	22
17	Cadillac	Eldorado	6.536	25.725	Automobile	39.665	4.6	275	108	75.5	200.6	3.843	19	22
19	Cadillac	Escalade	14.785		Truck	46.225	5.7	255	117.5	77	201.2	5.572	30	15
22	Chevrolet	Lumina	24.629	10.31	Automobile	18.89	3.1	175	107.5	72.5	200.9	3.33	16.6	25
23	Chevrolet	Monte Carlo	42.593	11.525	Automobile	19.39	3.4	180	110.5	72.7	197.9	3.34	17	27
25	Chevrolet	Corvette	17.947	36.225	Automobile	45.705	5.7	345	104.5	73.6	179.7	3.21	19.1	22
26	Chevrolet	Prizm	32.299	9.125	Automobile	13.96	1.8	120	97.1	66.7	174.3	2.398	13.2	33
28	Chevrolet	Impala	107.995		Automobile	18.89	3.4	180	110.5	73	200	3.389	17	27
31	Chrysler	Concorde	31.148	13.725	Automobile	22.245	2.7	200	113	74.4	209.1	3.452	17	26
32	Chrysler	Cirrus	32.306	12.64	Automobile	16.48	2	132	108	71	186	2.911	16	27
33	Chrysler	LHS	13.462	17.325	Automobile	28.34	3.5	253	113	74.4	207.7	3.564	17	23
36	Dodge	Neon	76.034	7.75	Automobile	12.64	2	132	105	74.4	174.4	2.567	12.5	29
37	Dodge	Avenger	4.734	12.545	Automobile	19.045	2.5	163	103.7	69.1	190.2	2.879	15.9	24
38	Dodge	Stratus	71.186	10.185	Automobile	20.23	2.5	168	108	71	186	3.058	16	24
42	Dodge	Ram Wagon	16.767	15.51	Truck	21.315	3.9	175	109.6	78.8	192.6	4.245	32	15

Listwise deletion was for only Obs #34 as the variables with missing data in this obs are assumed MCAR. Also 1 case out of 91 cases is < 5%

*Other imputation methods which are **not suitable for linear models** include*

***Arbitrary Imputation** (replace missing values arbitrary values e.g. 999, -1)*

***End Distribution Imputation** (replace missing values with an extreme value)*

*Popular univariate approach if a variable is **not MCAR** in linear models is*

***Missing Indicator** (create a new binary variable M against the feature X, if Xi is not 0, then Mi=0 and if Xi is missing, then Mi= 1)*

There are several sophisticated Multivariate imputation techniques.

*We will explore **kNN** and **Regression** methods for the feature **'resale'***

1.10 Scaling

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

Training After Scaling

	manufact	sales_t	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
Min	5.131	0.1087	1.49866		2.27203	1.5	92	92.7	65.7	152	2.24	11.9	15
Max	203.824	6.29263	4.43602		4.44853	5.7	327.5	123.9	79.9	224.25	5.11125	24.825	33
Mean	52.2016	3.23347	2.98057		3.25634	3.17582	192.72	107.743	71.5615	188.076	3.44006	18.0308	23.3556
SD	52.2344	1.31047	0.58441		0.46848	1.00192	55.2124	6.45855	3.47482	12.7205	0.61174	3.30656	4.06424
Q1	15.0563	2.57681	2.60017		2.93864	2.4	150	104.4	68.75	179	3.015	16.2	21
Q3	71.3893	4.22221	3.33451		3.5854	3.8	221	112.2	73.6	197.1	3.8535	19.65	26
IQR	56.333	1.6454	0.73434		0.64676	1.4	71	7.8	4.85	18.1	0.8385	3.45	5
Median	38.379	3.22311	2.94995		3.23081	3	185	107.5	71.5	188.5	3.389	17.9	23

#	manufact	sales_t	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
4	8.588	2.15037	3.60011	0	3.73768	3.5	210	114.6	71.4	196.6	3.85	18	22
5	13.519	3.01539	3.28002	0	3.17764	1.8	150	102.6	68.2	178	2.998	16.4	27
6	13.519	2.9328	3.34348	0	3.5249	2.8	200	108.7	76.1	192	3.561	18.5	22
7	13.519	0.32208	3.91268	0	4.12714	4.2	310	113	74	198.2	3.902	23.7	21
8	14.489	2.98301	3.15803	0	3.29547	2.5	170	107.3	68.4	176	3.179	16.6	26
9	14.489	2.22257	3.5643	0	3.50856	2.8	193	107.3	68.5	176	3.197	16.6	24
11	71.3893	4.51702	2.64033	0	3.08991	3.1	175	109	72.7	194.6	3.368	17.5	25
12	71.3893	3.6725	2.74616	0	3.23081	3.8	240	109	72.7	196.2	3.543	17.5	23
14	71.3893	4.42194	2.71539	0	3.32809	3.8	205	112.2	73.5	200	3.591	17.5	25
15	25.2483	4.15465	3.29349	0	3.68626	4.6	275	115.3	74.5	207.2	3.978	18.5	22
16	25.2483	2.76902	3.50072	0	3.79493	4.6	275	112.2	75	201	3.4514	18.5	22
17	25.2483	1.87733	3.44222	0	3.68048	4.6	275	108	75.5	200.6	3.843	19	22
19	25.2483	2.69362	3.73944	1	3.83353	5.7	255	117.5	77	201.2	5.11125	24.825	15
22	45.0926	3.20393	2.43254	0	2.93864	3.1	175	107.5	72.5	200.9	3.33	16.6	25
23	45.0926	3.7517	2.55381	0	2.96476	3.4	180	110.5	72.7	197.9	3.34	17	27
25	45.0926	2.88743	3.82872	0	3.82222	5.7	327.5	104.5	73.6	179.7	3.21	19.1	22
26	45.0926	3.47504	2.30018	0	2.6362	1.8	120	97.1	66.7	174.3	2.398	13.2	33
28	45.0926	4.6821	2.63099	0	2.93864	3.4	180	110.5	73	200	3.389	17	27
31	25.6387	3.43876	2.74496	0	3.10212	2.7	200	113	74.4	209.1	3.452	17	26

Robust Estimate

z - score

#	manufact	sales_t	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	m
4	-0.5288	-0.8265	1.06013	0	1.02746	0.32355	0.31298	1.06171	-0.0465	0.67007	0.67013	-0.0093	-0
5	-0.4413	-0.1664	0.51241	0	-0.168	-1.3732	-0.7737	-0.7963	-0.9674	-0.7921	-0.7226	-0.4932	0.
6	-0.4413	-0.2294	0.62099	0	0.57326	-0.3751	0.13186	0.1482	1.3061	0.30845	0.1977	0.14191	-0
7	-0.4413	-2.2216	1.59497	0	1.8588	1.02221	2.12416	0.81398	0.70175	0.79585	0.75513	1.71454	-0
8	-0.4241	-0.1911	0.30367	0	0.08353	-0.6745	-0.4115	-0.0686	-0.9098	-0.9494	-0.4267	-0.4327	0.
9	-0.4241	-0.7714	0.99884	0	0.53839	-0.3751	0.00508	-0.0686	-0.8811	-0.9494	-0.3973	-0.4327	0.
11	0.58599	0.97945	-0.5822	0	-0.3553	-0.0757	-0.3209	0.19465	0.32763	0.51284	-0.1178	-0.1605	0.
12	0.58599	0.33502	-0.4011	0	-0.0545	0.62298	0.85633	0.19465	0.32763	0.63862	0.16828	-0.1605	-0
14	0.58599	0.90691	-0.4538	0	0.15317	0.62298	0.22242	0.69011	0.55786	0.93735	0.24675	-0.1605	0.
15	-0.2331	0.70294	0.53546	0	0.9177	1.42144	1.49025	1.1701	0.84564	1.50337	0.87937	0.14191	-0
16	-0.2331	-0.3544	0.89005	0	1.14968	1.42144	1.49025	0.69011	0.98954	1.01597	0.01854	0.14191	-0
17	-0.2331	-1.0349	0.78996	0	0.90536	1.42144	1.49025	0.03981	1.13343	0.98452	0.65869	0.29312	-0
19	-0.2331	-0.412	1.29854	1	1.23206	2.51933	1.12801	1.51073	1.5651	1.03169	2.73188	2.05477	-2
22	0.11918	-0.0225	-0.9378	0	-0.6782	-0.0757	-0.3209	-0.0376	0.27007	1.0081	-0.1799	-0.4327	0.
23	0.11918	0.39545	-0.7302	0	-0.6224	0.22375	-0.2304	0.4269	0.32763	0.77227	-0.1636	-0.3117	0.
25	0.11918	-0.2641	1.4513	0	1.20791	2.51933	2.44112	-0.5021	0.58664	-0.6585	-0.3761	0.32337	-0
26	0.11918	0.18434	-1.1642	0	-1.3237	-1.3732	-1.3171	-1.6479	-1.3991	-1.083	-1.7034	-1.461	2.
28	0.11918	1.10542	-0.5982	0	-0.6782	0.22375	-0.2304	0.4269	0.41397	0.93735	-0.0835	-0.3117	0.
31	-0.2262	0.15665	-0.4032	0	-0.3292	-0.4749	0.13186	0.81398	0.81687	1.65273	0.01952	-0.3117	0.

1.11 Variable Selection

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

Correlation

Correlation
Matrix & VIF

	manufact	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg	VIF
manufact	1	-0.381324	0.24629659	-0.384495	-0.019135	-0.2849273	0.1195606	0.11652188	0.14237923	-0.02562885	0.04636487	0.042116207	1.54509076
resale_t	-0.381324	1	0.06661696	0.8726467	0.576469	0.7517863	0.2213264	0.27979151	0.18731982	0.556089618	0.56905168	-0.56694667	4.90812452
type	0.2462966	0.066617	1	0.0187673	0.2572904	-0.0114791	0.2922478	0.23181466	0.02625178	0.543149941	0.55938439	-0.60176986	4.39303407
price_t	-0.3844946	0.8726467	0.01876728	1	0.6726757	0.89490067	0.2992064	0.37426632	0.24621695	0.629332108	0.56803817	-0.61049777	**14.413581**
engine_s	-0.0191354	0.576469	0.25729044	0.6726757	1	0.82090646	0.5626728	0.62752243	0.56335592	0.795093077	0.70235143	-0.73102797	6.77690905
horsepow	-0.2849273	0.7517863	-0.01147905	0.8949007	0.8209065	1	0.415055	0.5044314	0.43064755	0.640497355	0.56120852	-0.60936859	**12.649885**
wheelbas	0.1195606	0.2213264	0.29224782	0.2992064	0.5626728	0.41505499	1	0.70047191	0.8050845	0.681351584	0.60890674	-0.46563484	4.07642412
width	0.1165219	0.2797915	0.23181466	0.3742663	0.6275224	0.5044314	0.7004719	1	0.67295045	0.680095737	0.61155516	-0.53889124	2.70083931
length	0.1423792	0.1873198	0.02625178	0.246217	0.5633559	0.43064755	0.8050845	0.67295045	1	0.587233955	0.5209756	-0.33022456	5.5170498
curb_wgt	-0.0256288	0.5560896	0.54314994	0.6293321	0.7950931	0.64049736	0.6813516	0.68009574	0.58723396	1	0.86848427	-0.8334419	**11.034125**
fuel_cap	0.0463649	0.5690517	0.55938439	0.5680382	0.7023514	0.56120852	0.6089067	0.61155516	0.5209756	0.86848427	1	-0.83713504	6.20084461
mpg	0.0421162	-0.5669467	-0.60176986	-0.610498	-0.731028	-0.6093686	-0.465635	-0.5388912	-0.3302246	-0.8334419	-0.837135	1	5.91349111

* Price and horsepow are highly correlated with all features
But we will keep them for now as they are important variables (we need to ensure less multicollinearity before final regression

1.12 Backward Selection

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

1. Fit the full model
2. Remove the variable with the largest p-value (if > 0.05)
3. Fit the model with remaining variable
4. Go to step 2.continue until desired number of features is obtained

**In our example, we will stick with 3 features*

Final Features

manufact
price_t
wheelbas

Backward Selection

#	sales_t	manufact	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
4	-0.8265	-0.5288	1.06013	0	1.02746	0.32355	0.31298	1.06171	-0.0465	0.67007	0.67013	-0.0093	-0.3335
5	-0.1664	-0.4413	0.51241	0	-0.168	-1.3732	-0.7737	-0.7963	-0.9674	-0.7921	-0.7226	-0.4932	0.89671
6	-0.2294	-0.4413	0.62099	0	0.57326	-0.3751	0.13186	0.1482	1.3061	0.30845	0.1977	0.14191	-0.3335
7	-2.2216	-0.4413	1.59497	0	1.8588	1.02221	2.12416	0.81398	0.70175	0.79585	0.75513	1.71454	-0.5796
8	-0.1911	-0.4241	0.30367	0	0.08353	-0.6745	-0.4115	-0.0686	-0.9098	-0.9494	-0.4267	-0.4327	0.65066
9	-0.7714	-0.4241	0.99884	0	0.53839	-0.3751	0.00508	-0.0686	-0.8811	-0.9494	-0.3973	-0.4327	0.15856
11	0.97945	0.58599	-0.5822	0	-0.3553	-0.0757	-0.3209	0.19465	0.32763	0.51284	-0.1178	-0.1605	0.40461
12	0.33502	0.58599	-0.4011	0	-0.0545	0.62298	0.85633	0.19465	0.32763	0.63862	0.16828	-0.1605	-0.0875
14	0.90691	0.58599	-0.4538	0	0.15317	0.62298	0.22242	0.69011	0.55786	0.93735	0.24675	-0.1605	0.40461
15	0.70294	-0.2331	0.53546	0	0.9177	1.42144	1.49025	1.1701	0.84564	1.50337	0.87937	0.14191	-0.3335
16	-0.3544	-0.2331	0.89005	0	1.14968	1.42144	1.49025	0.69011	0.98954	1.01597	0.01854	0.14191	-0.3335
17	-1.0349	-0.2331	0.78996	0	0.90536	1.42144	1.49025	0.03981	1.13343	0.98452	0.65869	0.29312	-0.3335
19	-0.412	-0.2331	1.29854	1	1.23206	2.51933	1.12801	1.51073	1.5651	1.03169	2.73188	2.05477	-2.0559
22	-0.0225	0.11918	-0.9378	0	-0.6782	-0.0757	-0.3209	-0.0376	0.27007	1.0081	-0.1799	-0.4327	0.40461
23	0.39545	0.11918	-0.7302	0	-0.6224	0.22375	-0.2304	0.4269	0.32763	0.77227	-0.1636	-0.3117	0.89671
25	-0.2641	0.11918	1.4513	0	1.20791	2.51933	2.44112	-0.5021	0.58664	-0.6585	-0.3761	0.32337	-0.3335
26	0.18434	0.11918	-1.1642	0	-1.3237	-1.3732	-1.3171	-1.6479	-1.3991	-1.083	-1.7034	-1.461	2.373
28	1.10542	0.11918	-0.5982	0	-0.6782	0.22375	-0.2304	0.4269	0.41397	0.93735	-0.0835	-0.3117	0.89671
31	0.15665	-0.2262	-0.4032	0	-0.3292	-0.4749	0.13186	0.81398	0.81687	1.65273	0.01952	-0.3117	0.65066
32	0.1845	-0.2262	-0.5576	0	-0.9695	-1.1736	-1.0997	0.03981	-0.1616	-0.1632	-0.8648	-0.6142	0.89671
33	-0.4835	-0.2262	0.0361	0	0.18772	0.32355	1.09179	0.81398	0.81687	1.54267	0.20261	-0.3117	-0.0875
36	0.83765	0.65539	-1.4657	0	-1.5358	-1.1736	-1.0997	-0.4247	0.81687	-1.0751	-1.4272	-1.6727	1.38881
37	-1.281	0.65539	-0.5717	0	-0.6607	-0.6745	-0.5383	-0.626	-0.7084	0.16694	-0.9172	-0.6444	0.15856
38	0.78738	0.65539	-0.9604	0	-0.5319	-0.6745	-0.4477	0.03981	-0.1616	-0.1632	-0.6245	-0.6142	0.15856
42	-0.316	0.65539	-0.173	1	-0.4203	0.72278	-0.3209	0.28755	2.08312	0.35562	1.31583	2.05477	-2.0559

FULL MODEL * drop horsepow

Regression Statistics	
Multiple R	0.8148886
R Square	0.6640433
Adjusted R Square	0.6123577
Standard Error	0.6226093
Observations	91

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	12	59.7639	4.980325	12.84773	5.01E-14
Residual	78	30.2361	0.387642		
Total	90	90			

	Coefficients	*andard Err*	*t Stat*	*P-value*
Intercept	-0.151854	0.103396	-1.46867	0.145945
manufact	0.4451964	0.087979	5.060274	2.72E-06
resale_t	0.1824723	0.145396	1.255003	0.213225
type	0.1615815	0.310438	0.520494	0.604193
price_t	-0.725121	0.249161	-2.91025	0.004705
engine_s	0.043319	0.170848	0.253553	0.800508
horsepow	0.0362187	0.23342	0.155166	**0.877092**
wheelbas	0.2122913	0.132506	1.602131	0.113169
width	-0.100808	0.107856	-0.93465	0.352853
length	0.0906958	0.154152	0.588355	0.557994

SUMMARY OUTPUT * drop engine_s

Regression Statistics	
Multiple R	0.814825
R Square	0.66394
Adjusted R S	0.617146
Standard Err	0.618752
Observation	91

ANOVA

	df	*SS*	*MS*	*F*
Regression	11	59.75457	5.432233	14.1888
Residual	79	30.24543	0.382854	
Total	90	90		

	Coefficients	*andard Err*	*t Stat*	*P-value*
Intercept	-0.15185	0.102756	-1.47774	0.143453
manufact	0.443347	0.086627	5.117873	2.12E-06
resale_t	0.180191	0.143754	1.253465	0.213734
type	0.163271	0.308325	0.529542	0.597915
price_t	-0.69803	0.176665	-3.95116	0.000168
engine_s	0.060204	0.130896	0.459933	**0.646828**
wheelbas	0.211616	0.131613	1.607859	0.111856
width	-0.09845	0.106118	-0.92773	0.356371
length	0.096522	0.148583	0.649614	0.517826
curb_wgt	0.336808	0.206514	1.630918	0.106888

1.13 Forward Selection

PEAKS2TAILS
http://peaks2tails.com

1. Select the feature most correlated with Y
2. Perform OLS obtain resdual,
3. find the next feature most correlated with residual
4. Go to step 2.continue until desired number of features is obtained or a desired performance measure (R^2) attains minimum threshold

**In our example, we will stick with 3 features*

Final Features

manufact
price_t
wheelbas

Forward Selection

	Corr	0.6463	-0.4814	0.24883	-0.5609	-0.1172	-0.4244	0.23459	0.0739	0.2359	-0.0311	-0.0384	0.15513
#	sales_t	manufact	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg
4	-0.8265	-0.5288	1.06013	0	1.02746	0.32355	0.31298	1.06171	-0.0465	0.67007	0.67013	-0.0093	-0.3335
5	-0.1664	-0.4413	0.51241	0	-0.168	-1.3732	-0.7737	-0.7963	-0.9674	-0.7921	-0.7226	-0.4932	0.89671
6	-0.2294	-0.4413	0.62099	0	0.57326	-0.3751	0.13186	0.1482	1.3061	0.30845	0.1977	0.14191	-0.3335
7	-2.2216	-0.4413	1.59497	0	1.8588	1.02221	2.12416	0.81398	0.70175	0.79585	0.75513	1.71454	-0.5796
8	-0.1911	-0.4241	0.30367	0	0.08353	-0.6745	-0.4115	-0.0686	-0.9098	-0.9494	-0.4267	-0.4327	0.65066
9	-0.7714	-0.4241	0.99884	0	0.53839	-0.3751	0.00508	-0.0686	-0.8811	-0.9494	-0.3973	-0.4327	0.15856
11	0.97945	0.58599	-0.5822	0	-0.3553	-0.0757	-0.3209	0.19465	0.32763	0.51284	-0.1178	-0.1605	0.40461
12	0.33502	0.58599	-0.4011	0	-0.0545	0.62298	0.85633	0.19465	0.32763	0.63862	0.16828	-0.1605	-0.0875
14	0.90691	0.58599	-0.4538	0	0.15317	0.62298	0.22242	0.69011	0.55786	0.93735	0.24675	-0.1605	0.40461
15	0.70294	-0.2331	0.53546	0	0.9177	1.42144	1.49025	1.1701	0.84564	1.50337	0.87937	0.14191	-0.3335
16	-0.3544	-0.2331	0.89005	0	1.14968	1.42144	1.49025	0.69011	0.98954	1.01597	0.01854	0.14191	-0.3335
17	-1.0349	-0.2331	0.78996	0	0.90536	1.42144	1.49025	0.03981	1.13343	0.98452	0.65869	0.29312	-0.3335
19	-0.412	-0.2331	1.29854	1	1.23206	2.51933	1.12801	1.51073	1.5651	1.03169	2.73188	2.05477	-2.0559
22	-0.0225	0.11918	-0.9378	0	-0.6782	-0.0757	-0.3209	-0.0376	0.27007	1.0081	-0.1799	-0.4327	0.40461
23	0.39545	0.11918	-0.7302	0	-0.6224	0.22375	-0.2304	0.4269	0.32763	0.77227	-0.1636	-0.3117	0.89671
25	-0.2641	0.11918	1.4513	0	1.20791	2.51933	2.44112	-0.5021	0.58664	-0.6585	-0.3761	0.32337	-0.3335
26	0.18434	0.11918	-1.1642	0	-1.3237	-1.3732	-1.3171	-1.6479	-1.3991	-1.083	-1.7034	-1.461	2.373
28	1.10542	0.11918	-0.5982	0	-0.6782	0.22375	-0.2304	0.4269	0.41397	0.93735	-0.0835	-0.3117	0.89671
31	0.15665	-0.2262	-0.4032	0	-0.3292	-0.4749	0.13186	0.81398	0.81687	1.65273	0.01952	-0.3117	0.65066
32	0.1845	-0.2262	-0.5576	0	-0.9695	-1.1736	-1.0997	0.03981	-0.1616	-0.1632	-0.8648	-0.6142	0.89671
33	-0.4835	-0.2262	0.0361	0	0.18772	0.32355	1.09179	0.81398	0.81687	1.54267	0.20261	-0.3117	-0.0875
36	0.83765	0.65539	-1.4657	0	-1.5358	-1.1736	-1.0997	-0.4247	0.81687	-1.0751	-1.4272	-1.6727	1.38881
37	-1.281	0.65539	-0.5717	0	-0.6607	-0.6745	-0.5383	-0.626	-0.7084	0.16694	-0.9172	-0.6444	0.15856
38	0.78738	0.65539	-0.9604	0	-0.5319	-0.6745	-0.4477	0.03981	-0.1616	-0.1632	-0.6245	-0.6142	0.15856
42	-0.316	0.65539	-0.173	1	-0.4203	0.72278	-0.3209	0.28755	2.08312	0.35562	1.31583	2.05477	-2.0559
45	1.05676	0.65539	0.10003	1	0.02906	2.02029	0.67521	1.23203	0.03985	0.42637	1.5594	2.05477	-1.5638

slope	0.696968
Intercept	-0.17102

		corr	-0.3079	0.11749	-0.409	-0.1373	-0.314878	0.20616	-0.0018	0.18855	-0.019	-0.089
sales_t	manufact	residual	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_c
-0.8265	-0.52884	-0.2869										
-0.16641	-0.4413	0.31218										
-0.22944	-0.4413	0.24915										
-2.22164	-0.4413	-1.743										
-0.19113	-0.42409	0.27546										
-0.77141	-0.42409	-0.3048										
0.979454	0.585986	0.74206										
0.335018	0.585986	0.09762										
0.906905	0.585986	0.66951										
0.702937	-0.23309	1.03641										
-0.35441	-0.23309	-0.0209										
-1.03486	-0.23309	-0.7014										
-0.41196	-0.23309	-0.0785										
-0.02254	0.119177	0.06541										
0.39545	0.119177	0.48341										
-0.26406	0.119177	-0.1761										
0.184339	0.119177	0.27229										
1.105425	0.119177	1.19338										
0.156649	-0.22616	0.48529										
0.184504	-0.22616	0.51315										
-0.48349	-0.22616	-0.1548										
0.837654	0.655386	0.55189										
-1.28099	0.655386	-1.5668										
0.787378	0.655386	0.50161										
-0.31596	0.655386	-0.6017										
1.056762	0.655386	0.771										

1.14 Ridge Regression

PEAKS2TAILS
http://peaks2tails.com

Ridge Regression

Ridge Regression Minimizes MSE + penalized L2 norm of the beta coefficients (Regularization)

We need to choose a range of hyperparameter (penalization factor) and train the beta coefficients on Training Set and measure the performance on validation set.

λ = 100

Ridge Minimizer $$\sum_{i=1}^{n}(y_i - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$$

Analytical beta $$\hat{\beta}_{ridge} = (X'X + \lambda I_p)^{-1} X'Y$$

Fit model on Training Set — **Training Set** — **MSE** **0.4701**

		-0.0313	0.21754	-0.1091	0.04708	-0.1727	0.00657	-0.1147	0.10135	0.01979	0.09564	0.03396	0.0141	0.02669	Predicted	
#	sales_t	Const	manufact	resale_t	type	price_t	engine_s	horsepow	wheelbas	width	length	curb_wgt	fuel_cap	mpg	sales_t	Error
4	-0.8265	1	-0.5288	1.06013	0	1.02746	0.32355	0.31298	1.06171	-0.0465	0.67007	0.67013	-0.0093	-0.3335	-0.2888	-0.5377
5	-0.1664	1	-0.4413	0.51241	0	-0.168	-1.3732	-0.7737	-0.7963	-0.9674	-0.7921	-0.7226	-0.4932	0.89671	-0.2577	0.09127
6	-0.2294	1	-0.4413	0.62099	0	0.57326	-0.3751	0.13186	0.1482	1.3061	0.30845	0.1977	0.14191	-0.3335	-0.2415	0.01209
7	-2.2216	1	-0.4413	1.59497	0	1.8588	1.02221	2.12416	0.81398	0.70175	0.79585	0.75513	1.71454	-0.5796	-0.6525	-1.5691
8	-0.1911	1	-0.4241	0.30367	0	0.08353	-0.6745	-0.4115	-0.0686	-0.9098	-0.9494	-0.4267	-0.4327	0.65066	-0.2474	0.05625
9	-0.7714	1	-0.4241	0.99884	0	0.53839	-0.3751	0.00508	-0.0686	-0.8811	-0.9494	-0.3973	-0.4327	0.15856	-0.4592	-0.3122
11	0.97945	1	0.58599	-0.5822	0	-0.3553	-0.0757	-0.3209	0.19465	0.32763	0.51284	-0.1178	-0.1605	0.40461	0.33712	0.64234
12	0.33502	1	0.58599	-0.4011	0	-0.0545	0.62298	0.85633	0.19465	0.32763	0.63862	0.16828	-0.1605	-0.0875	0.14358	0.19144
14	0.90691	1	0.58599	-0.4538	0	0.15317	0.62298	0.22242	0.69011	0.55786	0.93735	0.24675	-0.1605	0.40461	0.2853	0.6216
15	0.70294	1	-0.2331	0.53546	0	0.9177	1.42144	1.49025	1.1701	0.84564	1.50337	0.87937	0.14191	-0.3335	-0.1585	0.86145
16	-0.3544	1	-0.2331	0.89005	0	1.14968	1.42144	1.49025	0.69011	0.98954	1.01597	0.01854	0.14191	-0.3335	-0.3589	0.00451
17	-1.0349	1	-0.2331	0.78996	0	0.90536	1.42144	1.49025	0.03981	1.13343	0.98452	0.65869	0.29312	-0.3335	-0.348	-0.6869
19	-0.412	1	-0.2331	1.29854	1	1.23206	2.51933	1.12801	1.51073	1.5651	1.03169	2.73188	2.05477	-2.0559	-0.1527	-0.2593
22	-0.0225	1	0.11918	-0.9378	0	-0.6782	-0.0757	-0.3209	-0.0376	0.27007	1.0081	-0.1799	-0.4327	0.40461	0.34688	-0.3694
23	0.39545	1	0.11918	-0.7302	0	-0.6224	0.22375	-0.2304	0.4269	0.32763	0.77227	-0.1636	-0.3117	0.89671	0.34724	0.04821
25	-0.2641	1	0.11918	1.4513	0	1.20791	2.51933	2.44112	-0.5021	0.58664	-0.6585	-0.3761	0.32337	-0.3335	-0.7552	0.49117
26	0.18434	1	0.11918	-1.1642	0	-1.3237	-1.3732	-1.3171	-1.6479	-1.3991	-1.083	-1.7034	-1.461	2.373	0.17891	0.00543
28	1.10542	1	0.11918	-0.5982	0	-0.6782	0.22375	-0.2304	0.4269	0.41397	0.93735	-0.0835	-0.3117	0.89671	0.36268	0.74274
31	0.15665	1	-0.2262	-0.4032	0	-0.3292	-0.4749	0.13186	0.81398	0.81687	1.65273	0.01952	-0.3117	0.65066	0.27242	-0.1158
32	0.1845	1	-0.2262	-0.5576	0	-0.9695	-1.1736	-1.0997	0.03981	-0.1616	-0.1632	-0.8648	-0.6142	0.89671	0.23731	-0.0528
33	-0.4835	1	-0.2262	0.0361	0	0.18772	0.32355	1.09179	0.81398	0.81687	1.54267	0.20261	-0.3117	-0.0875	0.00634	-0.4898
36	0.83765	1	0.65539	-1.4657	0	-1.5358	-1.1736	-1.0997	-0.4247	0.81687	-1.0751	-1.4272	-1.6727	1.38881	0.49015	0.3475
37	-1.281	1	0.65539	-0.5717	0	-0.6607	-0.6745	-0.5383	-0.626	-0.7084	0.16694	-0.9172	-0.6444	0.15856	0.24754	-1.5285

Validation Set — **MSE** **0.5917**

#	sales_t	manufact	resale_t	type	price_t	engine_s	horsepow	wheelbas	w
1	-0.3091	-0.5288	-0.0723	0	-0.4019	-1.3732	-0.9549	-1.0131	-
2	0.33567	-0.5288	0.29675	0	0.19222	0.02413	0.58465	0.0553	-
3	-0.4474	-0.5288	0.13209	0	-0.0545	0.02413	0.58465	-0.1305	-
13	0.07127	0.58599	0.32669	0	0.44464	0.62298	0.22242	0.93785	0
21	1.27644	0.11918	-0.7794	0	-0.9624	-0.0757	-0.4115	-0.115	-
24	0.0305	0.11918	-0.5014	0	-0.1371	0.62298	0.13186	-1.0285	0
27	-0.1137	0.11918	-2.2088	0	-2.2057	-2.1716	-2.4944	-2.2672	-
39	0.94942	0.65539	-0.6124	0	-0.3044	-0.4749	0.16808	0.81398	0
40	-2.3845	0.65539	2.38993	0	2.10944	2.71894	2.44112	-1.7872	1
41	1.67249	0.65539	-0.2286	1	-0.6147	2.02029	0.67521	2.50167	2
49	0.2471	2.93691	-1.224	0	-0.8988	-0.6745	-0.4115	-0.1924	-
58	1.57445	1.21095	-1.0225	0	-1.4948	-1.5728	-1.5707	-0.7034	-
60	0.80869	1.21095	0.07774	1	-0.4984	-1.1736	-0.8462	-0.7034	-
70	1.39113	0.30494	0.19206	1	0.076	0.82259	0.0413	-0.2853	0
72	-0.5281	-0.2918	1.21529	0	0.80282	-0.1755	0.58465	0.38045	-
74	-1.0539	-0.2918	1.66254	0	1.56409	0.82259	1.76193	0.69011	0
81	0.39451	-0.414	-0.9225	0	-0.6605	-0.7743	-0.7013	-1.075	-
94	0.06442	-0.4745	1.71384	0	1.39534	0.02413	0.51221	0.58173	-
97	-0.8808	-0.4745	0.90444	0	0.86377	-0.8741	-0.0493	-2.0504	-
105	0.05625	0.32783	-0.1889	1	0.03626	0.12394	-0.4115	0.69011	0
109	-2.3845	0	-0.7769	0	-0.7641	-0.0757	-0.7737	-0.115	-
114	-0.0309	0	-0.2062	1	-0.0507	0.22375	-0.1398	1.89782	0
115	0.19454	-0.4963	-1.4657	0	-1.5358	-1.1736	-1.0997	-0.4247	0

1.15 Model Building

PEAKS2TAILS
http://peaks2tails.com

We will choose the best model between the two candidate models we identified i.e. (1) Forward Selection , (2) LARS

Final Features

Model 1	Model 2
manufact	manufact
price_t	price_t
wheelbas	resale_t

Fitting the model

Model 1

Coeff	-0.1093	0.445312	-0.5025396	0.3355869

#	sales_t	manufact	price_t	wheelbas
4	-0.8265	-0.528837	1.0274562	1.0617149
5	-0.1664	-0.441304	-0.1679839	-0.796286
6	-0.2294	-0.441304	0.5732563	0.1481977
7	-2.2216	-0.441304	1.8588044	0.8139814
8	-0.1911	-0.424085	0.0835335	-0.068569
9	-0.7714	-0.424085	0.5383922	-0.068569
11	0.97945	0.585986	-0.3552549	0.1946477
12	0.33502	0.585986	-0.0544935	0.1946477
14	0.90691	0.585986	0.1531691	0.6901147
15	0.70294	-0.233092	0.9176985	1.1700983
16	-0.3544	-0.233092	1.1496778	0.6901147
17	-1.0349	-0.233092	0.9053567	0.0398143
19	-0.412	-0.233092	1.2320592	1.5107318
22	-0.0225	0.119177	-0.6781602	-0.037602
23	0.39545	0.119177	-0.6223945	0.4268979
25	-0.2641	0.119177	1.2079105	-0.502103
26	0.18434	0.119177	-1.323738	-1.64787
28	1.10542	0.119177	-0.6781602	0.4268979
31	0.15665	-0.226161	-0.3291877	0.8139814
32	0.1845	-0.226161	-0.9694999	0.0398143
33	-0.4835	-0.226161	0.1877181	0.8139814

MODEL 1 OUTPUT

Regression Statistics	
Multiple R	0.792114
R Square	0.627445
Adjusted R	0.614598
Standard Er	0.620808
Observatio	91

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	56.47002	18.82334	48.84079	1.33006E-18
Residual	87	33.52998	0.385402		
Total	90	90			

	Coefficient	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	-0.10927	0.067926	-1.60863	0.111321	-0.244278221	0.025743	-0.24428	0.025743
X Variable 1	0.445312	0.079316	5.614431	2.32E-07	0.287663346	0.60296	0.287663	0.60296
X Variable 2	-0.50254	0.076523	-6.56718	3.59E-09	-0.654637128	-0.35044	-0.65464	-0.35044
X Variable 3	0.335587	0.071151	4.716564	9.07E-06	0.194167176	0.477007	0.194167	0.477007

Model 2

Coeff	-0.1356	0.55262	-0.4793	0.13229

#	sales_t	manufact	price_t	resale_t
4	-0.8265	-0.5288	1.02746	1.06013
5	-0.1664	-0.4413	-0.168	0.51241
6	-0.2294	-0.4413	0.57326	0.62099
7	-2.2216	-0.4413	1.8588	1.59497
8	-0.1911	-0.4241	0.08353	0.30367
9	-0.7714	-0.4241	0.53839	0.99884
11	0.97945	0.58599	-0.3553	-0.5822
12	0.33502	0.58599	-0.0545	-0.4011
14	0.90691	0.58599	0.15317	-0.4538
15	0.70294	-0.2331	0.9177	0.53546
16	-0.3544	-0.2331	1.14968	0.89005
17	-1.0349	-0.2331	0.90536	0.78996
19	-0.412	-0.2331	1.23206	1.29854
22	-0.0225	0.11918	-0.6782	-0.9378
23	0.39545	0.11918	-0.6224	-0.7302
25	-0.2641	0.11918	1.20791	1.4513
26	0.18434	0.11918	-1.3237	-1.1642
28	1.10542	0.11918	-0.6782	-0.5982
31	0.15665	-0.2262	-0.3292	-0.4032
32	0.1845	-0.2262	-0.9695	-0.5576
33	-0.4835	-0.2262	0.18772	0.0361

MODEL 2 OUTPUT

Regression Statistics	
Multiple R	0.732334
R Square	0.536312
Adjusted R Squa	0.520323
Standard Error	0.692587
Observations	91

ANOVA

	df	*SS*	*MS*
Regression	3	48.26812	16.0893
Residual	87	41.73188	0.47967
Total	90	90	

	Coefficient	*andard Err*	*t Stat*
Intercept	-0.1356	0.075589	-1.7938
X Variable 1	0.552616	0.085733	6.44574
X Variable 2	-0.47931	0.150484	-3.1851
X Variable 3	0.132287	0.15027	0.8803

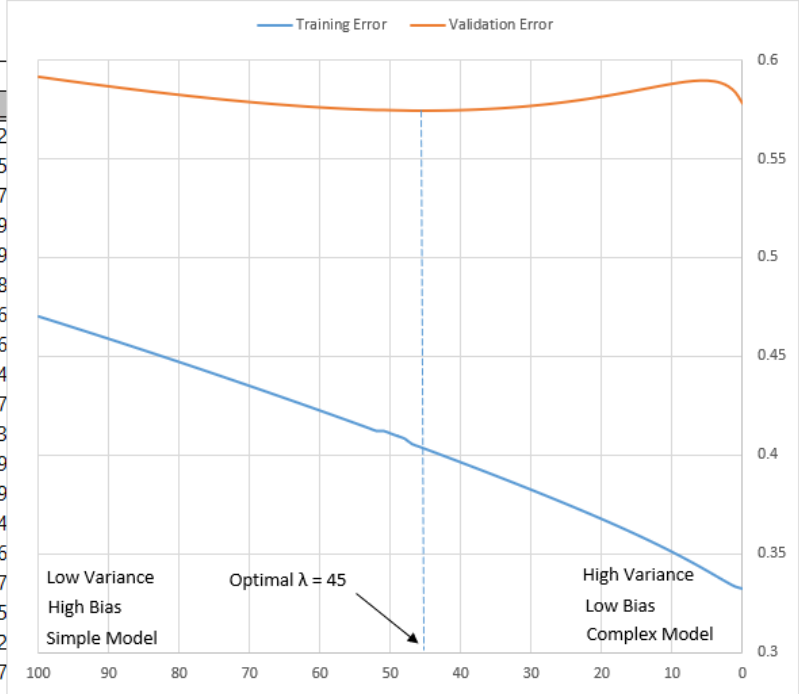
1.16 Hyperparameter Tuning

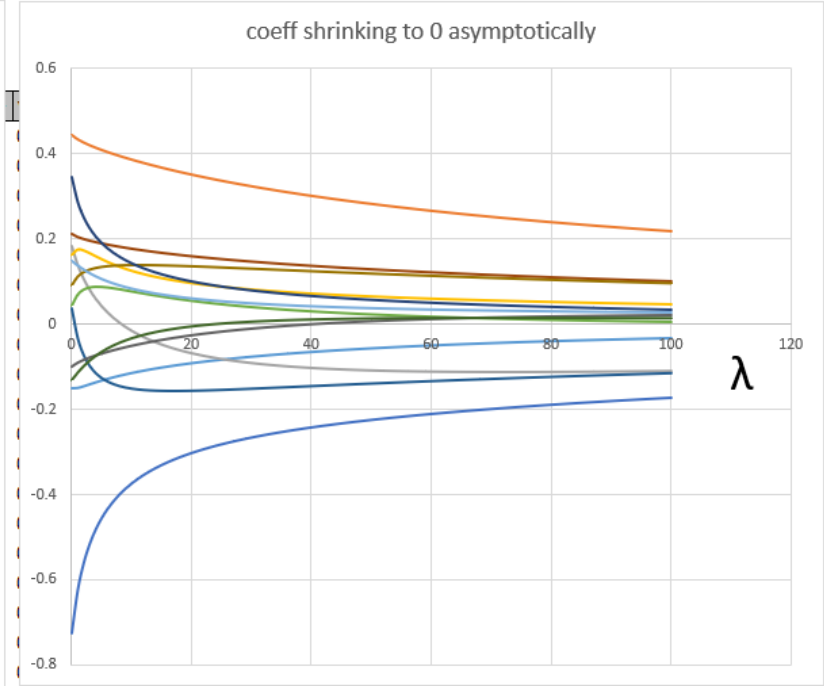
PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

Ridge Regression (Hyperparameter Tuning)

λ	Const	manufact	resale_t	type	price_t	engine_									
0	-0.1519	0.4452	0.18247	0.16158	-0.7251	0.04332									
1	-0.1511	0.435	0.1379	0.17444	-0.6238	0.06905									
2	-0.147	0.42735	0.1054	0.17215	-0.5602	0.08037									
3	-0.1423	0.42089	0.07995	0.16589	-0.5153	0.08519									
4	-0.1377	0.41511	0.05922	0.1588	-0.4814	0.08669									
5	-0.1333	0.40978	0.04188	0.15187	-0.4547	0.08638									
6	-0.1292	0.40478	0.02711	0.14542	-0.4328	0.08506									
7	-0.1253	0.40004	0.01434	0.13953	-0.4146	0.08316									
8	-0.1217	0.39552	0.00319	0.13419	-0.399	0.08094									
9	-0.1184	0.39117	-0.0066	0.12934	-0.3855	0.07857									
10	-0.1152	0.38699	-0.0154	0.12493	-0.3736	0.07613									
11	-0.1123	0.38295	-0.0232	0.12092	-0.3631	0.07369									
12	-0.1095	0.37905	-0.0302	0.11725	-0.3537	0.07129									
13	-0.1069	0.37526	-0.0365	0.11388	-0.3452	0.06894									
14	-0.1043	0.37159	-0.0423	0.11079	-0.3375	0.06666									
15	-0.102	0.36802	-0.0475	0.10793	-0.3304	0.06447									
16	-0.0997	0.36454	-0.0522	0.10529	-0.3239	0.06235									
17	-0.0975	0.36116	-0.0566	0.10283	-0.3179	0.06032									
18	-0.0954	0.35787	-0.0606	0.10054	-0.3123	0.05837									
19	-0.0935	0.35466	-0.0643	0.0984	-0.3071	0.0565	-0.1559674	0.1614857	-0.0294	0.13525	0.10359	-0.0071	0.06132	0.365939639	0.582329468
20	-0.0916	0.35153	-0.0677	0.09639	-0.3023	0.05471	-0.1557231	0.1600114	-0.0274	0.13476	0.1007	-0.0054	0.05972	0.367496762	0.581730829
21	-0.0897	0.34847	-0.0708	0.09451	-0.2977	0.053	-0.1554103	0.1585843	-0.0255	0.13426	0.09799	-0.0038	0.05822	0.369036626	0.581155095
22	-0.088	0.34549	-0.0737	0.09274	-0.2934	0.05135	-0.1550409	0.1572013	-0.0237	0.13373	0.09545	-0.0023	0.05682	0.370560545	0.580603444
23	-0.0863	0.34257	-0.0764	0.09106	-0.2893	0.04977	-0.1546247	0.1558597	-0.022	0.13319	0.09305	-0.001	0.0555	0.372069702	0.580076709
24	-0.0846	0.33972	-0.0789	0.08948	-0.2855	0.04826	-0.1541699	0.154557	-0.0204	0.13264	0.0908	0.00025	0.05426	0.373565161	0.579575444
25	-0.0831	0.33693	-0.0812	0.08799	-0.2818	0.04681	-0.153683	0.1532908	-0.0188	0.13209	0.08867	0.00138	0.05309	0.375047881	0.579099984
26	0.0815	0.3342	0.0833	0.08656	0.2784	0.04542	0.1531696	0.152059	0.0173	0.13152	0.08665	0.00243	0.05199	0.376518726	0.578650484





2.1 Classification Master Pipeline

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

Classification Master Pipeline

1 Data Preparation

Load Data → Exploratory Data analysis → Split Data set → Missing Value Imputation → Categorical Encoding → Variable Transformation → Outlier Engineering → Feature Scaling

Training Set / Validation Set / Testing Set

Exploratory Data analysis

- Missing Value Analysis
- Cardinality
- Distribution
- Box Plots
- Outliers
- Duplicate Check

Split Data set

Random Sampling
- Training Set
- Validation Set
- Test Set

Resampling (Cross Validation)
- K Fold
- Leave one Out
- Leave p out
- Repeated K Fold

Missing Value Imputation

Univariate Imputations
- Mean
- Median
- Mode
- Missing Indicator
- Discretization + missing cateogry

Multiple Imputation
- Likelihood
- EM
- Bayesian
- kNN
- mice
- Regression
- Regression + noise

Categorical Encoding

- One hot encoding
- One hot encoding Top categories
- Count Frequency Encoding
- Ordinal Encoding
- Mean Encoding
- WOE Encoding
- Rare Label Encoding
- Binary Hashing Encoding
- Discretization (numerical) + encoding

Variable Transformation

Normal Transformation
- Box Cox
- Yeo Johnson

Discretization
- Equal Width
- Equal Frequency
- k-Means
- Decision Trees
- Domain Knowledge

Dimension Reduction
- PCA

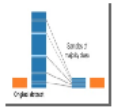
Outlier Engineering

- Trimming
- Winsorization
- 3 sigma approach

Feature Scaling

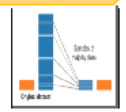
- standardization
- Mean Normalization
- MinMax Scaling
- MaxAbs Scaling
- Robust Scaling
- Unit Norm

2.2 Classification Master Pipeline



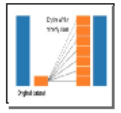
show all

2 Class Balancer



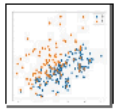
Under Sampling

- **Fixed**
  - Random
  - NearMiss
  - Instance Hardness
- **Cleaning**
  - CNN
  - Tomek Links
  - One sided selection
  - EDD
  - Neighborhood Cleaning Rule



Over Sampling

- ROS
- ROS (with smoothing)
- SMOTE
- SMOTE-NC
- SMOTE-N
- SVM Smote
- k-Means SMOTE
- Borderline SMOTE
- ADASYN



Over and Under Sampling

(Combine an Oversampling and an undersampling Technique)

3 Variable Selection

Basic Filters

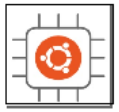
- Remove Constants
- Remove Quasi-Constant
- Remove Duplicates

Statistical Filters

- Correlation
- Multicollinearity
- t-Tests
- Chi-Square tests
- ANOVA

Wrapper

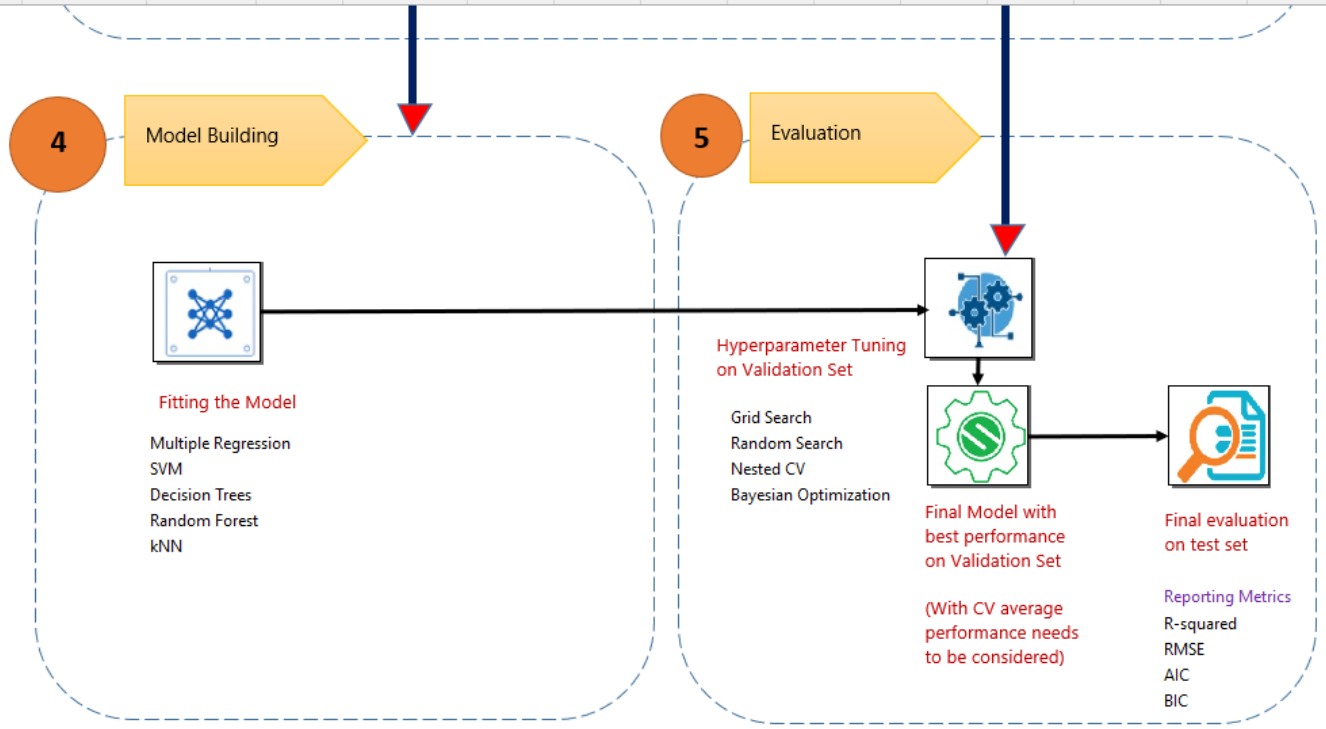
- Forward Selection
- Backward Selection
- Stepwise
- Least Angle



Embedded Methods

- Ridge
- LASSO
- Elastic Net

2.3 Classification Master Pipeline



2.4 SMOTE Over Sampling

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

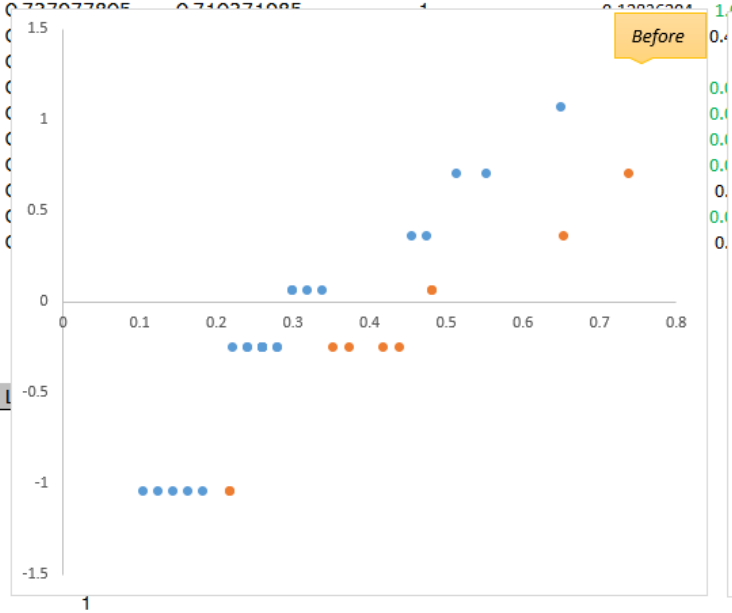
Synthetic Minority OverSampling Technique (SMOTE)

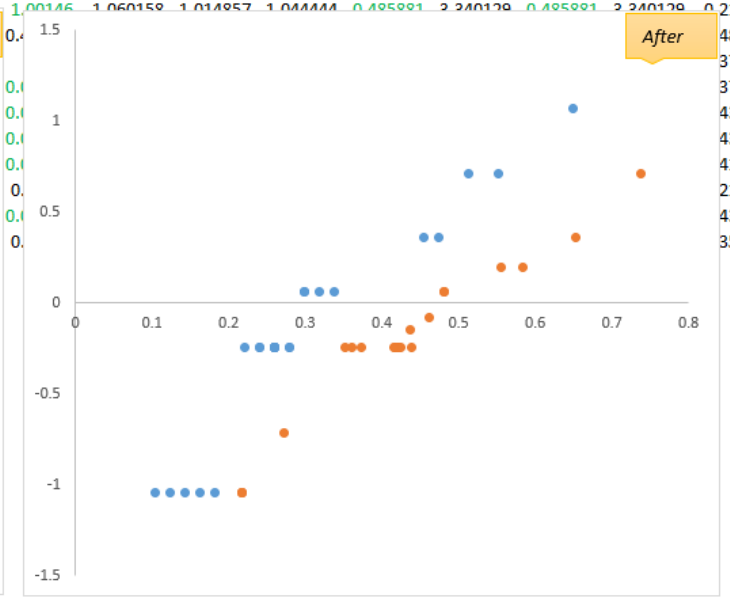
The algorithms works as follows:
1) Separate Minority Class from the Training Data
2) Train k-NN to each sample in Minority Class (Typically k = 5)
3) Determines how many samples to be synthesized
4) Randomly Select a minority sample
5) Randomly select 1 neighbor
6) Find a random factor (0 < factor < 1)
7) Create a New Sample by the following formula

New Sample = Original Sample - factor x (Original sample - Neighbor)

Minority Class

Interest_Rate	Loan_Grade	Loan_Status	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	Random Neighbor	random factor
0.737977805	0.710371985	1			1.00146	1.060158	1.014857	1.044444	0.485881	3.340129	0.485881	3.340129	0.217016 -1.041406377	0.295851795
0.652281134	0.362495	1											0.480888 0.062463157	0.561562796
0.438347719	-0.24445	1											0.373613 -0.244448288	0.258673125
0.352651048	-0.24445	1											0.373613 -0.244448288	0.385422187
0.41676942	-0.24445	1											0.438348 -0.244448288	0.344992593
0.373612824	-0.24445	1											0.438348 -0.244448288	0.655212093
0.480887793	0.062463	1											0.416769 -0.244448288	0.673896405
0.21701603	-1.04141	1											0.217016 -1.041406377	0.183422756
0.480887793	0.062463	1											0.438348 -0.244448288	0.466735798
0.21701603	-1.04141	1											0.352651 -0.244448288	0.410908322
0.163378545	-1.04141	0												
0.260172626	-0.24445	0												
0.552404439	0.710372	0												
0.221331689	-0.24445	0												
0.104808878	-1.04141	0												
0.260172626	-0.24445	0												





Interest_I	Loan_Gra	L
0.738	0.7104	
0.6523	0.3625	
0.4383	-0.244	
0.3527	-0.244	
0.4168	-0.244	
0.3736	-0.244	
0.4809	0.0625	
0.217	-1.041	1
0.4809	0.0625	1
0.217	-1.041	1
0.5839	0.1921	1
0.556	0.194	1
0.4216	-0.244	1
0.3607	-0.244	1
0.4242	-0.244	1
0.416	-0.244	1

2.5 Adaptive Synthetic Method

PEAKS2TAILS
http://peaks2tails.com

Adaptive Synthetic Method (ADASYN)

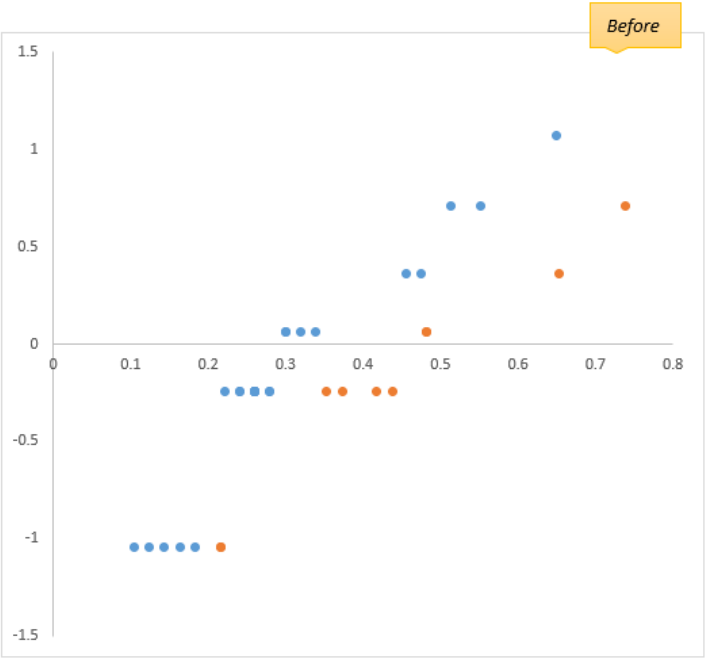
Works like SMOTE, but kNN is trained on the entire data set (Majority + Minority), then number of samples generated from a minority example is proportional to how many Nearest neighbors to that example are from majority class.

Minority Class

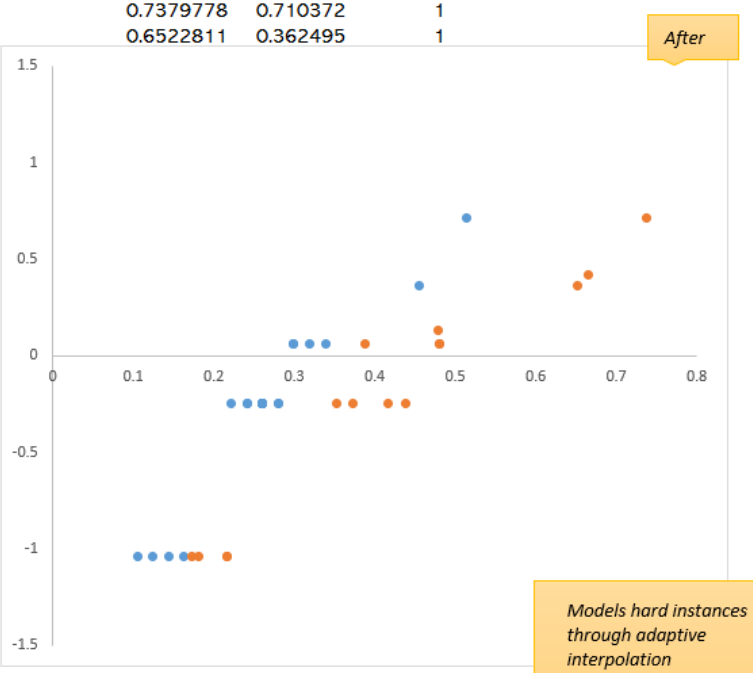
Interest_Rate	Loan_Grade	Loan_Status
0.737977805	0.710372	1
0.652281134	0.362495	1
0.438347719	-0.24445	1
0.352651048	-0.24445	1
0.41676942	-0.24445	1
0.373612824	-0.24445	1
0.480887793	0.062463	1
0.21701603	-1.04141	1
0.480887793	0.062463	1
0.21701603	-1.04141	1

Majority Class

Interest_Rate	Loan_Grade	Loan_Status
0.163378545	-1.04141	0
0.260172626	-0.24445	0
0.552404439	0.710372	0
0.221331689	-0.24445	0
0.104808878	-1.04141	0
0.260172626	-0.24445	0



Interest_Rate	Loan_Grade	Loan_Status
0.7379778	0.710372	1
0.6522811	0.362495	1



0.5135635	0.710372	0
0.3187423	0.062463	0
0.2601726	-0.244448	0
0.2996301	0.062463	0
0.2996301	0.062463	0

Models hard instances through adaptive interpolation

2.6 Random Under Sampling

PEAKS2TAILS
http://peaks2tails.com

Random Under Sampling

*A few observations are shown only with **Interest rate** vs **Loan grade** for better visibility.*

Current Balance	
0	11252
1	1953
Balancing Ratio	17.36%

Desired Balance	
0	1953
1	1953
Balancing Ratio	100.00%

Randomly Undersample from Majority Class until a balancing ratio (say 50:50) is achieved

#	Emp_length	Annual_Inc	Loan_Amt	Loan_Ter	Loan_Purpose	Interest_Rate	Loan_Grade	Region	Loan_Status
5	0.2631579	-0.06614518	0.3061224	0	0.160090465	0.163378545	-1.041406377	-0.084807274	0
7	0	-0.09348365	0.3877551	0	-0.022452846	0.260172626	-0.244448288	0.122009744	0
8	0.5789474	-0.09348365	0.7959184	0	-0.374239526	0.552404439	0.710371985	-0.106412293	0
9	0.2631579	-0.06614518	0.3469388	0	-0.022452846	0.435881628	0.36249539	-0.106412293	1
10	0.5789474	-0.26212979	0.4693878	0	-0.374239526	0.221331689	-0.244448288	-0.084807274	0
11	0	-0.26212979	0.0204082	0	0.158548445	0.104808878	-1.041406377	0.122009744	0
12	0.0526316	-0.26212979	0.8918367	0	-0.060580897	0.533292232	0.710371985	-0.106412293	1
13	0.0526316	0.326005066	0.2244898	0	-0.022452846	0.260172626	-0.244448288	-0.106412293	0
14	0	0.326005066	0.0204082	0	-0.022452846	0.260172626	-0.244448288	-0.084807274	0
15	0.0526316	-0.00397048	0.3877551	0	0.012529595	0.241060419	-0.244448288	0.100015393	1
17	0.0526316	-0.00397048	0.2163265	0	-0.060580897	0.123921085	-1.041406377	0.100015393	0
19	0.1578947	-0.00397048	1	0	-0.022452846	0.260172626	-0.244448288	-0.084807274	0
22	0.1578947	-0.06614518	0.8367347	0	-0.022452846	0.279901356	-0.244448288	0.100015393	0
23	0.5789474	0.326005066	0.1836735	0	-0.022452846	0.454993835	0.36249539	-0.106412293	0
24	0	-0.26212979	0.4693878	0	-0.673295672	0.474722565	0.36249539	-0.106412293	0
25	0	-0.06614518	0.7142857	0	0.547206434	0.241060419	-0.244448288	-0.106412293	0
26	0.3684211	0.326005066	0.9	0	-0.022452846	0.649815043	1.069820014	-0.084807274	0
30	0.1578947	0.326005066	0.1428571	0	0.158548445	0.143649815	-1.041406377	0.100015393	0
32	1	-0.09348365	0.3602041	0	-0.022452846	0.338471023	0.062463157	0.100015393	0
36	0	0.326005066	0.5918367	0	-0.175598612	0.260172626	-0.244448288	-0.106412293	0
38	0.1578947	-0.09348365	0.5918367	0	-0.175598612	0.182490752	-1.041406377	0.100015393	0
39	0.0526316	-0.09348365	1	0	0.728149631	0.552404439	0.710371985	0.122009744	1
41	0	0.326005066	0.1010204	0	-0.022452846	0.279901356	-0.244448288	-0.084807274	0
42	0.3684211	-0.00397048	0.1020408	0	0.158548445	0.241060419	-0.244448288	-0.106412293	0
43	0.6842105	-0.00397048	0.0938776	0	-0.673295672	0.416152898	0.36249539	-0.106412293	1

Majority Class before Undersampling

Count 11252

#	Loan_Stat	rnd	rnd(fixe
5	0	Discard	Discard
7	0	Discard	Keep
8	0	Discard	Keep
10	0	Discard	Discard
11	0	Discard	Discard
13	0	Discard	Discard
14	0	Discard	Discard
17	0	Discard	Keep
19	0	Discard	Discard
22	0	Keep	Discard
23	0	Discard	Discard
24	0	Discard	Discard
25	0	Discard	Keep
26	0	Discard	Discard
30	0	Discard	Discard
32	0	Discard	Discard
36	0	Keep	Keep
38	0	Discard	Discard
41	0	Discard	Discard
42	0	Discard	Discard
46	0	Discard	Keep
47	0	Keep	Discard
48	0	Discard	Discard
49	0	Discard	Keep
52	0	Keep	Discard

Majority Class after Undersampling

Count 1953

#	Loan_Status
7	0
8	0
17	0
25	0
36	0
46	0
49	0
85	0
98	0
106	0
122	0
139	0
141	0
183	0
189	0
192	0
207	0
234	0
238	0
239	0
240	0
254	0
255	0
274	0
286	0

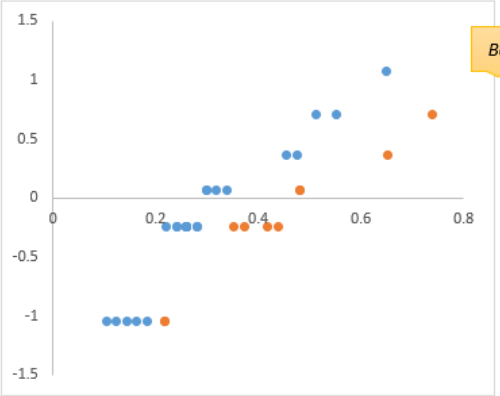
2.7 Condensed Nearest Neighbours

PEAKS2TAILS
http://peaks2tails.com

Condensed Nearest Neighbor

*A few observations are shown only with **Interest rate** vs **Loan grade** for tracking the process.*

The algorithms works as follows:
1) Put all minority class observations in a group, typically group O
2) Add 1 sample (at random) from the majority class to group O
3) Train a KNN with group O
4) Take a sample of the majority class that is not in group O yet
5) Predict its class with the KNN from point 3
6) If the prediction was correct, go to 4 and repeat
7) If the prediction was incorrect, add that sample to group O, go to 3 and repeat
8) Continue until all samples of the majority class were either assigned to O or left out
9) Final version of Group O is our undersampled dataset



Minority Class

Interest_Rate	Loan_Grade	Loan_Status
0.737977805	0.710371985	1
0.652281134	0.36249539	1
0.438347719	-0.244448288	1
0.352651048	-0.244448288	1
0.41676942	-0.244448288	1
0.373612824	-0.244448288	1
0.480887793	0.062463157	1
0.21701603	-1.041406377	1
0.480887793	0.062463157	1
0.21701603	-1.041406377	1

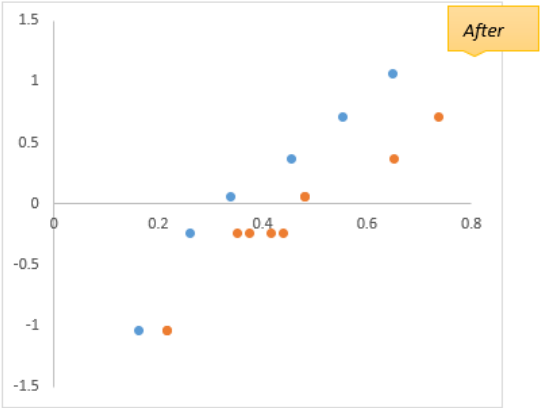
Majority Class

seed

Interest_Rate	Loan_Grade	Loan_Status
0.163378545	-1.041406377	0
0.260172626	-0.244448288	0
0.552404439	0.710371985	0
0.221331689	-0.244448288	0
0.104808878	-1.041406377	0
0.260172626	-0.244448288	0
0.260172626	-0.244448288	0

Group O

Interest_Rate	Loan_Grade	Loan_Status
0.73797781	0.710372	1
0.65228113	0.3624954	1
0.43834772	-0.2444483	1
0.35265105	-0.2444483	1
0.41676942	-0.2444483	1
0.37361282	-0.2444483	1
0.48088779	0.0624632	1
0.21701603	-1.0414064	1
0.48088779	0.0624632	1
0.21701603	-1.0414064	1
0.16337855	-1.0414064	0
0.26017263	-0.2444483	0
0.55240444	0.710372	0
0.45499383	0.3624954	0
0.64981504	1.06982	0
0.33847102	0.0624632	0



Models noisy boundary, data size varies

Classify Majority Class (based on 1-NN model on Group O)

		Class	1	1	1	1	1	1	1	
Interest_Rate	Loan_Grade	Loan_Status	d1	d2	d3	d4	d5	d6	d7	d
0.26017263	-0.2444483	0	1.13998	0.52213	0.031746	0.008552	0.024523	0.012869	0.14291	0.63
0.55240444	0.710372	0	0.03444	0.130993	0.924691	0.951583	0.930079	0.943648	0.4249	3.18
0.22133169	-0.2444483	0	1.1786	0.554098	0.047096	0.017245	0.038196	0.02319	0.161564	0.63
0.10480888	-1.0414064	0	3.46963	2.270666	0.74639	0.696568	0.732462	0.707398	1.359963	0.0

2.8 Tomek Links

PEAKS2TAILS
http://peaks2tails.com

Tomek Links

If two nearest neighbors have different class, they are called Tomek links.

The algorithm removes either only majority observations from the Tomek links or both observations fro the Tomek links. In our example, we will only remove obs from the majority class.

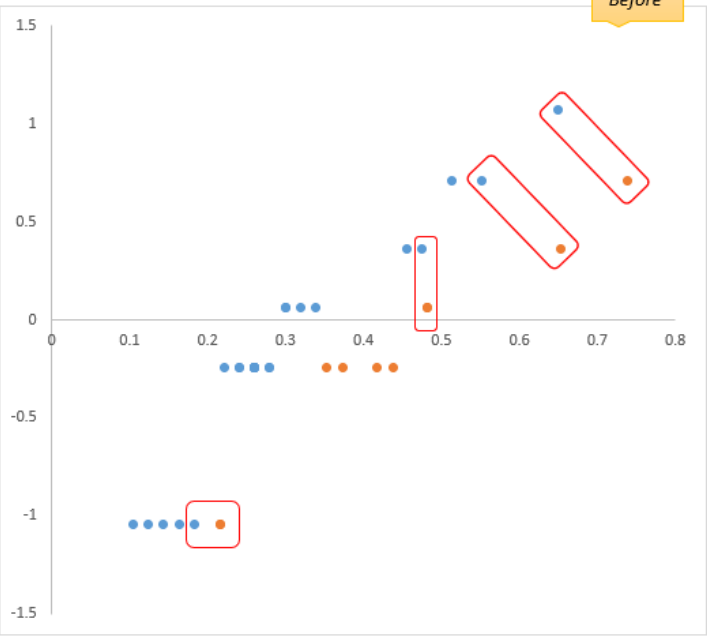
Minority Class

Interest_Rate	Loan_Grade	Loan_Status
0.737977805	0.710372	1
0.652281134	0.362495	1
0.438347719	-0.24445	1
0.352651048	-0.24445	1
0.41676942	-0.24445	1
0.373612824	-0.24445	1
0.480887793	0.062463	1
0.21701603	-1.04141	1
0.480887793	0.062463	1
0.21701603	-1.04141	1

Majority Class

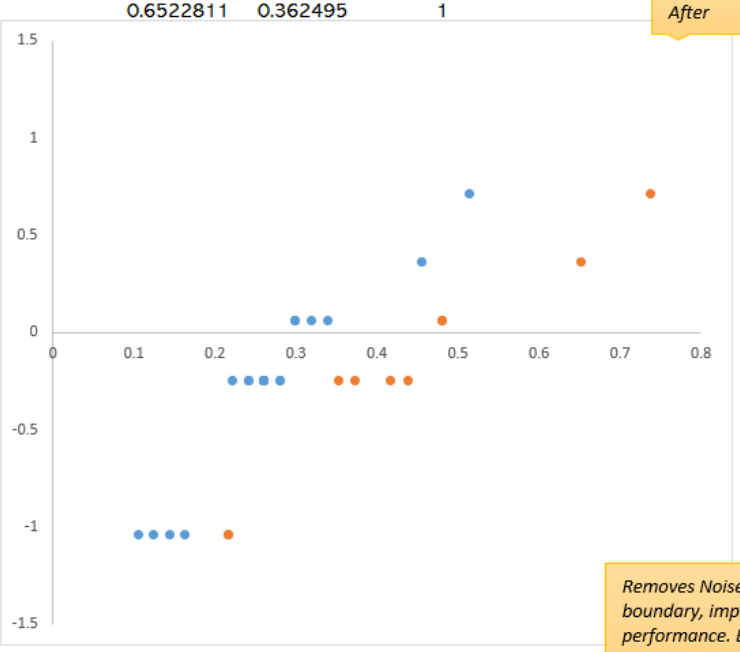
Interest_Rate	Loan_Grade	Loan_Status
0.163378545	-1.04141	0
0.260172626	-0.24445	0
~~0.552404439~~	~~0.710372~~	0
0.221331689	-0.24445	0
0.104808878	-1.04141	0
0.260172626	-0.24445	0

Before



Interest_Rate	Loan_Grade	Loan_Status
0.7379778	0.710372	1
0.6522811	0.362495	1

After



0.5135635	0.710372	0
0.3187423	0.062463	0
0.2601726	-0.244448	0
0.2996301	0.062463	0
0.2996301	0.062463	0

Removes Noise from the boundary, improves performance. But, mis classify hard instances

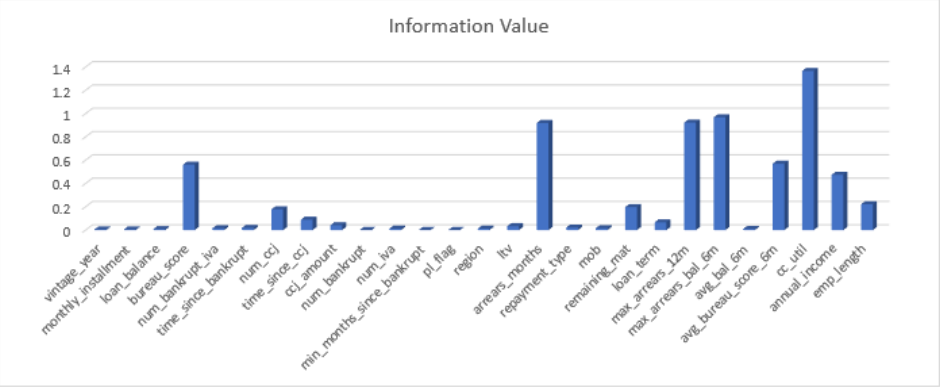
3.1 Information Value

PEAKS2TAILS

Univariate Analysis

vintage_year	Bad	Good
2001	0	2
2002	36	635
2003	57	1061
2004	77	1623
2005	226	3847
2006	385	6929
2007	155	3121
2008	0	0
2009	0	0
2010	0	0
2011	0	0
2012	2	20
2013	0	14
2014	1	11
2015	0	3
Total	939	17266

vintage_year	Bad	Good	%Bad	%Good	WOE	IV
2001-02	36	637	0.03834	0.03689	-0.0384	5.55421E-05
2003	57	1061	0.0607	0.06145	0.01224	9.14548E-06
2004	77	1623	0.082	0.094	0.13655	0.001638243
2005	226	3847	0.24068	0.22281	-0.0772	0.00137923
2006	385	6929	0.41001	0.40131	-0.0215	0.000186666
2007	155	3121	0.16507	0.18076	0.0908	0.001424783
2008 and above	3	48	0.00319	0.00278	-0.1391	5.77027E-05
Total	939	17266			Total	0.004751312



monthly_installment	Bad	Good
500	444	8503
1000	405	7422
1500	70	1083
2000	17	201
2500	1	35
3000	2	11
3500	0	5
4000	0	4
4500	0	0
5000	0	1

monthly_installment	Bad	Good	%Bad	%Good	WOE	IV
Less then 500	444	8503	0.47284	0.4925	0.04073	0.000800555
500 - 1000	405	7422	0.43131	0.42989	-0.0033	4.7016E-06
1000-1500	70	1083	0.07455	0.06273	-0.1726	0.002040325
1500 and Above	20	257	0.0213	0.01489	-0.3583	0.002297865
Total	939	17265			Total	0.005143446

loan_balance	Bad	Good
100000	548	10605
200000	314	5510
300000	51	867
400000	15	182
500000	9	70
600000	1	22
700000	0	3
800000	1	3
900000	0	3
1000000	0	1

loan_balance	Bad	Good	%Bad	%Good	WOE	IV
Less than 100000	548	10605	0.5836	0.61421	0.05113	0.001565153
100000-200000	314	5510	0.3344	0.31912	-0.0468	0.000714092
200000-300000	51	867	0.05431	0.05021	-0.0785	0.000321616
300000 and Above	26	284	0.02769	0.01645	-0.5208	0.005854078
Total	939	17266			Total	0.008454939

bureau_score	Bad	Good

bureau_score	Bad	Good	%Bad	%Good	WOE	IV

Variables	IV
vintage_year	0.00475
monthly_installment	0.00514
loan_balance	0.00845
bureau_score	0.56076
num_bankrupt_iva	0.01517
time_since_bankrupt	0.0182
num_ccj	0.17735
time_since_ccj	0.08907
ccj_amount	0.04142
num_bankrupt	0.001
num_iva	0.01172
min_months_since_bankrupt	0.00083
pl_flag	0.00025
region	0.01221
ltv	0.03341
arrears_months	0.9191
repayment_type	0.02158
mob	0.01549
remaining_mat	0.19698
loan_term	0.0665
max_arrears_12m	0.92283
max_arrears_bal_6m	0.96793
avg_bal_6m	0.0091
avg_bureau_score_6m	0.56911
cc_util	1.36593

Most Prominent variables (IV > 0.4)

bureau_score

arrears_months (dropped since it is repeat

max_arrears_12m

max_arrears_bal_6m

avg_bureau_score_6m (dropped since it is repeat

cc_util

annual_income

Medium Important (included as part of expert judgement)

num_ccj

emp_length

months_since_recent_cc_delinq

3.2 Weight Of Evidence Tool

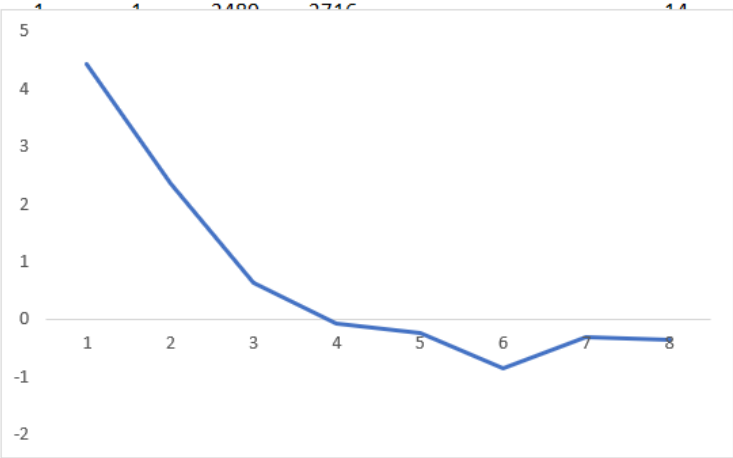
MONTHLY _INCOME	Status
7200	0
1086	0
554.3	1
505.77	1
400	1
100	1
2391	1
1064	0
1520	0
232	1
1053	0
1330	1
2743	1
275	1
2931	0
1732	0
2593	0
1091	0
2574	0
285	1
292	0
2365	1
1253	0
1822	1
2217	1
623	1
1426	1
745	1
662	1
1192	0
1850	1
2884	0
2876	1
2180	0

n	12

Input 2-20
Min 2
Max 20

N	Flag	Location	L	
1	1	0.0%	10	
2	1	8.3%	309	561
3	1	16.7%	561	815
4	1	25.0%	815	1013
5	1	33.3%	1013	1261
6	1	41.7%	1261	1501
7	1	50.0%	1501	1711
8	1	58.3%	1711	1959
9	1	66.7%	1959	2271
10	1	75.0%	2271	2489
11	1	[illegible]	2489	[illegible]
12				

Bad	Good	% Bad	% Good	WOE	IV
60	1	0.19802	0.00235849	4.43035	0.86685
53	7	0.17492	0.01650943	2.36038	0.3739
35	26	0.11551	0.06132075	0.63325	0.03432
24	36	0.07921	0.08490566	-0.0695	0.0004
22	39	0.07261	0.09198113	-0.2365	0.00458
14	46	0.0462	0.10849057	-0.8536	0.05317
21	40	0.06931	0.09433962	-0.3084	0.00772
20	40	0.06601	0.09433962	-0.3571	0.01012
16	45	0.05281	0.10613208	-0.6981	0.03723
14	46	0.0462	0.10849057	-0.8536	0.05317
[illegible]	47	0.0462	0.11084906	-0.8751	0.05657
[illegible]	51	0.033	0.12028302	-1.2932	0.11287
	424				1.61089



3.3 Weight Of Evidence

WOE Transformed (Training Set)

Default	bureau_score	max_arrears_12m	max_arrears_bal_6m	cc_util	annual_income	num_ccj	emp_length	months_since_recent_cc_delinq
0	0.848988348	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.519371295	-0.379586857
1	0.848988348	0.446897907	0.605966823	-0.7399926	0.324421315	0.182549	-0.382216378	-0.359844634
0	1.075105467	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.88879147	1.036190088
0	0.848988348	0.446897907	0.605966823	1.14072133	0.056834275	0.182549	-0.180192003	-0.359844634
1	-0.232100523	-1.708298136	-1.371963137	-1.9129446	-0.641267459	-0.2321	-0.306787205	-0.359844634
0	-0.100424502	0.446897907	0.605966823	1.14072133	-0.641267459	0.182549	-0.860408385	1.036190088
0	-0.100424502	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.627726273	1.036190088
0	1.075105467	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	-0.180192003	-0.359844634
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.439573656	-0.359844634
0	0.848988348	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.520370796	-0.379586857
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.519371295	-0.379586857
0	0.848988348	0.446897907	0.605966823	-0.7399926	-0.641267459	0.182549	-0.306787205	-0.359844634
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.211585475	-0.359844634
0	1.075105467	0.446897907	0.605966823	-0.7399926	0.324421315	0.182549	0.211585475	-0.359844634
0	-1.178027211	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.627726273	-0.379586857
0	0.848988348	0.446897907	0.605966823	-1.9129446	0.056834275	0.182549	0.519371295	1.036190088
0	-0.100424502	0.446897907	0.605966823	-0.7399926	0.324421315	0.182549	-0.306787205	-0.359844634
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.439573656	-0.379586857
0	-0.684965808	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.519371295	1.036190088
0	0.848988348	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.211585475	-0.359844634
0	-0.684965808	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.211585475	1.036190088
0	-0.100424502	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.520370796	-0.359844634
0	1.075105467	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	0.211585475	-0.379586857
1	0.848988348	0.446897907	0.605966823	1.14072133	-0.641267459	0.182549	-0.382216378	1.036190088
0	-1.178027211	0.446897907	0.605966823	-1.9129446	0.659648047	0.182549	0.520370796	-0.379586857
0	1.075105467	0.446897907	0.605966823	1.14072133	-0.641267459	0.182549	0.211585475	-0.379586857
0	-0.100424502	0.446897907	0.605966823	-0.7399926	0.056834275	0.182549	0.211585475	-0.379586857
0	1.075105467	0.446897907	0.605966823	1.14072133	0.324421315	0.182549	-0.306787205	-0.359844634
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.439573656	-0.359844634
0	-0.100424502	0.446897907	0.605966823	1.14072133	-0.641267459	0.182549	0.211585475	-0.379586857
0	-0.100424502	0.446897907	0.605966823	-1.9129446	-0.641267459	-0.84072	-0.382216378	-0.359844634
0	-0.100424502	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	-0.180192003	1.036190088
0	-0.100424502	0.446897907	0.605966823	-0.7399926	0.659648047	0.182549	0.520370796	-0.379586857
0	-0.100424502	0.446897907	-1.371963137	1.14072133	0.324421315	-0.84072	0.520370796	-0.379586857
0	-0.100424502	0.446897907	0.605966823	1.14072133	0.056834275	0.182549	0.627726273	1.036190088
0	0.848988348	0.446897907	0.605966823	1.14072133	0.659648047	0.182549	0.439573656	-0.359844634
0	-1.178027211	-1.708298136	-1.371963137	-1.9129446	0.324421315	-0.9376	-0.306787205	-0.379586857

3.4 Stepwise Logistic Regression

PEAKS2TAILS

Stepwise Regression (3 variables)

3 variables selected so far are :

cc_util, max_arrears_12m, annual_income

min AIC
5019.264266

	-2.91237698	-0.981180025	-0.958784912	-0.985164084		
Default	Intercept	cc_util	max_arrears_12m	annual_income	P(1)	ylnP(1) + (1-y)ln(1-P(1))
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
1	-2.91237698	-0.739992552	0.446897907	0.324421315	0.050479	-2.986195999
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.056834275	0.010813	-0.010872256
1	-2.91237698	-1.912944649	-1.708298136	-0.641267459	0.774555	-0.255467104
0	-2.91237698	1.14072133	0.446897907	-0.641267459	0.021283	-0.021512294
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	-0.739992552	0.446897907	-0.641267459	0.120996	-0.128965297
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	-0.739992552	0.446897907	0.324421315	0.050479	-0.051797732
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	-1.912944649	0.446897907	0.056834275	0.179476	-0.197811707
0	-2.91237698	-0.739992552	0.446897907	0.324421315	0.050479	-0.051797732
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
1	-2.91237698	1.14072133	0.446897907	-0.641267459	0.021283	-3.849867559
0	-2.91237698	-1.912944649	0.446897907	0.659648047	0.107765	-0.114025964
0	-2.91237698	1.14072133	0.446897907	-0.641267459	0.021283	-0.021512294
0	-2.91237698	-0.739992552	0.446897907	0.056834275	0.06472	-0.066908977
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	1.14072133	0.446897907	-0.641267459	0.021283	-0.021512294
0	-2.91237698	-1.912944649	0.446897907	-0.641267459	0.303189	-0.361240653
0	-2.91237698	1.14072133	0.446897907	0.659648047	0.006	-0.006018116
0	-2.91237698	-0.739992552	0.446897907	0.659648047	0.036804	-0.037498312
0	-2.91237698	1.14072133	0.446897907	0.324421315	0.008328	-0.008363304

min AIC
5324.541447

	-2.9067	-0.981943205	-0.917025532	-0.482702242		
Default	Intercept	cc_util	max_arrears_12m	num_ccj	P(1)	ylnP(1) + (1-y)ln(1-P(1))
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
1	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-2.744443629
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
1	-2.9067	-1.912944649	-1.708298136	-0.232100523	0.657077	-0.419954432
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-0.066443328
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-0.066443328
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-1.912944649	0.446897907	0.182549259	0.178546	-0.196679643
0	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-0.066443328
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
1	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-4.53553359
0	-2.9067	-1.912944649	0.446897907	0.182549259	0.178546	-0.196679643
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-0.066443328
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-1.912944649	0.446897907	-0.840719323	0.26264	-0.304678526
0	-2.9067	1.14072133	0.446897907	0.182549259	0.010721	-0.010779071
0	-2.9067	-0.739992552	0.446897907	0.182549259	0.064284	-0.066443328
0	-2.9067	1.14072133	0.446897907	-0.840719323	0.01745	-0.017603891

	-2
Default	Int
0	-2
1	-2
0	-2
0	-2
1	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
1	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2
0	-2

3.5 Scores Normalisation

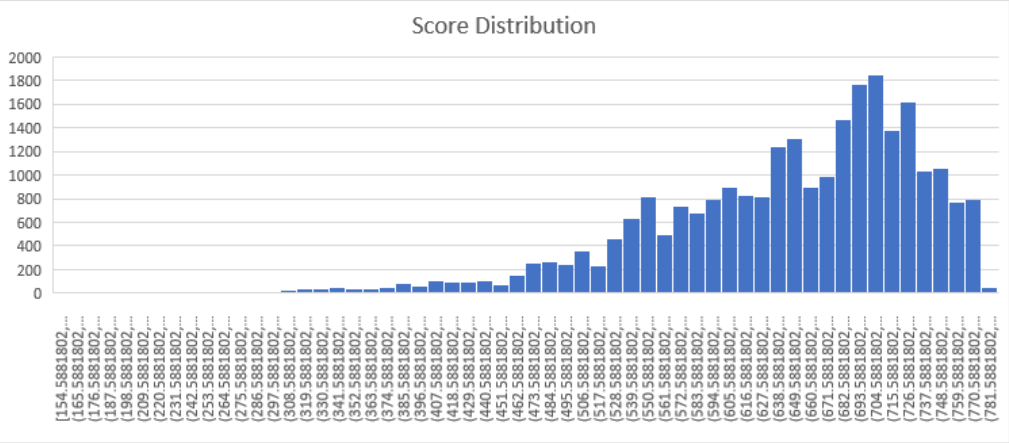
PEAKS2TAILS

Score Normalization

Goal is to score the entire data set
Odds 72 : 1 is anchored at score 660 as per standard
An increment of score 40 doubles the odds of not defaulting

So we have
ln (odds) = a + b * Score
ln(72) = a + 660b
ln(36) = a + 620b

Solving	b =	0.01732868
	a =	-7.16026236



As expected, the score is concentrated at high scores indicating low credit risk

Note that , we are able to derive scores from PD estimates
If softwares produce score as direct output, it's easy to backtrack PD using the same

IRB in BASEL requires accounts to be mapped to a rating.
IFRS/CECL does not require mapping to a rating

Training + Testing Data Set

	-2.91589321	-0.652455435	-0.785562379	-0.924812	-0.865525003	-0.14029063	-0.337886572	-0.528973758			
Default	Intercept	bureau_score	max_arrears_12m	cc_util	annual_income	num_ccj	emp_length	months_since_recent_cc_delinq	P(1)	ln(odds)	Score
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.324421315	0.182549259	0.519371295	-0.379586857	0.005726	5.156941	710.7988
1	-2.91589321	0.848988348	0.446897907	-0.739993	0.324421315	0.182549259	-0.382216378	-0.359844634	0.042151	3.123443	593.45
0	-2.91589321	1.075105467	0.446897907	1.1407213	0.659648047	0.182549259	0.88879147	1.036190088	0.001549	6.468351	786.4773
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.056834275	0.182549259	-0.180192003	-0.359844634	0.009019	4.699408	684.3955
1	-2.91589321	-0.232100523	-1.708298136	-1.912945	-0.641267459	-0.23210052	-0.306787205	-0.359844634	0.773509	-1.22823	342.3243
0	-2.91589321	-0.100424502	0.446897907	1.1407213	-0.641267459	0.182549259	-0.860408385	1.036190088	0.018264	3.984364	643.1319
0	-2.91589321	-0.100424502	0.446897907	1.1407213	0.659648047	0.182549259	0.627726273	1.036190088	0.003636	5.61316	737.1261
0	-2.91589321	1.075105467	0.446897907	1.1407213	0.659648047	0.182549259	-0.180192003	-0.359844634	0.004639	5.36869	723.0183
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.659648047	0.182549259	0.439573656	-0.359844634	0.004361	5.430569	726.5892
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.324421315	0.182549259	0.520370796	-0.379586857	0.005724	5.157279	710.8182
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.659648047	0.182549259	0.519371295	-0.379586857	0.00429	5.447089	727.5425
0	-2.91589321	0.848988348	0.446897907	-0.739993	-0.641267459	0.182549259	-0.306787205	-0.359844634	0.090044	2.313101	546.687
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.659648047	0.182549259	0.211585475	-0.359844634	0.004709	5.353535	722.1437
0	-2.91589321	1.075105467	0.446897907	-0.739993	0.324421315	0.182549259	0.211585475	-0.359844634	0.030131	3.471612	613.5421
0	-2.91589321	-1.178027211	0.446897907	1.1407213	0.324421315	0.182549259	0.627726273	-0.379586857	0.020412	3.871016	636.5908
0	-2.91589321	0.848988348	0.446897907	-1.912945	0.056834275	0.182549259	0.519371295	1.036190088	0.054672	2.850179	577.6806
0	-2.91589321	-0.100424502	0.446897907	-0.739993	0.324421315	0.182549259	-0.306787205	-0.359844634	0.073817	2.52948	559.1737
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.659648047	0.182549259	0.439573656	-0.379586857	0.004407	5.420126	725.9866
0	-2.91589321	-0.684965808	0.446897907	1.1407213	0.324421315	0.182549259	0.519371295	1.036190088	0.007355	4.905014	696.2606
0	-2.91589321	0.848988348	0.446897907	1.1407213	0.324421315	0.182549259	0.211585475	-0.359844634	0.006284	5.063388	705.4

3.6 Checking Discriminatory Power

PEAKS2TAILS

Cut-off 1

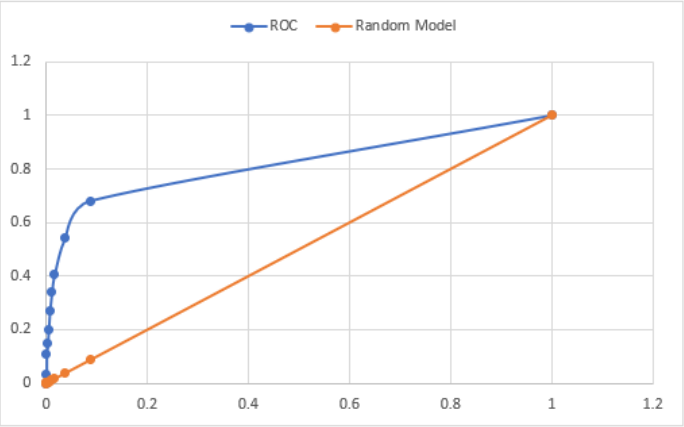
Discrimination

We will test the discriminatory power of the Model on Testing data set
Most popular metric in this regard are Accuracy ratios and Gini coefficients

		Predicted	
		1	0
Actual	1	0	423
	0	0	7278

TPR	0
SPC	1
FPR	0
ACC	0.945072

Default	bureau_score	max_arrears_12m	cc_util	annual_income	num_ccj	emp_length	months_since_recent_cc_delinq	P(1)	Default (Pred)
0	1.075105467	0.446897907	1.14072133	0.659648047	0.182549259	-0.382216378	1.036190088	0.002379	0
0	0.848988348	0.446897907	-0.739992552	0.659648047	0.182549259	0.439573656	-0.379586857	0.024583	0
1	-0.684965808	-2.643118019	-0.739992552	-0.641267459	-0.840719323	-0.382216378	-0.379586857	0.784928	0
0	0.848988348	0.446897907	-1.912944649	0.659648047	0.182549259	0.439573656	-0.359844634	0.068721	0
0	-0.100424502	-1.708298136	1.14072133	0.324421315	0.182549259	0.519371295	-0.379586857	0.054969	0
0	0.848988348	0.446897907	1.14072133	-0.641267459	0.182549259	-0.860408385	1.036190088	0.009914	0
0	-0.100424502	0.446897907	1.14072133	0.659648047	-0.840719323	0.627726273	-0.359844634	0.008739	0
0	1.075105467	0.446897907	1.14072133	0.324421315	0.182549259	0.519371295	1.036190088	0.002344	0
0	0.848988348	0.446897907	-3.025007735	0.659648047	0.182549259	0.211585475	-0.379586857	0.183836	0
0	-1.178027211	0.446897907	-3.025007735	0.659648047	0.182549259	0.520370796	-0.379586857	0.432325	0
0	-0.684965808	0.446897907	1.14072133	-0.641267459	0.182549259	-0.655185064	-0.379586857	0.051008	0
0	0.848988348	0.446897907	1.14072133	0.659648047	0.182549259	0.520370796	-0.359844634	0.004245	0
0	1.075105467	0.446897907	1.14072133	0.659648047	0.182549259	0.211585475	1.036190088	0.001947	0
0	-1.178027211	0.446897907	1.14072133	0.324421315	0.182549259	-0.860408385	1.036190088	0.01603	0
0	1.075105467	0.446897907	1.14072133	0.659648047	0.182549259	-0.306787205	-0.359844634	0.00484	0
0	0.848988348	0.446897907	-1.912944649	0.324421315	0.182549259	0.519371295	-0.359844634	0.087598	0
0	-0.684965808	0.446897907	1.14072133	0.324421315	0.182549259	-0.180192003	1.036190088	0.009298	0
0	-0.100424502	0.446897907	1.14072133	-0.641267459	0.182549259	-0.306787205	1.036190088	0.015196	0
0	-0.684965808	-2.643118019	1.14072133	0.324421315	-0.840719323	-0.306787205	1.036190088	0.113557	0
0	1.075105467	0.446897907	-0.739992552	0.324421315	0.182549259	0.519371295	-0.379586857	0.027514	0
0	0.848988348	0.446897907	1.14072133	-0.641267459	-0.840719323	-0.382216378	1.036190088	0.009739	0
0	1.075105467	0.446897907	1.14072133	0.324421315	0.182549259	0.519371295	-0.379586857	0.004945	0
0	0.848988348	0.446897907	1.14072133	0.659648047	0.182549259	0.88879147	1.036190088	0.001795	0
0	-0.100424502	0.446897907	1.14072133	0.659648047	0.182549259	0.519371295	-0.359844634	0.00786	0
0	-0.232100523	0.446897907	1.14072133	0.324421315	-0.232100523	-0.655185064	-0.359844634	0.017863	0
0	-0.232100523	0.446897907	1.14072133	-0.641267459	-0.232100523	-0.180192003	1.036190088	0.016789	0
0	-0.100424502	0.446897907	-0.739992552	0.324421315	-0.840719323	-0.860408385	-0.379586857	0.100795	0
0	1.075105467	0.446897907	-0.739992552	0.324421315	0.182549259	-0.306787205	-0.359844634	0.035693	0
0	-0.100424502	0.446897907	1.14072133	0.324421315	0.182549259	-0.180192003	-0.359844634	0.013235	0
0	-0.100424502	0.446897907	1.14072133	-0.641267459	0.182549259	-0.306787205	1.036190088	0.015196	0
0	1.075105467	0.446897907	-1.912944649	-0.641267459	0.182549259	-0.306787205	-0.379586857	0.203359	0
0	-0.100424502	0.446897907	1.14072133	0.324421315	0.182549259	0.520370796	-0.359844634	0.010475	0
0	-0.100424502	0.446897907	1.14072133	0.324421315	0.182549259	-0.306787205	1.036190088	0.006645	0



AUC	0.811977
Gini	0.623953

Cutoff	TPR	FPR	Random		area
0	1	1	1		0.76716276
0.1	0.6809	0.0872	0.0872		0.0305575
0.2	0.5414	0.0372	0.0372		0.0098118
0.3	0.4066	0.0165	0.0165		0.00205315
0.4	0.34	0.011	0.011		0.001219
0.5	0.2695	0.007	0.007		0.000538315
0.6	0.1986	0.0047	0.0047		0.000364875
0.7	0.1489	0.0026	0.0026		0.000242535
0.8	0.1064	0.0007	0.0007		0.000020565
0.9	0.0307	0.0004	0.0004		0.00000614
1	0	0	0	Total	0.81197664

3.7 Bayes Decision Rule

PEAKS2TAILS

Bayes Decision Rule

Prior Class Probabilities

P(0) = 0.5
P(1) = 0.5

Cost Matrix

State	Prediction 0	Prediction 1
0	-2	5
1	6	0

Class Distributions

	μ	σ
H1 :	65	8
H0 :	55	5

Intuitively, Our decision rule will be of the form

$$\delta(X) = \begin{cases} 1, & X >= t \\ 0, & X < t \end{cases}$$

where t is the threshold or cut-off

Loss

Lij = Loss associated with choosing class I when true state of nature is j

Conditional Risks

R (0) = Expected Loss if Class 0 is chosen
R (1) = Expected Loss if Class 1 is chosen

$$R(0) = P(0 \mid X) L_{00} + P(1 \mid X) L_{01} = P(0) P(X \mid 0) L_{00} + P(1) P(X \mid 1) L_{01}$$
$$R(1) = P(0 \mid X) L_{10} + P(1 \mid X) L_{11} = P(0) P(X \mid 0) L_{10} + P(1) P(X \mid 1) L_{11}$$

P (1 | X) and P (0 | X) are posterior probabilities
and proportional to Likelihood x Prior

Bayesian Decision Rule , choose the action that minimizes Risk

i.e. if R (0) < R(1) , choose **0** or else **1**

Bayesian Decision Rule

Choose 0 if , R (0) < R(1)
R (0) is the expected loss when 0 is chosen and R (1) is the expected loss when 1 is chosen

R (0) =	$P(0) P(t \mid 0) L_{00} + P(1) P(t \mid 1) L_{01}$	**0.094898**	R (0) - R (1)
R (1) =	$P(0) P(t \mid 0) L_{10} + P(1) P(t \mid 1) L_{11}$	**0.094818**	8.01E-05

t = 61.09798

Use Goal Seek for t
so that R (0) = R (1)

Bayes Decision Rule :

$$\delta(X) = \begin{cases} 1, & X >= 61.1 \\ 0, & X < 61.1 \end{cases}$$

3.8 Minimax Decision Rule

PEAKS2TAILS

Minimax Decision Rule (Prior Probabilities are not known)

What if priors are not known ?

Minimax will sweep through all priors and
calculate maximum risk of an action
and choose the action that has lowest maximum risk

Cost Matrix

State	Prediction 0	Prediction 1
0	-2	5
1	6	0

Class Distributions

	μ	σ
H1 :	65	8
H0 :	55	5

Intuitively, Our decision rule will be of the form

$$\delta(X) = \begin{cases} 1, & X >= t \\ 0, & X < t \end{cases}$$

where t is the threshold or cut-off

Loss

Lij = Loss associated with choosing class I when true state of nature is j

Conditional Risks

R (0) = Expected Loss if Class 0 is chosen
R (1) = Expected Loss if Class 1 is chosen

$$R(0) = P(0 \mid X)\, L_{00} + P(1 \mid X)\, L_{01} = P(0)\, P(X \mid 0)\, L_{00} + P(1)\, P(X \mid 1)\, L_{01}$$
$$R(1) = P(0 \mid X)\, L_{10} + P(1 \mid X)\, L_{11} = P(0)\, P(X \mid 0)\, L_{10} + P(1)\, P(X \mid 1)\, L_{11}$$

P (1 | X) and P (0 | X) are posterior probabilities
and proportional to Likelihood x Prior

Minimax decision rule

For all possible Priors, calculate $R(0)_{max}$ and $R(1)_{max}$ and choose the lower between these

Minimax Decision Rule

P(0) = 1 P(1) = 0

For a Person with Weight = 61

$R(0)_{max}$ = -0.078
$R(1)_{max}$ = 0.1942

Solve using Excel Solver
one at a time with DV = P(0)

$R(0)_{max}$	$R(1)_{max}$
0.264	0.1942

Choose 1

3.9 Neyman Pearson Rule

PEAKS2TAILS

Minimax Decision Rule (Prior Probabilities are not known)

What if priors are not known ?

Minimax will sweep through all priors and
calculate maximum risk of an action
and choose the action that has lowest maximum risk

Cost Matrix

		Prediction	
		0	**1**
State	**0**	-2	5
	1	6	0

Class Distributions

	μ	σ
H1 :	65	8
H0 :	55	5

Intuitively, Our decision rule will be of the form

$$\delta(X) = \begin{cases} 1, & X >= t \\ 0, & X < t \end{cases}$$

where t is the threshold or cut-off

Loss

Lij = Loss associated with choosing class I when true state of nature is j

Conditional Risks

R (0) = Expected Loss if Class 0 is chosen
R (1) = Expected Loss if Class 1 is chosen

$$R(0) = P(0 \mid X)\, L_{00} + P(1 \mid X)\, L_{01} = P(0)\, P(X \mid 0)\, L_{00} + P(1)\, P(X \mid 1)\, L_{01}$$
$$R(1) = P(0 \mid X)\, L_{10} + P(1 \mid X)\, L_{11} = P(0)\, P(X \mid 0)\, L_{10} + P(1)\, P(X \mid 1)\, L_{11}$$

P (1 | X) and P (0 | X) are posterior probabilities
and proportional to Likelihood x Prior

Minimax decision rule

For all possible Priors, calculate $R(0)_{max}$ and $R(1)_{max}$ and choose the lower between these

Minimax Decision Rule

P(0) = 1 P(1) = 0

For a Person with Weight = 61

$R(0)_{max}$ = -0.078
$R(1)_{max}$ = 0.1942

Solve using Excel Solver
one at a time with DV = P(0)

$R(0)_{max}$	$R(1)_{max}$
0.264	0.1942

Choose 1

4.1 Variable Clustering PCA

Extracting the First Principal Component

-0.4014201	0.6573516	0.145825687	-0.337163677	0.88646318	-0.56981581	-0.230684114	0.84753852	-0.248294399
X1	X2	X3	X4	X5	X6	X7	X8	X9
calories	protein	fat	sodium	fiber	carbo	sugars	potass	vitamins
-1.89	1.33	-0.01	-0.35	3.29	-2.24	-0.21	2.58	-0.15
0.67	0.42	3.96	-1.73	-0.06	-1.54	0.24	0.55	-1.26
-1.89	1.33	-0.01	1.20	2.87	-1.78	-0.43	3.14	-0.15
-2.92	1.33	-1.01	-0.23	4.97	-1.54	-1.56	3.28	-0.15
0.16	-0.50	0.98	0.48	-0.48	-0.14	0.24	-1.36	-0.15
0.16	-0.50	0.98	0.24	-0.27	-0.96	0.69	-0.37	-0.15
0.16	-0.50	-1.01	-0.41	-0.48	-0.84	1.59	-0.93	-0.15
1.19	0.42	0.98	0.60	-0.06	0.80	0.24	0.06	-0.15
-0.87	-0.50	-0.01	0.48	0.78	0.09	-0.21	0.41	-0.15
-0.87	0.42	-1.01	0.60	1.19	-0.37	-0.43	1.32	-0.15
0.67	-1.41	0.98	0.72	-0.90	-0.61	1.14	-0.86	-0.15
0.16	3.16	0.98	1.55	-0.06	0.56	-1.33	0.13	-0.15
0.67	-1.41	1.97	0.60	-0.90	-0.37	0.47	-0.72	-0.15
0.16	0.42	0.98	-0.23	-0.06	-0.37	0.02	0.13	-0.15
0.16	-1.41	-0.01	0.24	-0.90	-0.61	1.37	-0.58	-0.15
0.16	-0.50	-1.01	1.44	-0.90	1.73	-0.88	-1.00	-0.15
-0.35	-0.50	-1.01	1.55	-0.48	1.50	-1.11	-0.86	-0.15
0.16	-1.41	-1.01	-0.83	-0.48	-0.37	1.14	-1.07	-0.15
0.16	-1.41	-0.01	0.24	-0.90	-0.61	1.37	-0.44	-0.15
0.16	0.42	1.97	-0.23	0.78	-1.07	0.02	0.90	-0.15
-0.35	0.42	-1.01	-0.95	-0.48	1.50	-1.56	-1.36	-1.26
0.16	-0.50	-1.01	0.72	-0.48	1.50	-0.88	-0.93	-0.15
-0.35	-0.50	-0.01	-0.23	-0.06	-0.84	0.69	0.34	-0.15
-0.35	-0.50	-1.01	0.36	-0.48	0.80	-0.43	-0.23	-0.15
0.16	-0.50	-0.01	-0.41	-0.48	-0.84	1.37	-0.93	-0.15
0.16	-1.41	-1.01	0.48	-0.48	-0.14	0.92	-1.00	-0.15
-0.35	0.42	-1.01	-1.90	0.36	-0.14	0.02	0.06	-0.15
0.67	0.42	0.98	0.00	1.19	-0.61	0.69	1.46	-0.15
0.67	0.42	-1.01	0.96	1.19	-0.14	1.14	1.32	-0.15

(Use Excel Solver)

(9 x 1)

	w		Xw
	-0.25		5.07
	0.41		1.67
	0.09		4.68
	-0.21		6.44
X	0.55	=	-1.19
	-0.35		-0.28
	-0.14		-0.90
	0.52		-0.46
	-0.15		0.57
			1.72
			-1.67
			1.05
			-1.47
			0.45
			-1.42
			-2.11
			-1.59
			-1.36
			-1.34
			1.64
			-0.72
			-1.61
			0.29
			-0.86
			-0.78
			-1.65
			0.85
			1.64
			0.96

Maximize	203.0634564
Variance Explained	29.30%

4.2 Variable Clustering PCA

Extracting the Second Principal Component

0.804642895	0.1558846	0.7146273	0.314039572	0.0828767	-0.2152934	0.7051399	0.31898341	0.328882718
X1	X2	X3	X4	X5	X6	X7	X8	X9
calories	protein	fat	sodium	fiber	carbo	sugars	potass	vitamins
-1.89	1.33	-0.01	-0.35	3.29	-2.24	-0.21	2.58	-0.15
0.67	0.42	3.96	-1.73	-0.06	-1.54	0.24	0.55	-1.26
-1.89	1.33	-0.01	1.20	2.87	-1.78	-0.43	3.14	-0.15
-2.92	1.33	-1.01	-0.23	4.97	-1.54	-1.56	3.28	-0.15
0.16	-0.50	0.98	0.48	-0.48	-0.14	0.24	-1.36	-0.15
0.16	-0.50	0.98	0.24	-0.27	-0.96	0.69	-0.37	-0.15
0.16	-0.50	-1.01	-0.41	-0.48	-0.84	1.59	-0.93	-0.15
1.19	0.42	0.98	0.60	-0.06	0.80	0.24	0.06	-0.15
-0.87	-0.50	-0.01	0.48	0.78	0.09	-0.21	0.41	-0.15
-0.87	0.42	-1.01	0.60	1.19	-0.37	-0.43	1.32	-0.15
0.67	-1.41	0.98	0.72	-0.90	-0.61	1.14	-0.86	-0.15
0.16	3.16	0.98	1.55	-0.06	0.56	-1.33	0.13	-0.15
0.67	-1.41	1.97	0.60	-0.90	-0.37	0.47	-0.72	-0.15
0.16	0.42	0.98	-0.23	-0.06	-0.37	0.02	0.13	-0.15
0.16	-1.41	-0.01	0.24	-0.90	-0.61	1.37	-0.58	-0.15
0.16	-0.50	-1.01	1.44	-0.90	1.73	-0.88	-1.00	-0.15
-0.35	-0.50	-1.01	1.55	-0.48	1.50	-1.11	-0.86	-0.15
0.16	-1.41	-1.01	-0.83	-0.48	-0.37	1.14	-1.07	-0.15
0.16	-1.41	-0.01	0.24	-0.90	-0.61	1.37	-0.44	-0.15
0.16	0.42	1.97	-0.23	0.78	-1.07	0.02	0.90	-0.15
-0.35	0.42	-1.01	-0.95	-0.48	1.50	-1.56	-1.36	-1.26
0.16	-0.50	-1.01	0.72	-0.48	1.50	-0.88	-0.93	-0.15
-0.35	-0.50	-0.01	-0.23	-0.06	-0.84	0.69	0.34	-0.15
-0.35	-0.50	-1.01	0.36	-0.48	0.80	-0.43	-0.23	-0.15
0.16	-0.50	-0.01	-0.41	-0.48	-0.84	1.37	-0.93	-0.15
0.16	-1.41	-1.01	0.48	-0.48	-0.14	0.92	-1.00	-0.15
-0.35	0.42	-1.01	-1.90	0.36	-0.14	0.02	0.06	-0.15
0.67	0.42	0.98	0.00	1.19	-0.61	0.69	1.46	-0.15
0.67	0.42	-1.01	0.96	1.19	-0.14	1.14	1.32	-0.15

(Use Excel Solver)

(9 x 1)

	w		Xw
	1.000		-0.06
	0.194		3.92
	0.888		0.40
	0.390		-2.85
X	0.103	=	0.72
	-0.268		1.66
	0.876		0.15
	0.396		2.33
	0.409		-0.81
			-1.14
			2.22
			0.91
			2.46
			1.12
			0.95
			-2.05
			-2.56
			-0.76
			1.01
			2.58
			-4.41
			-2.20
			0.35
			-1.99
			0.84
			-0.48
			-1.86
			3.04
			1.86

Maximize	155.15		
Variance	22.4%		
Explained			
Constraint : w1.w2 = 0	-1.71191E-05	=	0

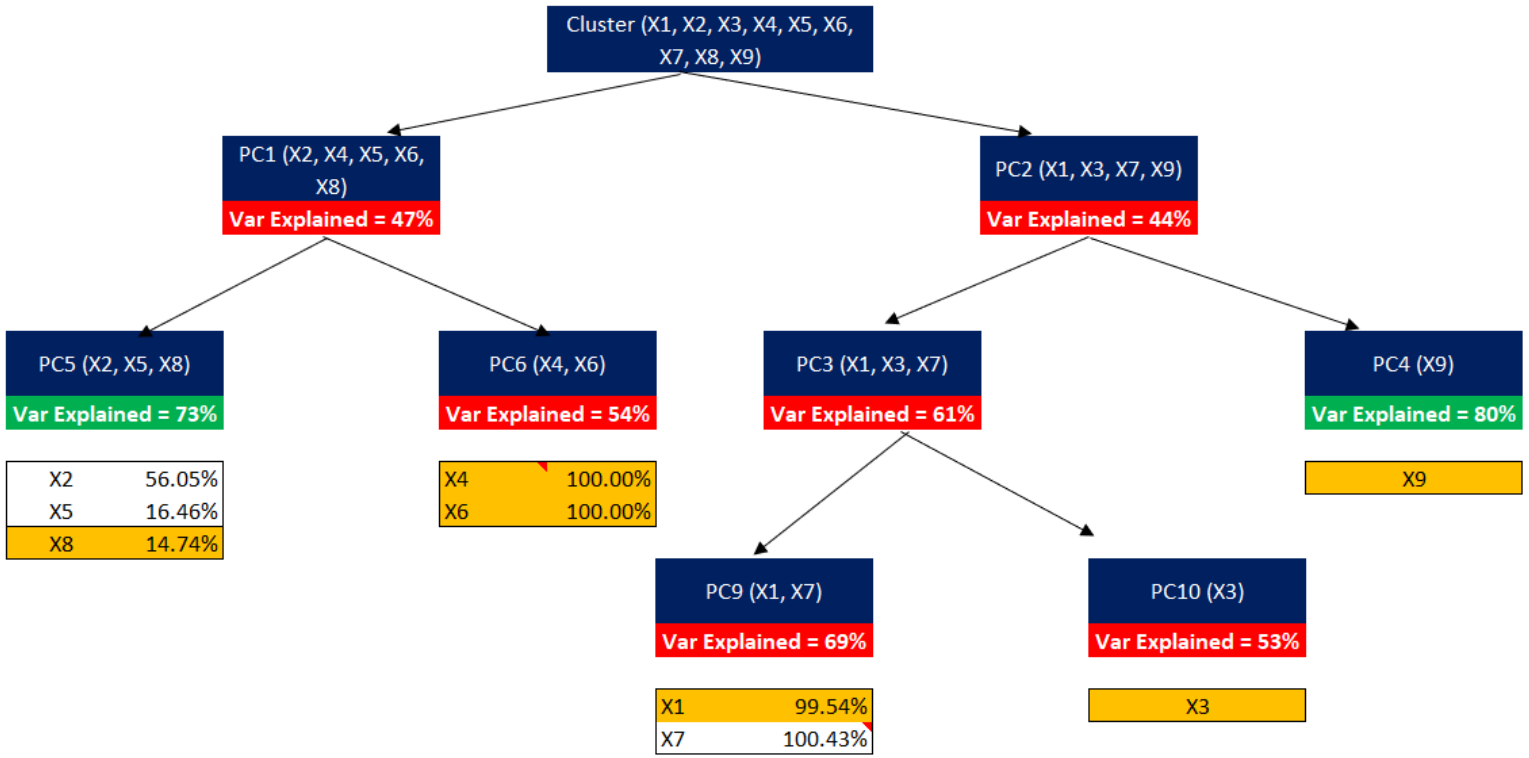
	1	2	Assigned to	1.00	2.00
X1	0.40	0.80	2		0.65
X2	0.66	0.16	1	0.43	
X3	0.15	0.71	2		0.51
X4	0.34	0.31	1	0.11	
X5	0.89	0.08	1	0.79	
X6	0.57	0.22	1	0.32	
X7	0.23	0.71	2		0.50
X8	0.85	0.32	1	0.72	
X9	0.25	0.33	2		0.11
				2.37	1.76
				0.47	0.44

Absolute Correlation with first principal component

Absolute Correlation with second principal component

A low value indicates Heterogeneous group

4.3 Variable Clustering PCA



5.1 Reject Inference KGB Model

customer	age	ed	employ	address	income	debtinc	creddebt	othdebt	default	Y	Sigmoid	Likelihood	LL	Default,P	Score
10012	28	2	7	2	44	17.7	2.99	4.8	0	0.477332	0.617118	0.38288232	-0.9600276	1	473.3501
10017	64	5	34	17	116	14.7	5.05	12	0	-5.91126	0.002701	0.99729853	-0.0027051	0	657.6858
10030	40	1	20	12	61	4.8	1.04	1.89	0	-5.19604	0.005508	0.99449205	-0.0055232	0	637.049
10039	30	1	11	3	27	34.5	1.75	7.56	0	0.253566	0.563054	0.43694609	-0.8279455	1	479.8066
10069	25	1	2	2	30	22.4	0.76	5.96	1	0.865219	0.70375	0.70374982	-0.3513324	1	462.1581
10071	35	1	2	9	38	10.9	1.46	2.68	1	-0.16042	0.45998	0.45997985	-0.7765726	1	491.7519
10096	26	3	2	4	38	11.9	0.95	3.57	1	-0.07013	0.482476	0.48247553	-0.7288251	1	489.1464
10128	25	1	4	2	30	14.4	1.05	3.27	0	-0.10957	0.472635	0.52736454	-0.6398632	1	490.2845
10129	65	4	29	14	189	5	3.36	6.09	0	-6.70121	0.001228	0.99877209	-0.0012287	0	680.4791
10140	21	3	0	0	23	3.9	0.31	0.59	0	-0.39732	0.401957	0.59804339	-0.514092	1	498.5872
10142	62	1	33	13	96	17.5	6.69	10.11	0	-4.25269	0.014026	0.98597359	-0.0141257	0	609.8296
10169	30	4	4	3	39	10.6	2.39	1.74	1	0.079721	0.51992	0.51991961	-0.6540811	1	484.8227
10197	38	5	7	7	124	19.1	9.52	14.16	0	3.823748	0.978621	0.02137873	-3.8453589	1	376.7929
10200	18	2	0	0	35	3.9	0.17	1.19	0	-0.41803	0.396989	0.60301135	-0.5058193	0	499.1847
10218	53	1	9	13	41	13.3	2.33	3.12	0	-1.49529	0.183129	0.81687078	-0.2022744	0	530.2679
10234	18	2	0	0	15	7.4	0.83	0.28	1	0.249435	0.562038	0.56203751	-0.5761867	1	479.9258
10351	18	2	0	0	56	15.7	1.84	6.95	1	1.514771	0.819767	0.8197672	-0.1987349	1	443.4159
10423	19	1	2	0	31	0.3	0.02	0.07	1	-1.23438	0.225417	0.22541653	-1.4898054	0	522.7395
10428	19	2	0	0	26	5.8	0.59	0.92	1	-0.03572	0.49107	0.49106993	-0.7111687	1	488.1538
10449	56	2	27	21	125	8.5	2.9	7.72	0	-6.09442	0.00225	0.99774966	-0.0022529	0	662.9708
10455	21	2	1	1	40	7.6	1.52	1.52	1	0.303825	0.575377	0.5753772	-0.5527294	1	478.3565
10474	25	1	8	3	37	8.3	0.95	2.12	0	-1.62353	0.164718	0.83528187	-0.179986	0	533.9683
10525	47	3	8	10	43	6.1	0.36	2.26	0	-2.79861	0.057399	0.9426009	-0.0591123	0	567.8739
10564	40	4	17	11	103	13.3	10.41	3.29	1	1.23223	0.774209	0.7742087	-0.2559138	1	451.5683
10567	41	2	16	11	56	3.4	0.9	1.01	0	-4.49501	0.011041	0.9889587	-0.0111027	0	616.8216
10746	23	4	0	2	118	6.4	0.29	7.26	0	-0.43671	0.392525	0.60747549	-0.4984434	0	499.7239
10775	45	5	1	4	58	15.4	2.37	6.57	1	0.892811	0.70947	0.70946989	-0.3432372	1	461.3619
10776	37	3	15	10	65	5	1.3	1.95	0	-3.85208	0.020794	0.97920609	-0.0210132	0	598.2706
10797	47	2	23	14	46	11.7	0.84	4.54	0	-5.55493	0.003853	0.99614655	-0.0038609	0	647.4043
10798	23	4	0	2	42	6.7	1.58	1.23	0	0.354067	0.587604	0.41239648	-0.8857701	1	476.9068
10800	18	2	0	0	19	5.2	0.32	0.67	1	-0.20441	0.449076	0.44907555	-0.8005641	1	493.0209
10802	19	2	0	0	21	2	0.28	0.14	1	-0.50728	0.375832	0.37583225	-0.9786124	0	501.7599
10805	41	1	3	12	23	10.6	0.15	2.29	1	-1.31623	0.211447	0.21144669	-1.5537824	0	525.1013
10824	42	2	3	12	76	5.2	0.72	3.23	1	-1.57755	0.171143	0.17114286	-1.7652566	0	532.6414
10839	29	4	2	3	45	3	0.67	0.68	0	-1.04152	0.260858	0.73914243	-0.3022646	0	517.1748
10870	24	4	0	2	41	8.4	0.79	2.66	0	0.069641	0.517403	0.48259682	-0.7285737	1	485.1136
10884	45	5	9	11	48	10.2	3.53	1.37	0	-1.04415	0.260351	0.73964937	-0.301579	0	517.2507
10887	39	4	8	9	259	7.2	1.55	17.1	0	-2.11689	0.107466	0.89253419	-0.1136905	0	548.2036
10893	40	2	15	6	46	10.6	0.82	4.06	0	-3.44406	0.030947	0.96905342	-0.0314355	0	586.4975
10927	21	3	0	0	17	17.6	1.37	1.62	1	1.289235	0.784018	0.78401763	-0.2433238	1	449.9236
10935	55	3	34	16	150	9.9	0.73	14.12	0	-8.49228	0.000205	0.999795	-0.000205	0	732.1583

b0	b1	b2	b3	b4	b5	b6	b7	b8	Sum
-0.4149156	-0.01531098	-0.02847339	-0.225346	-0.0386646	-0.00275587	0.076746213	0.557631	0.026705	-408.
			-0.6402615					-0.38821	
0.3977341			0.34518742					0.404148	
			-5.255%					0.641%	

LB	UB	Disb. Bads	Disb. Goods	Disb Bads %	Disb Goods %
0.0%	10.0%	7	196	3%	97%
10.0%	20.0%	14	94	13%	87%
20.0%	30.0%	28	90	24%	76%
30.0%	40.0%	49	55	47%	53%
40.0%	50.0%	39	58	40%	60%
50.0%	60.0%	38	34	53%	47%
60.0%	70.0%	33	26	56%	44%
70.0%	80.0%	38	12	76%	24%
80.0%	90.0%	37	5	88%	12%
90.0%	100.0%	46	1	98%	2%

5.2 Hard Cut-Off

M2 =IF(L2>493,0,1)

customer	age	ed	employ	address	income	debtinc	creddebt	othdebt	Y	Prob	Score	Flag
360036	24	4	0	1	56	9.3	1.55	3.66	0.586544	0.642572	470.1989	1
360188	19	2	0	0	55	7.9	0.61	3.73	0.131718	0.532882	483.3224	1
360230	28	4	2	3	22	6.2	0.76	0.61	-0.66891	0.33874	506.4238	0
360245	40	5	1	11	32	18.6	3.4	2.55	1.482958	0.815019	444.3339	1
360349	33	5	6	4	121	15.1	3.31	14.96	0.501401	0.622789	472.6556	1
360448	34	3	0	4	23	6.8	0.69	0.88	-0.30881	0.423405	496.0334	0
360449	19	2	0	0	30	4	0.52	0.68	-0.23033	0.44267	493.769	0
360504	44	1	17	14	107	13	3.83	10.08	-2.38152	0.084593	555.8391	0
360509	29	2	1	6	36	9.3	1.42	1.93	0.084692	0.52116	484.6793	1
360518	65	1	10	18	32	9.9	1.37	1.8	-2.9044	0.051936	570.9263	0
360585	45	2	18	11	59	8.9	2.74	2.51	-3.52701	0.028553	588.891	0
360684	33	2	4	8	79	12.6	8.55	1.4	3.366597	0.966644	389.9835	1
360740	32	3	3	5	34	12.4	2.18	2.03	0.268153	0.56664	479.3857	1
360763	32	4	5	6	67	8.4	3.38	2.25	0.027428	0.506856	486.3316	1
360917	23	3	0	1	34	9.2	1.25	1.87	0.468191	0.614955	473.6139	1
360925	25	5	0	2	60	8.4	1.25	3.79	0.260182	0.564681	479.6157	1
360927	22	4	0	1	23	6.2	0.35	1.08	-0.26786	0.433432	494.8518	0
360963	27	1	6	2	20	14.3	0.38	2.48	-0.96571	0.275737	514.9874	0
360968	43	4	0	10	94	19.9	6.32	12.38	3.549213	0.972056	384.7144	1
360970	53	4	26	20	249	5.8	7.58	6.86	-3.80362	0.021804	596.8722	0
361058	24	3	2	3	22	5.6	0.08	1.15	-0.99001	0.270909	515.6888	0
361072	43	2	13	13	62	24.2	8.07	6.93	1.809174	0.859262	434.9213	1
361083	25	1	4	1	33	13.2	1.63	2.73	0.137739	0.53438	483.1487	1
361101	26	3	0	3	24	32	4.42	3.26	3.927112	0.98068	373.8105	1
361105	18	2	0	0	34	16.3	1.87	3.67	1.550583	0.824998	442.3826	1
361146	46	4	14	10	57	4.7	0.25	2.43	-4.36668	0.012534	613.1187	0
361166	46	4	22	10	160	15.5	9.92	14.88	0.100333	0.525062	484.228	1
361198	52	2	22	17	87	11.3	0.71	9.12	-5.616	0.003626	649.1665	0
361201	35	3	3	7	54	11	1.16	4.78	-0.51301	0.374487	501.9255	0
361255	23	3	1	2	63	16.8	3.14	7.44	1.910203	0.871042	432.0062	1
361267	33	1	7	5	33	6.9	0.68	1.6	-1.85887	0.134834	540.7587	0
361299	18	2	0	0	28	8.1	0.84	1.42	0.303352	0.575262	478.3701	1
361305	31	2	11	8	37	5.7	0.82	1.28	-2.9077	0.051774	571.0214	0
361346	30	3	4	3	62	14.8	1.53	7.64	0.045142	0.511284	485.8205	1
361396	23	3	2	2	32	3	0.28	0.68	-1.06416	0.256515	517.8282	0
361437	42	2	1	5	50	9.6	2.6	2.2	0.573971	0.639679	470.5617	1
361440	21	2	0	1	21	6.3	0.83	0.49	0.06949	0.517365	485.1179	1
361454	35	2	15	8	55	10.2	3.37	2.24	-2.12698	0.106502	548.4946	0
361487	26	2	5	1	47	6.2	1.83	1.09	-0.63947	0.345367	505.5741	0
361612	75	4	50	18	460	19.6	26.69	63.47	3.1743	0.959856	395.5321	1
361630	22	3	0	2	32	3.7	0.31	0.87	-0.52263	0.372237	502.203	0

5.3 Parcelling

custom	age	ed	emplo	addre	incom	debtin	credde	othde	Y	Prob	Scor	Parcels	infer	Orde	Flag
360504	44	1	17	14	107	13	3.83	10.08	-2.38152	0.084593	555.8391	Parcel 1	5	1	1
360518	65	1	10	18	32	9.9	1.37	1.8	-2.9044	0.051936	570.9263	Parcel 1	5	2	1
360585	45	2	18	11	59	8.9	2.74	2.51	-3.52701	0.028553	588.891	Parcel 1	5	3	1
360970	53	4	26	20	249	5.8	7.58	6.86	-3.80362	0.021804	596.8722	Parcel 1	5	4	1
361146	46	4	14	10	57	4.7	0.25	2.43	-4.36668	0.012534	613.1187	Parcel 1	5	5	1
361198	52	2	22	17	87	11.3	0.71	9.12	-5.616	0.003626	649.1665	Parcel 1	5	6	0
361305	31	2	11	8	37	5.7	0.82	1.28	-2.9077	0.051774	571.0214	Parcel 1	5	7	0
361845	38	2	18	7	53	5.2	1.2	1.55	-4.41699	0.011927	614.5704	Parcel 1	5	8	0
362711	63	1	17	26	103	12.3	3.47	9.2	-3.40335	0.032191	585.3229	Parcel 1	5	9	0
362745	31	3	7	4	30	1.9	0.17	0.4	-2.53844	0.073207	560.3668	Parcel 1	5	10	0
362909	45	2	24	10	86	9.7	1.74	6.6	-5.30184	0.004958	640.1017	Parcel 1	5	11	0
363019	65	1	21	23	42	1.6	0.29	0.38	-6.88125	0.001026	685.6738	Parcel 1	5	12	0
363544	43	4	13	12	127	3	0.42	3.39	-4.37567	0.012423	613.3783	Parcel 1	5	13	0
363832	54	1	13	18	105	13.6	1.63	12.65	-2.8945	0.052426	570.6405	Parcel 1	5	14	0
363882	42	4	9	9	49	7.3	1.15	2.43	-2.41659	0.081917	556.8509	Parcel 1	5	15	0
364013	55	5	21	14	91	6.6	2.01	3.99	-5.18982	0.005542	636.8696	Parcel 1	5	16	0
364071	50	4	27	12	181	2.3	1.79	2.37	-7.10352	0.000822	692.0872	Parcel 1	5	17	0
364304	39	4	9	12	78	4.1	0.35	2.85	-3.24704	0.037433	580.8129	Parcel 1	5	18	0
365110	47	2	10	10	39	6.7	0.22	2.39	-3.23836	0.037747	580.5623	Parcel 1	5	19	0
365236	67	2	7	25	58	11.4	1.59	5.03	-2.30571	0.090651	553.6516	Parcel 1	5	20	0
365265	51	2	22	11	65	18.1	4.91	6.86	-2.5045	0.075543	559.3876	Parcel 1	5	21	0
365285	45	2	26	12	100	7.5	1.7	5.81	-6.08069	0.002281	662.5745	Parcel 1	5	22	0
365291	46	2	23	14	99	3.2	1.81	1.36	-5.88204	0.002781	656.8428	Parcel 1	5	23	0
365477	51	3	10	17	144	4.9	0.92	6.14	-3.53575	0.028312	589.1432	Parcel 1	5	24	0
365536	62	1	32	19	86	6.3	3.97	1.45	-6.83935	0.00107	684.465	Parcel 1	5	25	0
365613	76	3	12	27	53	6.2	1.28	2.01	-4.31485	0.013192	611.6234	Parcel 1	5	26	0
365683	26	1	10	4	43	5.2	0.89	1.35	-2.43667	0.080419	557.4304	Parcel 1	5	27	0
365743	36	2	5	9	50	0.8	0.12	0.28	-2.49977	0.075874	559.2512	Parcel 1	5	28	0
365817	42	2	17	7	66	8.7	1.73	4.01	-3.65886	0.025115	592.6954	Parcel 1	5	29	0
366109	35	1	16	6	93	4.7	2.86	1.51	-3.07723	0.044056	575.9132	Parcel 1	5	30	0
366365	70	2	33	17	150	13.3	5.17	14.78	-5.75234	0.003165	653.1005	Parcel 1	5	31	0
366410	40	1	17	5	72	7.3	0.95	4.31	-4.07336	0.016735	604.6553	Parcel 1	5	32	0
366563	33	3	12	9	48	3.7	0.93	0.84	-3.36502	0.033407	584.217	Parcel 1	5	33	0
366610	32	2	12	9	27	1.8	0.16	0.32	-3.85245	0.020787	598.2811	Parcel 1	5	34	0
366658	41	2	21	9	56	4.8	1.15	1.54	-5.2834	0.005049	639.5698	Parcel 1	5	35	0
367033	64	1	36	15	145	6.6	3.27	6.3	-8.01709	0.00033	718.4474	Parcel 1	5	36	0
367082	50	2	21	10	88	1.4	0.21	1.02	-6.34705	0.001749	670.2602	Parcel 1	5	37	0
367368	27	2	9	3	31	6.3	0.27	1.69	-2.4356	0.080498	557.3997	Parcel 1	5	38	0
367383	67	1	40	24	88	6.3	1.42	4.12	-10.2682	3.47E-05	783.3997	Parcel 1	5	39	0
367435	36	4	12	9	35	8.1	1.05	1.78	-2.9739	0.048619	572.9316	Parcel 1	5	40	0
367497	46	1	20	14	48	2.7	0.94	0.36	-5.5872	0.003732	648.3354	Parcel 1	5	41	0

LB	UB	Parcel	Disb. Bads	Disb. Goods	Total Rejects	Inferred Bads	Inferred Goods	Parceling Inferred Bads
0.0%	10.0%	Parcel 1	3.4%	96.6%	125	5	120	5
10.0%	20.0%	Parcel 2	13.0%	87.0%	75	10	65	10
20.0%	30.0%	Parcel 3	23.7%	76.3%	71	17	54	17
30.0%	40.0%	Parcel 4	47.1%	52.9%	78	37	41	37
40.0%	50.0%	Parcel 5	40.2%	59.8%	77	31	46	31
50.0%	60.0%	Parcel 6	52.8%	47.2%	66	35	31	35
60.0%	70.0%	Parcel 7	55.9%	44.1%	30	17	13	17
70.0%	80.0%	Parcel 8	76.0%	24.0%	27	21	6	21
80.0%	90.0%	Parcel 9	88.1%	11.9%	29	26	3	26
90.0%	100.0%	Parcel 10	97.9%	2.1%	22	22	0	22

5.4 Fuzzy Augmentation

customer	age	ed	employ	address	income	debtinc	creddebt	othdebt	Y	Prob	Parcels	Flag	Weight
360036	24	4	0	1	56	9.3	1.55	3.66	0.586544	0.642572	Parcel 7	0	0.440678
360036	24	4	0	1	56	9.3	1.55	3.66	0.586544	0.642572	Parcel 7	1	0.559322
360188	19	2	0	0	55	7.9	0.61	3.73	0.131718	0.532882	Parcel 6	0	0.472222
360188	19	2	0	0	55	7.9	0.61	3.73	0.131718	0.532882	Parcel 6	1	0.527778
360230	28	4	2	3	22	6.2	0.76	0.61	-0.66891	0.33874	Parcel 4	0	0.528846
360230	28	4	2	3	22	6.2	0.76	0.61	-0.66891	0.33874	Parcel 4	1	0.471154
360245	40	5	1	11	32	18.6	3.4	2.55	1.482958	0.815019	Parcel 9	0	0.119048
360245	40	5	1	11	32	18.6	3.4	2.55	1.482958	0.815019	Parcel 9	1	0.880952
360349	33	5	6	4	121	15.1	3.31	14.96	0.501401	0.622789	Parcel 7	0	0.440678
360349	33	5	6	4	121	15.1	3.31	14.96	0.501401	0.622789	Parcel 7	1	0.559322
360448	34	3	0	4	23	6.8	0.69	0.88	-0.30881	0.423405	Parcel 5	0	0.597938
360448	34	3	0	4	23	6.8	0.69	0.88	-0.30881	0.423405	Parcel 5	1	0.402062
360449	19	2	0	0	30	4	0.52	0.68	-0.23033	0.44267	Parcel 5	0	0.597938
360449	19	2	0	0	30	4	0.52	0.68	-0.23033	0.44267	Parcel 5	1	0.402062
360504	44	1	17	14	107	13	3.83	10.08	-2.38152	0.084593	Parcel 1	0	0.965517
360504	44	1	17	14	107	13	3.83	10.08	-2.38152	0.084593	Parcel 1	1	0.034483
360509	29	2	1	6	36	9.3	1.42	1.93	0.084692	0.52116	Parcel 6	0	0.472222
360509	29	2	1	6	36	9.3	1.42	1.93	0.084692	0.52116	Parcel 6	1	0.527778
360518	65	1	10	18	32	9.9	1.37	1.8	-2.9044	0.051936	Parcel 1	0	0.965517
360518	65	1	10	18	32	9.9	1.37	1.8	-2.9044	0.051936	Parcel 1	1	0.034483
360585	45	2	18	11	59	8.9	2.74	2.51	-3.52701	0.028553	Parcel 1	0	0.965517
360585	45	2	18	11	59	8.9	2.74	2.51	-3.52701	0.028553	Parcel 1	1	0.034483
360684	33	2	4	8	79	12.6	8.55	1.4	3.366597	0.966644	Parcel 10	0	0.021277
360684	33	2	4	8	79	12.6	8.55	1.4	3.366597	0.966644	Parcel 10	1	0.978723
360740	32	3	3	5	34	12.4	2.18	2.03	0.268153	0.56664	Parcel 6	0	0.472222
360740	32	3	3	5	34	12.4	2.18	2.03	0.268153	0.56664	Parcel 6	1	0.527778
360763	32	4	5	6	67	8.4	3.38	2.25	0.027428	0.506856	Parcel 6	0	0.472222
360763	32	4	5	6	67	8.4	3.38	2.25	0.027428	0.506856	Parcel 6	1	0.527778
360917	23	3	0	1	34	9.2	1.25	1.87	0.468191	0.614955	Parcel 7	0	0.440678
360917	23	3	0	1	34	9.2	1.25	1.87	0.468191	0.614955	Parcel 7	1	0.559322
360925	25	5	0	2	60	8.4	1.25	3.79	0.260182	0.564681	Parcel 6	0	0.472222
360925	25	5	0	2	60	8.4	1.25	3.79	0.260182	0.564681	Parcel 6	1	0.527778
360927	22	4	0	1	23	6.2	0.35	1.08	-0.26786	0.433432	Parcel 5	0	0.597938
360927	22	4	0	1	23	6.2	0.35	1.08	-0.26786	0.433432	Parcel 5	1	0.402062
360963	27	1	6	2	20	14.3	0.38	2.48	-0.96571	0.275737	Parcel 3	0	0.762712
360963	27	1	6	2	20	14.3	0.38	2.48	-0.96571	0.275737	Parcel 3	1	0.237288
360968	43	4	0	10	94	19.9	6.32	12.38	3.549213	0.972056	Parcel 10	0	0.021277
360968	43	4	0	10	94	19.9	6.32	12.38	3.549213	0.972056	Parcel 10	1	0.978723
360970	53	4	26	20	249	5.8	7.58	6.86	-3.80362	0.021804	Parcel 1	0	0.965517
360970	53	4	26	20	249	5.8	7.58	6.86	-3.80362	0.021804	Parcel 1	1	0.034483
361058	24	3	2	3	22	5.6	0.08	1.15	-0.99001	0.270909	Parcel 3	0	0.762712

LB	UB	Parcel	Disb Bads %	Disb Goods %
0.0%	10.0%	Parcel 1	3.4%	96.6%
10.0%	20.0%	Parcel 2	13.0%	87.0%
20.0%	30.0%	Parcel 3	23.7%	76.3%
30.0%	40.0%	Parcel 4	47.1%	52.9%
40.0%	50.0%	Parcel 5	40.2%	59.8%
50.0%	60.0%	Parcel 6	52.8%	47.2%
60.0%	70.0%	Parcel 7	55.9%	44.1%
70.0%	80.0%	Parcel 8	76.0%	24.0%
80.0%	90.0%	Parcel 9	88.1%	11.9%
90.0%	100.0%	Parcel 10	97.9%	2.1%

5.5 KNN Inferencing

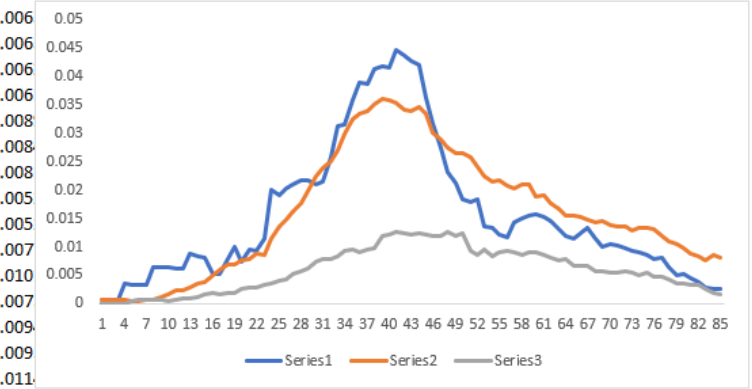
L4 =SQRT(($J4-L$2)^2+($K4-L$3)^2)

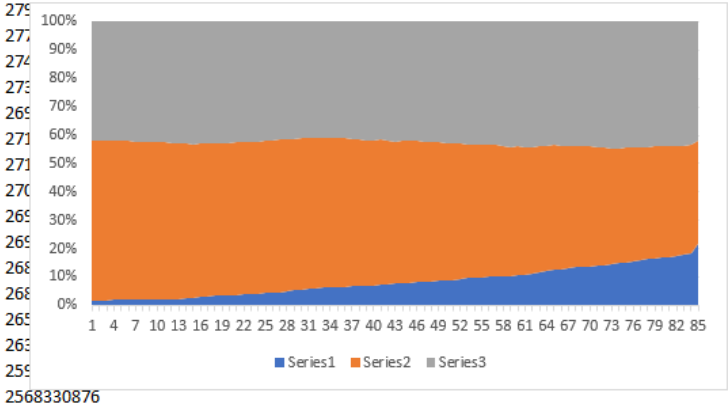
	A	B	C	D	E	F	G	H	I	J	K
1	Mean	2.654444	10.1772				k	25			
2	SD	1.151585	6.92562								ed
3	customer	ed	debtinc	default			customer	ed	debtinc	ed	debtinc
4	10012	-0.5683	1.08622	0		1	360036	4	9.3	1.168438	-0.12666
5	10017	2.036807	0.65305	0		2	360188	2	7.9	-0.5683	-0.32881
6	10030	-1.43667	-0.7764	0		3	360230	4	6.2	1.168438	-0.57428
7	10039	-1.43667	3.512	0		4	360245	5	18.6	2.036807	1.216176
8	10069	-1.43667	1.76486	1		5	360349	5	15.1	2.036807	0.710806
9	10071	-1.43667	0.10436	1		6	360448	3	6.8	0.30007	-0.48764
10	10096	0.30007	0.24875	1		7	360449	2	4	-0.5683	-0.89194
11	10128	-1.43667	0.60973	0		8	360504	1	13	-1.43667	0.407585
12	10129	1.168438	-0.7475	0		9	360509	2	9.3	-0.5683	-0.12666
13	10140	0.30007	-0.9064	0		10	360518	1	9.9	-1.43667	-0.04003
14	10142	-1.43667	1.05735	0		11	360585	2	8.9	-0.5683	-0.18442
15	10169	1.168438	0.06105	1		12	360684	2	12.6	-0.5683	0.349828
16	10197	2.036807	1.28837	0		13	360740	3	12.4	0.30007	0.32095
17	10200	-0.5683	-0.9064	0		14	360763	4	8.4	1.168438	-0.25662
18	10218	-1.43667	0.4509	0		15	360917	3	9.2	0.30007	-0.1411
19	10234	-0.5683	-0.401	1		16	360925	5	8.4	2.036807	-0.25662
20	10351	-0.5683	0.79744	1		17	360927	4	6.2	1.168438	-0.57428
21	10423	-1.43667	-1.4262	1		18	360963	1	14.3	-1.43667	0.595293
22	10428	-0.5683	-0.632	1		19	360968	4	19.9	1.168438	1.403885
23	10449	-0.5683	-0.2422	0		20	360970	4	5.8	1.168438	-0.63203
24	10455	-0.5683	-0.3721	1		21	361058	3	5.6	0.30007	-0.66091
25	10474	-1.43667	-0.2711	0		22	361072	2	24.2	-0.5683	2.024767
26	10525	0.30007	-0.5887	0		23	361083	1	13.2	-1.43667	0.436463
27	10564	1.168438	0.4509	1		24	361101	3	32	0.30007	3.15102
28	10567	-0.5683	-0.9786	0		25	361105	2	16.3	-0.5683	0.884076
29	10746	1.168438	-0.5454	0		26	361146	4	4.7	1.168438	-0.79086
30	10775	2.036807	0.75412	1		27	361166	4	15.5	1.168438	0.768563
31	10776	0.30007	-0.7475	0		28	361198	2	11.3	-0.5683	0.162119
32	10797	-0.5683	0.21988	0		29	361201	3	11	0.30007	0.118802
33	10798	1.168438	-0.5021	0		30	361255	3	16.8	0.30007	0.956272
34	10800	-0.5683	-0.7187	1		31	361267	1	6.9	-1.43667	-0.4732
35	10802	-0.5683	-1.1807	1		32	361299	2	8.1	-0.5683	-0.29993
36	10805	-1.43667	0.06105	1		33	361305	2	5.7	-0.5683	-0.64647
37	10824	-0.5683	-0.7187	1		34	361346	3	14.8	0.30007	0.667489
38	10839	1.168438	-1.0363	0		35	361396	3	3	0.30007	-1.03633
39	10870	1.168438	-0.2566	0		36	361437	2	9.6	-0.5683	-0.08335
40	10884	2.036807	0.00329	0		37	361440	2	6.3	-0.5683	-0.55984
41	10887	1.168438	-0.4299	0		38	361454	2	10.2	-0.5683	0.003289
42	10893	-0.5683	0.06105	0		39	361487	2	6.2	-0.5683	-0.57428

	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
2	-0.5683	2.036807	-1.43667	-1.43667	-1.43667	-1.43667	0.30007	-1.43667	1.168438	0.30007	-1.43667	1.168438	2.036807	-0.5683	-1.43667	-0.5683	-0.5683	-1.43667
3	1.086224	0.65305	-0.77642	3.511998	1.764863	0.104363	0.248754	0.609732	-0.74755	-0.90638	1.057346	0.061045	1.288372	-0.90638	0.450902	-0.40101	0.797441	-1.42619
4	2.118337	1.167055	2.684915	4.47509	3.219387	2.61533	0.946046	2.707186	0.620883	1.167055	2.861548	0.187709	1.660237	1.903736	2.668363	1.758272	1.967289	2.911243
5	1.415035	2.783995	0.976945	3.93775	2.266613	0.970414	1.042903	1.278643	1.786504	1.042903	1.635694	1.779957	3.066245	0.577565	1.167055	0.072196	1.126252	1.399391
6	2.402815	1.503461	2.612937	4.846052	3.501164	2.692049	1.196429	2.861548	0.17327	0.929707	3.073885	0.635322	2.055121	1.768205	2.799566	1.745359	2.213112	2.740862
7	2.608345	0.563126	4.004433	4.163631	3.516545	3.647075	1.988005	3.526018	2.147153	2.742533	3.477104	1.445127	0.072196	3.360328	3.556778	3.066245	2.638545	4.364298
8	2.632018	0.057757	3.778476	4.462253	3.629885	3.526018	1.79715	3.474945	1.697308	2.373086	3.490719	1.084552	0.577565	3.066245	3.483185	2.832438	2.606546	4.078206
9	1.797531	2.077844	1.760583	4.360433	2.844298	1.834864	0.736396	2.054382	0.90643	0.418735	2.324488	1.027191	2.484045	0.964056	1.974113	0.87268	1.550968	1.974113
10	1.978161	3.028789	0.876018	4.488731	2.795112	1.32162	1.433611	1.734669	1.742729	0.868489	2.133956	1.981018	3.397105	0.014439	1.59915	0.49093	1.689378	1.019551
11	1.102096	3.482137	1.184009	3.104413	1.357278	0.303222	1.743985	0.202148	2.84972	2.177786	0.649761	2.628054	3.583408	1.574979	0.043317	1.186543	0.951868	1.83377
12	1.212887	2.719289	1.084552	3.740845	2.08133	0.898575	0.946046	1.138571	1.844384	1.167055	1.468312	1.746852	2.964608	0.779713	1.042903	0.274344	0.924104	1.562953
13	1.422149	3.541947	0.736396	3.552026	1.804891	0.144391	1.760583	0.649761	2.699474	1.940829	1.097374	2.607066	3.718827	1.226631	0.49093	0.940409	1.206408	1.386157
14	1.270644	2.736409	1.050968	3.797047	2.133956	0.915128	0.970414	1.176751	1.825751	1.129285	1.515271	1.753998	2.992606	0.721957	1.075964	0.216587	0.981861	1.515271
15	0.736396	2.622694	1.422149	3.279235	1.660237	0.902395	0.874231	0.90643	2.054382	1.527126	1.120109	1.760583	2.769015	1.256204	0.874231	0.750835	0.447613	1.976939
16	1.157458	1.768205	2.054382	3.633049	2.258571	1.750191	0.072196	1.760583	1.376861	1.227326	1.886408	0.90643	1.988005	1.503461	1.741593	1.129285	0.990509	2.463481
17	2.19533	1.257599	2.65646	4.581378	3.297416	2.629997	1.00472	2.745385	0.49093	1.084552	2.917717	0.317661	1.772301	1.854305	2.699474	1.742729	2.031574	2.855604
18	1.503461	1.909695	1.849295	4.044923	2.578558	1.753998	0.389857	1.892091	1.059169	0.765274	2.110103	0.891587	2.249367	1.157458	1.834864	0.90643	1.278643	2.160485
19	2.930835	0.909665	3.512155	5.125181	4.018881	3.492182	1.808772	3.579886	0.997535	1.854305	3.713694	0.924647	1.544987	2.684915	3.5448	2.609105	2.810269	3.665095
20	2.402815	1.503461	2.612937	4.846052	3.501164	2.692049	1.196429	2.861548	0.17327	0.929707	3.073885	0.635322	2.055121	1.768205	2.799566	1.745359	2.213112	2.740862
21	0.997535	3.473955	1.371718	2.916705	1.16957	0.49093	1.770973	0.014439	2.930835	2.295924	0.462052	2.659323	3.541947	1.734669	0.144391	1.32162	0.891587	2.021478
22	1.76555	1.147962	3.397105	3.351227	2.629997	2.911243	1.445127	2.723464	2.151431	2.46807	2.628054	1.342839	0.876018	2.890253	2.773942	2.504774	1.839574	3.84654
23	2.443085	1.550968	2.609105	4.894851	3.540012	2.707186	1.236871	2.885924	0.115513	0.910675	3.104928	0.693078	2.10761	1.758272	2.821228	1.752036	2.249367	2.723464
24	1.951037	2.177786	1.740575	4.519892	2.983394	1.897868	0.909665	2.151928	0.87268	0.245465	2.443085	1.129285	2.610739	0.902395	2.062131	0.90643	1.697308	1.897868
25	0.938544	2.944179	2.932702	1.722184	0.90643	2.10761	1.976939	1.660237	3.271388	3.057069	1.299988	2.621538	2.707186	2.931144	1.797531	2.425774	1.227326	3.55853
26	1.084552	3.480221	1.212887	3.075535	1.3284	0.3321	1.746852	0.17327	2.861548	2.19533	0.620883	2.632018	3.57642	1.59915	0.014439	1.206408	0.940409	1.862648
27	2.239966	3.042386	4.294307	1.773855	2.222091	3.506904	2.902265	3.078051	3.994105	4.057396	2.720244	3.209674	2.546707	4.14928	3.210435	3.656632	2.508664	4.895616
28	0.202148	2.61533	1.873853	2.767677	1.236871	1.167055	1.075964	0.910675	2.382949	1.989921	0.885487	1.921884	2.636292	1.790452	0.970414	1.285083	0.086635	2.46807
29	2.557286	1.684918	2.605146	5.030029	3.649427	2.754634	1.354573	2.957744	0.043317	0.876018	3.194128	0.851909	2.253283	1.740575	2.885924	1.779957	2.353501	2.681457
30	1.76555	0.876018	3.028789	3.783254	2.78912	2.688446	1.01206	2.609944	1.516109	1.88666	2.621063	0.707517	1.01206	2.412816	2.624402	2.093836	1.736978	3.406391
31	0.924104	2.65096	1.278643	3.4606	1.822869	0.870287	0.87268	0.976945	1.960548	1.376861	1.247195	1.739676	2.838137	1.068496	0.915128	0.563126	0.635322	1.810187
32	1.299988	1.817052	1.95389	3.811828	2.392859	1.736797	0.129952	1.804791	1.226631	1.025178	1.974113	0.870287	2.093836	1.343523	1.768205	1.01206	1.102096	2.324488
33	0.878039	1.763009	2.453261	3.089983	1.915745	1.934426	0.707517	1.770973	1.912344	1.862648	1.739676	1.247195	1.768205	2.055121	1.808772	1.611294	0.882775	2.94828
34	1.784902	3.651503	0.303222	3.9852	2.238065	0.577565	1.880819	1.082935	2.619512	1.789943	1.530548	2.659323	3.894634	0.970414	0.924104	0.871365	1.539025	0.952983
35	1.386157	2.773942	0.990509	3.909588	2.239966	0.957872	1.027191	1.257599	1.793492	1.059169	1.611294	1.773855	3.051113	0.606444	1.147962	0.101074	1.097374	1.422149
36	1.732696	2.911243	0.878039	4.248169	2.562928	1.147962	1.247195	1.527126	1.739676	0.90643	1.912344	1.875323	3.245027	0.259904	1.399391	0.245465	1.443913	1.167055
37	0.964056	1.736797	2.258571	3.33279	2.054382	1.825751	0.418735	1.737698	1.660237	1.573865	1.779957	1.059169	1.844384	1.797531	1.750191	1.376861	0.878039	2.720244
38	2.293315	2.422861	1.756077	4.868627	3.295896	2.077844	1.285083	2.392859	0.915128	0.129952	2.720244	1.399391	2.901808	0.878039	2.286506	1.075964	2.028984	1.779957
39	1.16957	2.707186	1.111045	3.698724	2.042043	0.888425	0.929707	1.111045	1.859414	1.196429	1.433611	1.742729	2.944179	0.823031	1.019551	0.317661	0.880787	1.59915
40	1.646061	2.873617	0.894972	4.163401	2.481591	1.093264	1.186543	1.456694	1.746852	0.934962	1.835577	1.844384	3.194128	0.346539	1.332538	0.15883	1.357278	1.226631
41	1.082935	2.684915	1.167055	3.614568	1.963977	0.874231	0.902395	1.059169	1.892091	1.257599	1.365687	1.737698	2.904826	0.909665	0.976945	0.404296	0.794152	1.672561
42	1.6605	2.879741	0.891587	4.177523	2.495123	1.102096	1.196429	1.468312	1.745359	0.929707	1.848311	1.849295	3.202505	0.3321	1.343523	0.17327	1.371718	1.216476

6.1 Segmentation OLTV

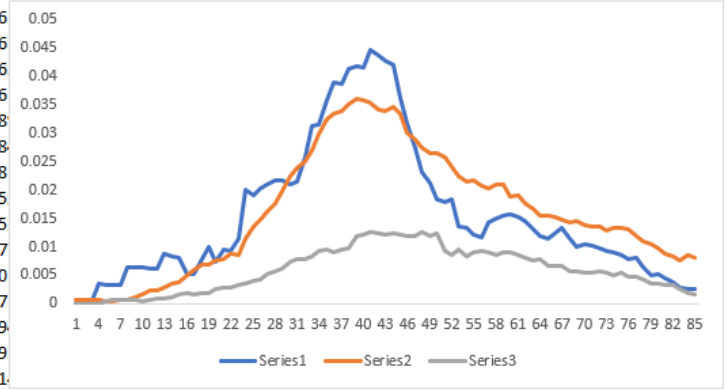
Date	O.FICO<350	350<=O.FICO	OFICO>=755	O.FICO<350	350<=O.FICO	OFICO>=755
01-May-06	0	0.00057192	0	113684205	3962587522	2944922103
01-Jun-06	0	0.00057996	0	114044658	3910153934	2918386775
01-Jul-06	0	0.00058746	0	118165884	3858166135	2882648396
01-Aug-06	0.00347222	0.00059356	0	127051774	3812140451	2849244429
01-Sep-06	0.00335571	0.00044937	0.00039355	130477458	3775222419	2821081319
01-Oct-06	0.00331126	0.00045317	0.00059465	132378480	3740922963	2798157681
01-Nov-06	0.00321543	0.00076617	0.00060048	134082290	3682037880	2818895249
01-Dec-06	0.00641026	0.00077459	0.00060569	133691408	3640577765	279
01-Jan-07	0.006				3608254542	277
01-Feb-07	0.006				3572684349	274
01-Mar-07	0.006				3543344618	273
01-Apr-07	0.006				3473494131	269
01-May-07	0.008				3478011138	271
01-Jun-07	0.008				3452873516	271
01-Jul-07	0.008				3414991106	270
01-Aug-07	0.005				3404644057	269
01-Sep-07	0.005				3401808209	269
01-Oct-07	0.007				3392047614	268
01-Nov-07	0.010				3371370364	268
01-Dec-07	0.007				3359374174	265
01-Jan-08	0.009				3333021338	263
01-Feb-08	0.009				3293878922	259
01-Mar-08	0.011				3276964303	2568330876
01-Apr-08	0.02013423	0.01154119	0.00363636	244974110	3282176341	2587897385
01-May-08	0.0190678	0.01370107	0.00413888	264756136	3312759607	2574286296
01-Jun-08	0.02028398	0.01478183	0.00437184	272903247	3339635928	2596792118
01-Jul-08	0.02095238	0.0162384	0.00533767	294257413	3352372272	2599983625
01-Aug-08	0.02169982	0.01774512	0.00561666	309063691	3357215034	2602071347
01-Sep-08	0.02173913	0.01988791	0.00613932	334664462	3333503583	2593230967
01-Oct-08	0.02100162	0.02248848	0.00747528	344811817	3289672507	2539657519
01-Nov-08	0.02160494	0.02388505	0.00780678	358950651	3253963748	2513093619
01-Dec-08	0.02571861	0.02519417	0.00792472	364142859	3217184032	2481714773
01-Jan-09	0.03129657	0.02708978	0.00837564	370406974	3167472829	2449393501
01-Feb-09	0.03162651	0.02999603	0.00940931	366455526	3104070430	2397911879
01-Mar-09	0.03598201	0.03248213	0.00946202	370868728	3041614857	2369541903
01-Apr-09	0.03892216	0.03343208	0.00894855	375211079	2983345237	2335787605
01-May-09	0.03880597	0.03400174	0.00953206	377520402	2933389945	2320762284
01-Jun-09	0.04148148	0.03515017	0.00971445	385667355	2920902459	2336016139
01-Jul-09	0.04184704	0.03604625	0.01183782	396775905	2897765404	2359707105
01-Aug-09	0.04154728	0.03580445	0.01227913	401228019	2885950988	2354954467
01-Sep-09	0.04481793	0.03541667	0.01270033	413025389	2897868232	2360983244

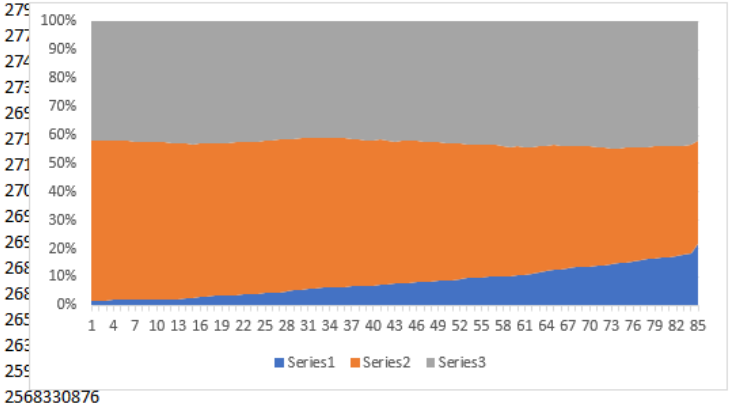




6.2 Segmentation OFICO

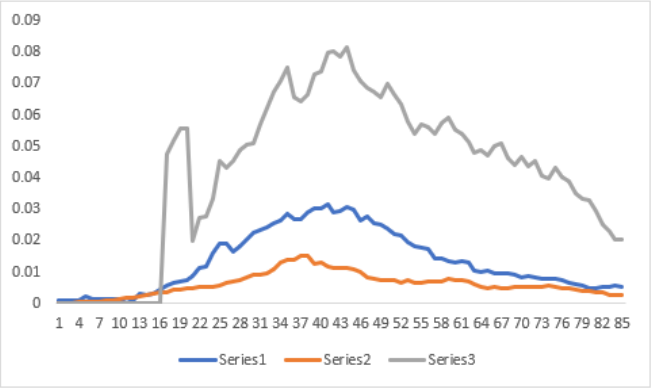
Date	O.FICO<350	350<=O.FICO	OFICO>=755	O.FICO<350	350<=O.FICO	OFICO>=755
01-May-06	0	0.00057192	0	113684205	3962587522	2944922103
01-Jun-06	0	0.00057996	0	114044658	3910153934	2918386775
01-Jul-06	0	0.00058746	0	118165884	3858166135	2882648396
01-Aug-06	0.00347222	0.00059356	0	127051774	3812140451	2849244429
01-Sep-06	0.00335571	0.00044937	0.00039355	130477458	3775222419	2821081319
01-Oct-06	0.00331126	0.00045317	0.00059465	132378480	3740922963	2798157681
01-Nov-06	0.00321543	0.00076617	0.00060048	134082290	3682037880	2818895249
01-Dec-06	0.00641026	0.00077459	0.00060569	133691408	3640577765	279
01-Jan-07	0.006				3608254542	277
01-Feb-07	0.006				3572684349	274
01-Mar-07	0.006				3543344618	273
01-Apr-07	0.006				3473494131	269
01-May-07	0.008				3478011138	271
01-Jun-07	0.008				3452873516	271
01-Jul-07	0.008				3414991106	270
01-Aug-07	0.005				3404644057	269
01-Sep-07	0.005				3401808209	269
01-Oct-07	0.007				3392047614	268
01-Nov-07	0.010				3371370364	268
01-Dec-07	0.007				3359374174	265
01-Jan-08	0.009				3333021338	263
01-Feb-08	0.009				3293878922	259
01-Mar-08	0.011				3276964303	2568330876
01-Apr-08	0.02013423	0.01154119	0.00363636	244974110	3282176341	2587897385
01-May-08	0.0190678	0.01370107	0.00413888	264756136	3312759607	2574286296
01-Jun-08	0.02028398	0.01478183	0.00437184	272903247	3339635928	2596792118
01-Jul-08	0.02095238	0.0162384	0.00533767	294257413	3352372272	2599983625
01-Aug-08	0.02169982	0.01774512	0.00561666	309063691	3357215034	2602071347
01-Sep-08	0.02173913	0.01988791	0.00613932	334664462	3333503583	2593230967
01-Oct-08	0.02100162	0.02248848	0.00747528	344811817	3289672507	2539657519
01-Nov-08	0.02160494	0.02388505	0.00780678	358950651	3253963748	2513093619
01-Dec-08	0.02571861	0.02519417	0.00792472	364142859	3217184032	2481714773
01-Jan-09	0.03129657	0.02708978	0.00837564	370406974	3167472829	2449393501
01-Feb-09	0.03162651	0.02999603	0.00940931	366455526	3104070430	2397911879
01-Mar-09	0.03598201	0.03248213	0.00946202	370868728	3041614857	2369541903
01-Apr-09	0.03892216	0.03343208	0.00894855	375211079	2983345237	2335787605
01-May-09	0.03880597	0.03400174	0.00953206	377520402	2933389945	2320762284
01-Jun-09	0.04148148	0.03515017	0.00971445	385667355	2920902459	2336016139
01-Jul-09	0.04184704	0.03604625	0.01183782	396775905	2897765404	2359707105
01-Aug-09	0.04154728	0.03580445	0.01227913	401228019	2885950988	2354954467
01-Sep-09	0.04481793	0.03541667	0.01270033	413025389	2897868232	2360983244



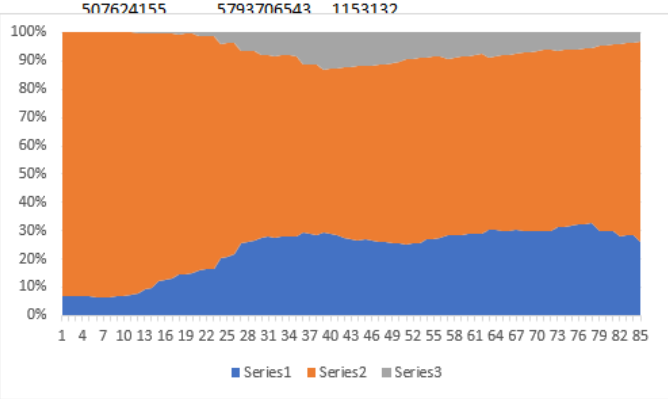


6.3 Segmentation ILTV

Date	ILTV<.69	.9<=ILTV<0	ILTV>=0.85
01-May-06	0.001098	0.000258	0
01-Jun-06	0.001117	0.000261	0
01-Jul-06	0.001171	0.000263	0
01-Aug-06	0.001179	0.000354	0
01-Sep-06	0.002353	0.000357	0
01-Oct-06	0.001272	0.000537	0
01-Nov-06	0.001277	0.000724	0
01-Dec-06	0.001269	0.000823	0
01-Jan-07	0.001248	0.001017	0
01-Feb-07	0.001244	0.001215	0
01-Mar-07	0	0.001892	0
01-Apr-07	0.001109	0.001841	0
01-May-07	0.002956	0.002245	0
01-Jun-07	0.00275	0.002668	0
01-Jul-07	0.003001	0.003067	0
01-Aug-07	0.004323	0.003619	0
01-Sep-07	0.005548	0.003744	0.047619
01-Oct-07	0.006519	0.004235	0.051724
01-Nov-07	0.007088	0.004262	0.055556
01-Dec-07	0.007634	0.004836	0.055556
01-Jan-08	0.008876	0.004998	0.020134
01-Feb-08	0.011216	0.005086	0.027211
01-Mar-08	0.011737	0.005171	0.027586
01-Apr-08	0.016019	0.005378	0.033097
01-May-08	0.019084	0.005677	0.045455
01-Jun-08	0.019284	0.006616	0.043269
01-Jul-08	0.016464	0.006986	0.045205
01-Aug-08	0.018147	0.007468	0.048747
01-Sep-08	0.020548	0.008402	0.05042
01-Oct-08	0.022534	0.009094	0.05079
01-Nov-08	0.023325	0.009346	0.056948
01-Dec-08	0.024299	0.00979	0.062212
01-Jan-09	0.025437	0.010984	0.067207
01-Feb-09	0.026439	0.013106	0.070755
01-Mar-09	0.028708	0.014034	0.075089
01-Apr-09	0.026941	0.01391	0.065719
01-May-09	0.026873	0.015116	0.064312
01-Jun-09	0.028846	0.015324	0.066359
01-Jul-09	0.03034	0.01261	0.073095
01-Aug-09	0.030239	0.012837	0.073751
01-Sep-09	0.031516	0.01155	0.07961



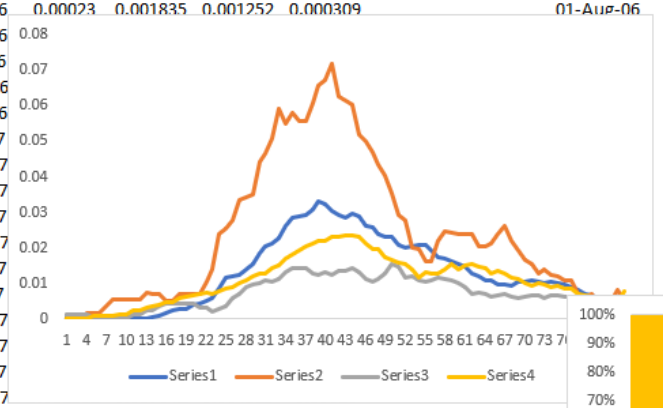
ILTV<.69	0.69<=ILTV<0.85	ILTV>=0.85
497691347	6523502484	
490433906	6452151461	
467302303	6391678112	
462856676	6325579977	
464151823	6262629373	
432942614	6238516509	
428561793	6206453625	
434749473	6137275104	
442321858	6077009453	
444926847	6011547975	
466939177	5942953017	
507624155	5793706543	1153132



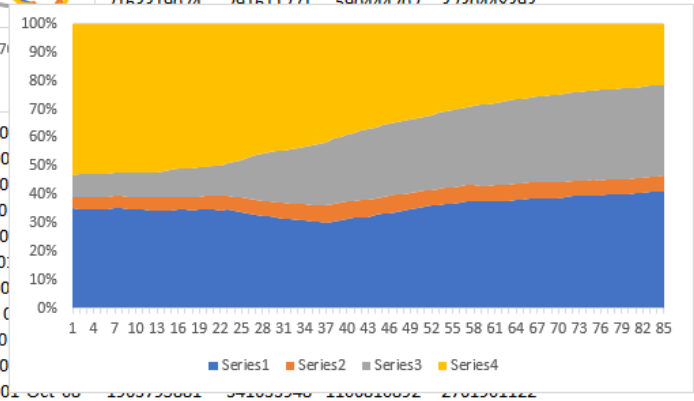
1645063701	4227131495	3.96E+08
1664481566	4203392682	3.94E+08
1706427189	3965411067	5.02E+08
1699257856	3927854137	4.99E+08
1678707761	3889848891	4.94E+08
1668512840	3833215860	4.86E+08
1637235713	3754296903	4.77E+08
1603415960	3705724539	4.73E+08
1670596880	3377882257	6.46E+08
1638718258	3358028630	6.35E+08
1600945936	3416050371	6.26E+08
1669912956	3243039553	7.41E+08
1633935595	3281760883	7.26E+08
1622009704	3342565767	7.07E+08

6.4 Segmentation Geography

Date	CA	FL	NY	OT
01-May-06	0.000445	0	0.001238	0.000149
01-Jun-06	0.000451	0	0.001242	0.000151
01-Jul-06	0.000456	0	0.001248	0.000153
01-Aug-06	0.00023	0.001835	0.001252	0.000309
01-Sep-06				
01-Oct-06				
01-Nov-06				
01-Dec-06				
01-Jan-07				
01-Feb-07				
01-Mar-07				
01-Apr-07				
01-May-07				
01-Jun-07				
01-Jul-07				
01-Aug-07				
01-Sep-07				
01-Oct-07				
01-Nov-07				
01-Dec-07	0.00394	0.006969	0.004306	0.006582
01-Jan-08	0.004502	0.006908	0.003202	0.007027
01-Feb-08	0.005153	0.010471	0.003151	0.007303
01-Mar-08	0.005769	0.013913	0.002064	0.007079
01-Apr-08	0.008691	0.024055	0.002899	0.007932
01-May-08	0.011621	0.025295	0.003656	0.008419
01-Jun-08	0.012017	0.027778	0.005957	0.008894
01-Jul-08	0.012442	0.0336	0.007109	0.010214
01-Aug-08	0.013885	0.034321	0.008889	0.010738
01-Sep-08	0.015634	0.034954	0.009894	0.012085
01-Oct-08	0.018692	0.044207	0.010302	0.012734
01-Nov-08	0.020589	0.046547	0.010796	0.012914
01-Dec-08	0.021124	0.050671	0.01054	0.014219
01-Jan-09	0.022834	0.058997	0.011104	0.015026
01-Feb-09	0.026307	0.054815	0.013089	0.016922
01-Mar-09	0.028489	0.058036	0.014388	0.018195
01-Apr-09	0.02898	0.055807	0.014474	0.01937
01-May-09	0.02918	0.055891	0.014221	0.020629
01-Jun-09	0.030702	0.06079	0.012739	0.021404
01-Jul-09	0.033147	0.065549	0.012594	0.021954
01-Aug-09	0.032258	0.067073	0.013258	0.02196
01-Sep-09	0.030568	0.071756	0.012563	0.023217

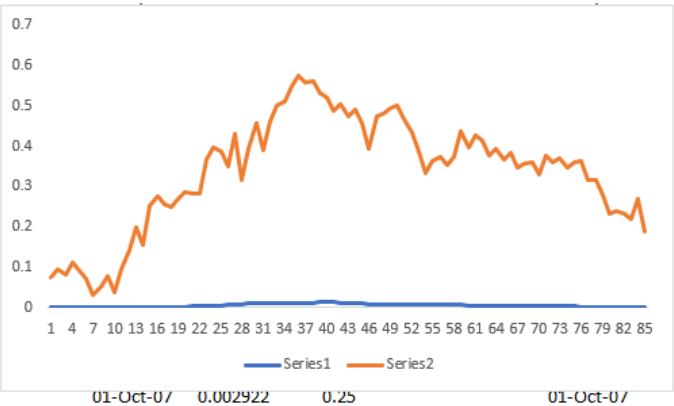


Date	CA	FL	NY	OT
01-May-06	2464866319	293830206	539696947	3722800358
01-Jun-06	2434820794	291660375	546069022	3670035177
01-Jul-06	2402179539	289819966	543438897	3623542013
01-Aug-06	2377319335	292351806	540071666	3578693846
	2346478300	293570214	538199604	3548533079
	2319778435	293343525	539113791	3519223372
	2332257878	291803729	536826605	3474127206
	2312757594	287125870	535830044	3436311068
	2276796411	286461411	537742159	3418331329
	2249996170	283727372	534451096	3388300184
	2218735313	284432532	545449495	3361274855
	2175699592	283438328	542683783	3300662127
	2189652275	286614087	561698945	3299942304
	2176247120	289431210	577470031	3272785539
01-Nov-08	1938042363	338589910	1123674441	2725701305
01-Dec-08	1897087642	338486410	1147997319	2679470293
01-Jan-09	1851324023	339956592	1171580431	2624412258
01-Feb-09	1808274905	339605563	1177136647	2543420721
01-Mar-09	1762445367	339398222	1203905878	2476276021
01-Apr-09	1740844857	334818905	1209866380	2408813779
01-May-09	1697258572	344163466	1246128207	2344122387
01-Jun-09	1719445994	346874276	1293288326	2282977358
01-Jul-09	1745342974	346245457	1323495183	2239164801
01-Aug-09	1770679502	347565180	1326601840	2197286952
01-Sep-09	1799721280	345742017	1358532686	2167880882

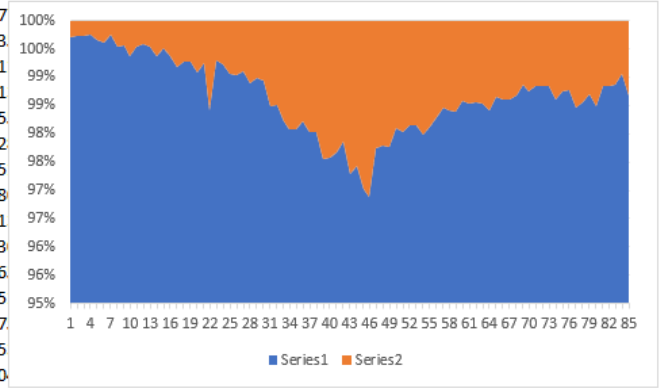


6.5 Segmentation Delinquent

Date	CUR	DLQ
01-May-06	7.99E-05	0.076923
01-Jun-06	8.08E-05	0.096774
01-Oct-07	0.002922	0.25
01-Nov-07	0.002933	0.268293
01-Dec-07	0.003045	0.287234
01-Jan-08	0.003626	0.282353
01-Feb-08	0.003684	0.284314
01-Mar-08	0.003725	0.365854
01-Apr-08	0.005198	0.395604
01-May-08	0.006191	0.38835
01-Jun-08	0.007058	0.351351
01-Jul-08	0.007647	0.429907
01-Aug-08	0.009148	0.317829
01-Sep-08	0.010052	0.393162
01-Oct-08	0.011119	0.457627
01-Nov-08	0.010868	0.390533
01-Dec-08	0.010902	0.460123
01-Jan-09	0.010426	0.5
01-Feb-09	0.011264	0.509901
01-Mar-09	0.011921	0.547264
01-Apr-09	0.012298	0.574468
01-May-09	0.012091	0.557214
01-Jun-09	0.012787	0.562814
01-Jul-09	0.013551	0.53211
01-Aug-09	0.013343	0.52
01-Sep-09	0.014298	0.486842

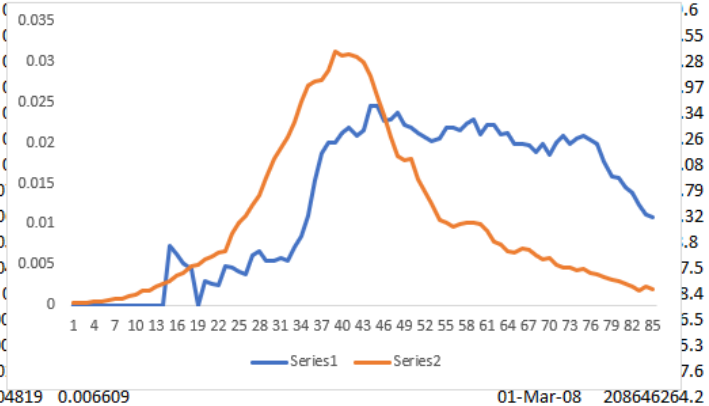


Date	CUR	DLQ
01-May-06	7000388629	20805201.53
01-Jun-06	6924451766	18133601.07
	6839688417	19291997.86
	6772366383	16070270.06
	6703545293	23235902.57
	6646131008	25328114.95
	6619165918	15849501.03
	6541584216	30440360.87
	6490863576	28467734.34
	6416068498	40406324.13
	6380006061	29886132.94
	6276479593	26004236.36
	630895187	
	627686243	
	624547531	
	626618901	
	625363845	
01-Oct-07	624318492	
01-Nov-07	621828835	
01-Dec-07	618139018	
01-Jan-08	615285881	
01-Feb-08	601869863	
01-Mar-08	604100396	
01-Apr-08	606791335	
01-May-08	609459657	
01-Jun-08	614993015	
01-Jul-08	619047060	
01-Aug-08	6198469830	69880241.49
01-Sep-08	6197845184	63553827.34
01-Oct-08	6108620079	65521763.74
01-Nov-08	6032549303	93458715.35
01-Dec-08	5973159004	89882660.54
01-Jan-09	5880240079	107033225
01-Feb-09	5755344330	113093505.4
01-Mar-09	5671541282	110484206.1
01-Apr-09	5592523287	101820633.6
01-May-09	5520320456	111352175.6
01-Jun-09	5531863291	110722662.5
01-Jul-09	5516399428	137848986.4
01-Aug-09	5505451361	136682112.8
01-Sep-09	5540351239	131525626.2

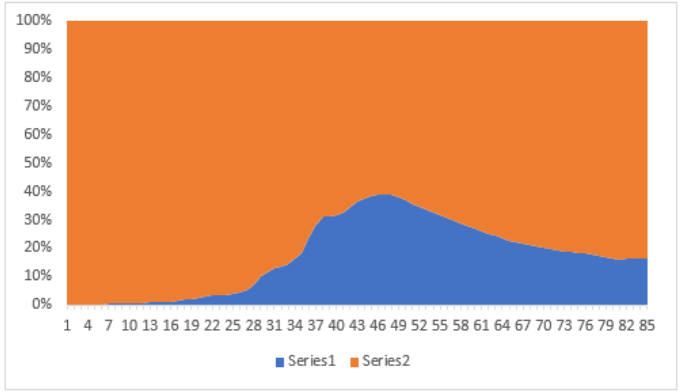


6.6 Segmentation MOB

Date	MOB >60	MOB< 60
01-May-06	0	0.00032
01-Jun-06	0	0.000323
01-Jul-06	0	0.000327
01-Aug-06	0	0.000413
01-Sep-06	0	0.000499
01-Oct-06	0	0.000587
01-Nov-06	0	0.000764
01-Dec-06	(	
01-Jan-07	(	
01-Feb-07	(	
01-Mar-07	(	
01-Apr-07	(	
01-May-07	(	
01-Jun-07	(	
01-Jul-07	0.00	
01-Aug-07	0.00	
01-Sep-07	0.00	
01-Oct-07	0.00	
01-Nov-07	(	
01-Dec-07	0.00	
01-Jan-08	0.00	
01-Feb-08	0.00	
01-Mar-08	0.004819	0.006609
01-Apr-08	0.004577	0.008763
01-May-08	0.004115	0.010246
01-Jun-08	0.003788	0.011082
01-Jul-08	0.006079	0.012372
01-Aug-08	0.006586	0.013588
01-Sep-08	0.005495	0.015622
01-Oct-08	0.00554	0.018063
01-Nov-08	0.005784	0.019298
01-Dec-08	0.005488	0.020748
01-Jan-09	0.007092	0.022505
01-Feb-09	0.008511	0.025118
01-Mar-09	0.011052	0.027159
01-Apr-09	0.015288	0.027574
01-May-09	0.018669	0.02771
01-Jun-09	0.020006	0.028958
01-Jul-09	0.020043	0.031214
01-Aug-09	0.021142	0.030691
01-Sep-09	0.021834	0.030982

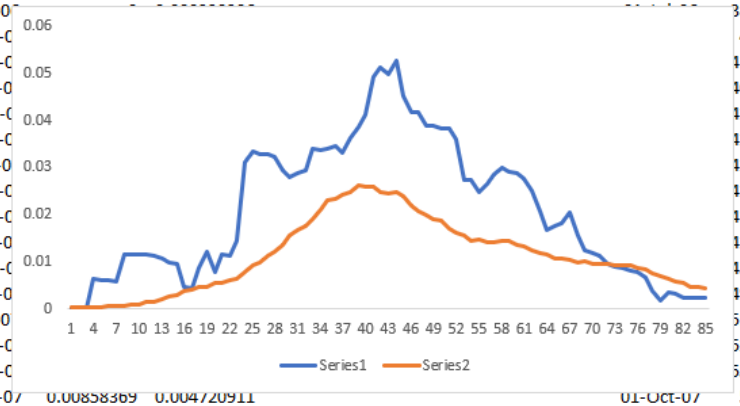


Date	MOB >60	MOB< 60
01-May-06	14960595.68	7.01E+09
01-Jun-06	15691683.54	6.93E+09
01-Jul-06	16161995.04	
01-Aug-06	18015833.97	
01-Sep-06	17970163.25	
01-Oct-06	18828939.8	
01-Nov-06	22152343.21	
	.6	
	.55	
	.28	
	.97	
	.34	
	.26	
	.08	
	.79	
	.32	
	.8	
	7.5	6.17E+09
	3.4	6.13E+09
	5.5	6.08E+09
	5.3	6.01E+09
	7.6	5.91E+09
01-Mar-08	208646264.2	5.87E+09
01-Apr-08	220829670.6	5.89E+09
01-May-08	244085943.4	5.91E+09
01-Jun-08	267464749	5.94E+09
01-Jul-08	332314090.5	5.91E+09
01-Aug-08	449261117.8	5.82E+09
01-Sep-08	630971355.4	5.63E+09
01-Oct-08	724184162.5	5.45E+09
01-Nov-08	785098869.6	5.34E+09
01-Dec-08	828017338.2	5.24E+09
01-Jan-09	857613626	5.13E+09
01-Feb-09	958122579.8	4.91E+09
01-Mar-09	1070044139	4.71E+09
01-Apr-09	1330303035	4.36E+09
01-May-09	1572345141	4.06E+09
01-Jun-09	1748557466	3.89E+09
01-Jul-09	1771096714	3.88E+09
01-Aug-09	1776199653	3.87E+09
01-Sep-09	1846625517	3.83E+09

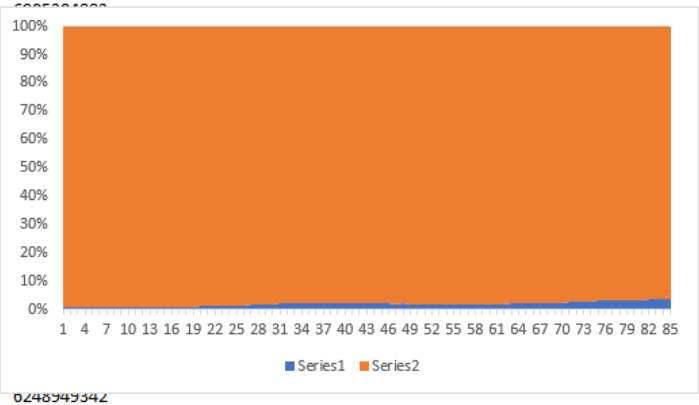


6.7 Segmentation Loan Size

Date	RG. BAL<$41	IRG. BAL>=$417
01-May-06	0	0.000322477
01-Jun-06	0	0.000326371
01-Jul-0		
01-Aug-0		
01-Sep-0		
01-Oct-0		
01-Nov-0		
01-Dec-0		
01-Jan-0		
01-Feb-0		
01-Mar-0		
01-Apr-0		
01-May-0		
01-Jun-0		
01-Jul-0		
01-Aug-0		
01-Sep-0		
01-Oct-07	0.00858369	0.004720911
01-Nov-07	0.0122449	0.00474551
01-Dec-07	0.00790514	0.005431729
01-Jan-08	0.01149425	0.005670542
01-Feb-08	0.0112782	0.006236208
01-Mar-08	0.01444043	0.006326034
01-Apr-08	0.03114187	0.00794897
01-May-08	0.03344482	0.009268389
01-Jun-08	0.03293413	0.009980237
01-Jul-08	0.03278689	0.011219222
01-Aug-08	0.03233831	0.012196341
01-Sep-08	0.02947846	0.013705533
01-Oct-08	0.02807775	0.015727796
01-Nov-08	0.02874743	0.016633746
01-Dec-08	0.02929688	0.017642087
01-Jan-09	0.03389831	0.019031142
01-Feb-09	0.03370787	0.021134594
01-Mar-09	0.03383459	0.022912132
01-Apr-09	0.03461539	0.023431953
01-May-09	0.03320313	0.024122807
01-Jun-09	0.03614458	0.02491015
01-Jul-09	0.03853955	0.026157697
01-Aug-09	0.04115226	0.02604957
01-Sep-09	0.04918033	0.025849994



Date	IRG. BAL<$417	IRG. BAL>=$417
01-May-06	36983658.72	6984210172
01-Jun-06	37280484.48	
	38507917.72	
	40406119.3	
	41613939.36	
	40707060.75	
	42778132.79	
	42505201.94	
	42886865.41	
	42286961.04	
	43278182.12	
	42996297.09	
	45226632.42	
	49518176.06	
	51451407.92	
	52132051.11	
	55831270.79	6248949342
01-Oct-07	58194017.2	6230403593
01-Nov-07	61729375.06	6202152560
01-Dec-07	63033884.6	6175651233
01-Jan-08	65340798.21	6133487437
01-Feb-08	66629283.17	6049351689
01-Mar-08	69487598.41	6013567952
01-Apr-08	73094912.74	6041952924
01-May-08	76174374.86	6075627663
01-Jun-08	85745374.83	6123585918
01-Jul-08	93876307.88	6152737002
01-Aug-08	102888120.6	6165461951
01-Sep-08	113406782.5	6147992229
01-Oct-08	118998224.3	6055143618
01-Nov-08	124840451.8	6001167567
01-Dec-08	130439544.3	5932602120
01-Jan-09	135201954.2	5852071350
01-Feb-09	136668830.2	5731769006
01-Mar-09	135975043.5	5646050444
01-Apr-09	132735714.5	5561608206
01-May-09	130390506.2	5501282125
01-Jun-09	126278817.3	5516307136
01-Jul-09	125271469.4	5528976945
01-Aug-09	123073825.6	5519059648
01-Sep-09	123532575.9	5548344290



6.8 Segmentation Using Decision Trees

PEAKS2TAILS

Node - 0

Default	Count	Pi
Yes	548	0.3653
No	952	0.6347
Total Count & Entropy	1500	0.6564
employ(min)	0	
employ(max)	63	
debtinc(min)	0	
debtinc(max)	40.7	
Age -1	577	
Age -2	418	
Age -3	283	
Age -4	222	

Split By Debtinc

Debtinc <= 5

DebtInc > 5

Node - 1

Admission	Count	Pi
Yes	73	0.183
No	326	0.817
Total Count & Entropy	399	0.4758

Node - 2

Admission	Count	Pi
Yes	475	0.4314
No	626	0.5686
Total Count & Entropy	1101	0.6837

Maximize Entropy Reduction 42.0066 Debtinc

Constraints

DebtInc <= 40.7

Debtinc >= 0

Node - 1

Admission	Count	Pi
Yes	537	0.394
No	826	0.606
Total Count & Entropy	1363	0.6705

Node - 0

Default	Count	Pi
Yes	548	0.3653

employ	debtinc	Age	default
7	17.7	2	0
34	14.7	4	0
20	4.8	3	0
11	34.5	2	0
2	22.4	1	1
2	10.9	2	1
2	11.9	1	1
4	14.4	1	0
29	5	4	0
0	3.9	1	0
33	17.5	4	0
4	10.6	2	1
7	19.1	3	0
0	3.9	1	0
9	13.3	4	0
0	7.4	1	1
0	15.7	1	1
2	0.3	1	1
0	5.8	1	1
27	8.5	4	0
1	7.6	1	1
8	8.3	1	0
8	6.1	3	0
17	13.3	3	1
16	3.4	3	0
0	6.4	1	0
1	15.4	3	1
15	5	2	0
23	11.7	3	0
0	6.7	1	0
0	5.2	1	1
0	2	1	1
3	10.6	3	1
3	5.2	3	1
2	3	2	0

Count 1500

6.9 Segmentation Using Decision Trees

PEAKS2TAILS

Variables	Type	Sign	Constraint	Count	Count of D 1	Count of ND 0	pd	pnd	Entropy
Combined				1500	548	952	0.365333	0.634666667	0.656425207
debtinc	Numerical	<=	9.028125	787	174	613	0.221093	0.778907243	0.528287552
debtinc	Numerical	>	9.028125	713	374	339	0.524544	0.47545582	0.691941863
employ	Numerical	<=	3.704210619	738	378	360	0.512195	0.487804878	0.692849709
employ	Numerical	>	3.704210619	762	170	592	0.223097	0.776902887	0.530800017
age	Categorical	=	1	577	291	286	0.504333	0.495667244	0.693109635
age	Categorical	<>	1	923	257	666	0.27844	0.72156013	0.591473791
age	Categorical	=	2	418	151	267	0.361244	0.638755981	0.654130602
age	Categorical	<>	2	1082	397	685	0.366913	0.633086876	0.65729233
age	Categorical	=	3	283	80	203	0.282686	0.717314488	0.595471829
age	Categorical	<>	3	1217	468	749	0.384552	0.615447823	0.666248726
age	Categorical	=	4	222	26	196	0.117117	0.882882883	0.361141422
age	Categorical	<>	4	1278	522	756	0.408451	0.591549296	0.676289693

Variable	Type	Category	Max	Min	EntropyN	EntropyO	Reduction	Constraint
debtinc	Numerical		40.7	0	909.11685	984.6378	75.52096	9.02813
employ	Numerical		63	0	915.79270	984.6378	68.84511	3.70421
age	Categorica	1	4	1	945.85457	984.6378	38.78324	1.0
age	Categorica	2			984.61689	984.6378	0.02092	2.0
age	Categorica	3			979.34323	984.6378	5.29458	3.0
age	Categorica	4			944.47162	984.6378	40.16619	4.0

1 Change Constraint
2 Solver
3 Advanced Filter in U1
4 Copy paste in O

Next Sheet	debtinc
Numerical	<=9.028125
Count	1500
	debtinc
Numerical	>9.028125

employ	debtinc	Age	default
7	17.7	2	0
34	14.7	4	0
20	4.8	3	0
11	34.5	2	0
2	22.4	1	1
2	10.9	2	1
2	11.9	1	1
4	14.4	1	0
29	5	4	0
0	3.9	1	0
33	17.5	4	0
4	10.6	2	1
7	19.1	3	0
0	3.9	1	0
9	13.3	4	0
0	7.4	1	1
0	15.7	1	1
2	0.3	1	1
0	5.8	1	1
27	8.5	4	0
1	7.6	1	1
8	8.3	1	0
8	6.1	3	0
17	13.3	3	1
16	3.4	3	0
0	6.4	1	0
1	15.4	3	1
15	5	2	0
23	11.7	3	0
0	6.7	1	0
0	5.2	1	1
0	2	1	1
3	10.6	3	1
3	5.2	3	1
2	3	2	0
0	8.4	1	0
9	10.2	3	0
8	7.2	3	0
15	10.6	3	0
0	17.6	1	1
34	9.9	4	0

Dr	1.1	employ	debtinc	Age	default
0.365333		20	4.8	3	0
		29	5	4	0
		0	3.9	1	0
		0	3.9	1	0
		0	7.4	1	1
		2	0.3	1	1
		0	5.8	1	1
		27	8.5	4	0
		1	7.6	1	1
		8	8.3	1	0
		8	6.1	3	0
		16	3.4	3	0
		0	6.4	1	0
		15	5	2	0
		0	6.7	1	0
		0	5.2	1	1
		0	2	1	1
		3	5.2	3	1
		2	3	2	0
		8	7.2	3	0
		6	3.3	2	1
		0	5.2	1	1
		24	6.8	4	0
		14	6.1	4	0
		5	6.8	2	0
		4	6.9	1	0
		23	4.5	4	0
		2	3	2	1
		20	3.4	4	0
		5	5.6	2	0
		2	8.2	1	0
		7	5.2	3	0
		6	2.5	2	0
		3	1.2	2	0
		0	2	3	0
		6	6.6	3	1
		0	8.2	1	1
		6	6.6	3	0
		14	6.5	3	0
		17	8.8	3	0

7.1 Master Rating System

Rating, Grade, Score and PD - Connecting the dots

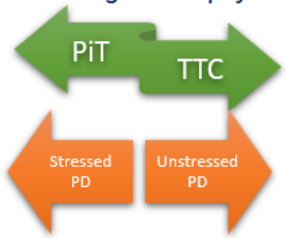
(The following diagram depicts overall IRB lanscape. In practice, the rating system(s) can be more complicated and customized based on size, class, usage)

Risk Driver

Obligor Data	
Static	(e.g. Age, Occupation)
Dynamic	(e.g. Income, Outstanding Debt)
Transaction Data*	
Static	(e.g. Collateral Seniority)
Dynamic	(e.g. limit usage)
Aggregate Data	
(e.g. GDP, Unemployment, Oil Prices)	

*Transaction Data is typically the main driver for LGD estimation

Rating Philosophy



PD Estimation Methods

- Historical average of observed default data
- Statistical Models
- Mapping with External Rating
- Structural Models (for Corporate Exposures)
- Reduced form Models (for Corporate Exposures)
- Expert Judgement

Output

- Distance to Default (DD)
- Probability of Default (PD)
- Score

Rating Assignment

Decision Trees (Segmentation)

Bank's Master Rating System (Indicative)

Grades/ Pools	Level	PD Band (%)		DD Band		Odds Band		Score Band	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1	Excellent	0.000	0.017	∞	3.58	∞	5881	∞	738
2	Prime	0.017	0.031	3.58	3.42	5881	3225	738	720
3	Good	0.031	0.089	3.42	3.12	3225	1123	720	690
4	Above Standard	0.089	0.578	3.12	2.53	1123	172	690	636
5	Standard	0.578	2.709	2.53	1.93	172	36	636	590
6	Below Standard	2.709	11.992	1.93	1.18	36	7	590	545
7	Needs Attention	11.992	99.999	1.18	-4.26	7	0	545	155
D	Bankrupt	100.000	100.000	-∞	-∞	0	0	0	0

Long-run PD (%)
0.01
0.023
0.06
0.3335
1.6435
7.3505
55.9955
100

Capital Requirement Calculation

Validation

Discrimination
Calibration

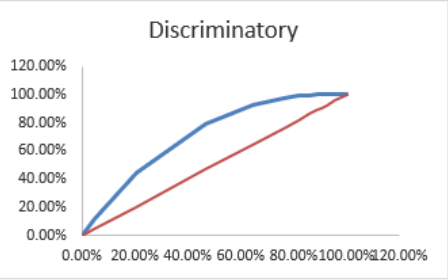
7.2 Validating Master Rating System

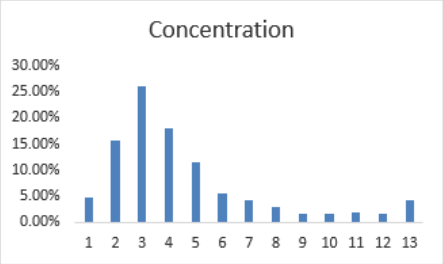
Median	LB	UB
17.408	0	23.02824
30.463	23.02824	40.29942
53.311	40.29942	70.52398
93.294	70.52398	123.417
163.265	123.417	215.9797
285.714	215.9797	377.9645
500.000	377.9645	661.4378
875.000	661.4378	1157.516
1531.250	1157.516	2025.653
2679.688	2025.653	3544.893
4689.453	3544.893	6203.563
8206.543	6203.563	9058.997
10000.000	9058.997	10000

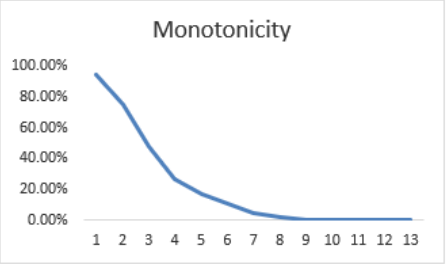
LB	UB	Grade
0.00%	0.23%	1
0.23%	0.40%	2
0.40%	0.71%	3
0.71%	1.23%	4
1.23%	2.16%	5
2.16%	3.78%	6
3.78%	6.61%	7
6.61%	11.58%	8
11.58%	20.26%	9
20.26%	35.45%	10
35.45%	62.04%	11
62.04%	90.59%	12
90.59%	100.00%	13

		0.00%			0.00%	0.00%			
13	70	4.67%	66	548	12.04%	4.67%	4.67%	96.40%	94.29%
12	307	20.47%	245	548	44.71%	20.47%	15.80%	75.37%	75.53%
11	700	46.67%	433	548	79.01%	46.67%	26.20%	47.51%	47.84%
10	969	64.60%	506	548	92.34%	64.60%	17.93%	27.76%	27.14%
9	1143	76.20%	535	548	97.63%	76.20%	11.60%	15.84%	16.67%
8	1227	81.80%	544	548	99.27%	81.80%	5.60%	8.92%	10.71%
7	1291	86.07%	547	548	99.82%	86.07%	4.27%	4.99%	4.69%
6	1336	89.07%	548	548	100.00%	89.07%	3.00%	3.02%	2.22%
5	1359	90.60%	548	548	100.00%	90.60%	1.53%	1.67%	0.00%
4	1384	92.27%	548	548	100.00%	92.27%	1.67%	0.97%	0.00%
3	1412	94.13%	548	548	100.00%	94.13%	1.87%	0.53%	0.00%
2	1436	95.73%	548	548	100.00%	95.73%	1.60%	0.30%	0.00%
1	1500	100.00%	548	548	100.00%	100.00%	4.27%	0.09%	0.00%

13	5.43%	3.39%	0.42%
12	15.87%	16.22%	0.00%
11	26.73%	26.02%	0.01%
10	17.25%	18.00%	0.01%
9	10.76%	12.48%	0.11%
8	5.64%	4.99%	0.03%
7	4.37%	5.53%	0.12%
6	3.41%	2.50%	0.12%
5	1.38%	1.96%	0.09%







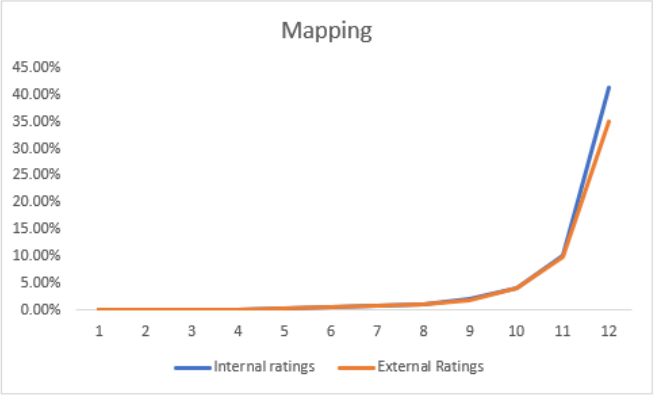
7.3 Mapping Internal & External Ratings

	External					
	Median	Min	Max	Median	Band Width	SD
AAA	0.03%	0.03%	0.04%	0.03%	0.01%	0.01%
AA+	0.06%	0.04%	0.07%	0.06%	0.03%	0.02%
AA	0.09%	0.07%	0.11%	0.09%	0.04%	0.02%
AA-	0.14%	0.11%	0.17%	0.14%	0.06%	0.03%
A+	0.20%	0.17%	0.23%	0.20%	0.06%	0.03%
A	0.26%	0.23%	0.31%	0.26%	0.08%	0.04%
A-	0.37%	0.31%	0.43%	0.37%	0.12%	0.06%
BBB+	0.51%	0.43%	0.63%	0.51%	0.20%	0.10%
BBB	0.78%	0.63%	0.96%	0.78%	0.33%	0.17%
BBB-	1.19%	0.96%	1.48%	1.19%	0.52%	0.27%
BB+	1.83%	1.48%	2.26%	1.83%	0.78%	0.40%
BB	2.80%	2.26%	3.43%	2.80%	1.17%	0.60%
BB-	4.21%	3.43%	4.73%	4.21%	1.30%	0.66%
B+	5.31%	4.73%	5.77%	5.31%	1.04%	0.53%
B	6.27%	5.77%	7.87%	6.27%	2.10%	1.07%
B-	9.87%	7.87%	10.76%	9.87%	2.89%	1.47%
CCC+	11.72%	10.76%	20.25%	11.72%	9.49%	4.84%
CC	35%	20.25%	42%	35.00%	21.75%	11.10%

	Internal				
	Median	Min	Max	Median	Band Width
1	0.02%	0%	0.02%	0.02%	0.02%
2	0.03%	0.02%	0.04%	0.03%	0.02%
3	0.06%	0.04%	0.08%	0.06%	0.04%
4	0.13%	0.08%	0.21%	0.13%	0.13%
5	0.26%	0.21%	0.32%	0.26%	0.11%
6	0.48%	0.32%	0.72%	0.48%	0.40%
7	0.80%	0.72%	0.89%	0.80%	0.17%
8	1.14%	0.89%	1.45%	1.14%	0.56%
9	2.04%	1.45%	2.86%	2.04%	1.41%
10	4.18%	2.86%	6.12%	4.18%	3.26%
11	10.20%	6.12%	17.00%	10.20%	10.88%
12	41.23%	17.00%	100.00%	41.23%	83.00%

	1	2	3	4	5	6	7	8	9	10	11	12
AAA	1145.4425	7819.2687	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA+	85.7108	381.8142	2606.4229	0.0749	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA	5.4499	25.9521	663.5485	286.3606	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AA-	0.6001	2.0474	42.8554	1235.5003	0.6001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A+	0.0000	0.0003	0.0374	95.4082	190.9071	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A	0.0000	0.0001	0.0060	6.1275	977.4086	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A-	0.0001	0.0001	0.0018	0.3001	129.7272	129.7272	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BBB+	0.0038	0.0061	0.0234	0.3808	19.4406	374.4267	6.8904	0.0000	0.0000	0.0000	0.0000	0.0000
BBB	0.0089	0.0116	0.0253	0.1375	2.0107	48.4436	235.2817	24.0932	0.0000	0.0000	0.0000	0.0000
BBB-	0.0090	0.0106	0.0173	0.0514	0.3228	4.1879	51.0422	147.7237	0.8876	0.0000	0.0000	0.0000
BB+	0.0032	0.0036	0.0051	0.0109	0.0418	0.3178	3.5192	22.2987	87.2178	0.0000	0.0000	0.0000
BB	0.0013	0.0014	0.0018	0.0030	0.0078	0.0351	0.2440	1.3988	29.7165	4.6181	0.0000	0.0000
BB-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0013	0.2850	60.0867	0.0000	0.0000
B+	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	7.7860	0.0000	0.0000
B	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0004	0.0154	5.5549	0.0446	0.0000
B-	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0158	26.3871	0.0000
CCC+	0.4446	0.4468	0.4535	0.4695	0.5005	0.5568	0.6477	0.7570	1.1168	2.4508	7.8433	0.0000
CC	0.0250	0.0251	0.0253	0.0258	0.0268	0.0285	0.0311	0.0342	0.0437	0.0760	0.2959	3.0709

1	2	3	4	5	6	7	8	9	10	11	12
AAA	AAA	AA+	AA-	A	BBB+	BBB	BBB-	BB+	BB-	B-	CC



8.1 Roll Rate Analysis

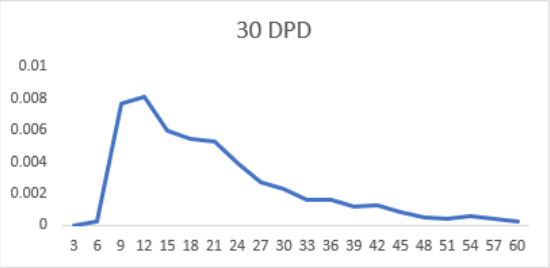
Count of Loan Id	Final State							
Initial State	0	1	2	3	4	5	6	Grand Total
0	13025	339	73	33	20	9	34	13533
1	373	226	63	28	11	11	23	735
2	72	49	27	16	8	5	19	196
3	56	20	12	12	2	5	14	121
4	50	15	4	2		3	16	90
5	32	8	4		3	2	12	61
6	33	7	6	3	3	3	90	145
Grand Total	**13641**	**664**	**189**	**94**	**47**	**38**	**###**	**14881**

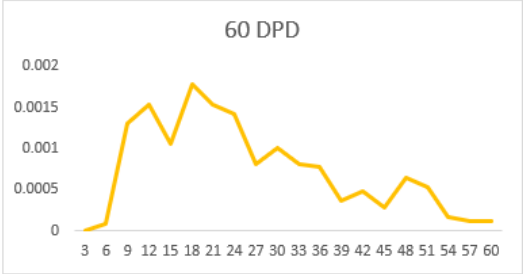
Delinquency	0	0-29	30-59	60-89	90-119	120-149	150+	Grand Total	Roll Back	Roll Forward
Current	13025	339	73	33	20	9	34	13533		
0-29	373	226	63	28	11	11	23	735	0.507483	0.18503401
30-59	72	49	27	16	8	5	19	196	0.617347	0.24489796
60-89	56	20	12	12	2	5	14	121	0.727273	0.17355372
90-119	50	15	4	2		3	16	90	0.788889	0.21111111
120-149	32	8	4		3	2	12	61	0.770492	0.19672131
150+	33	7	6	3	3	3	90	145	0.37931	
Grand Total	13641	664	189	94	47	38	208	14881		

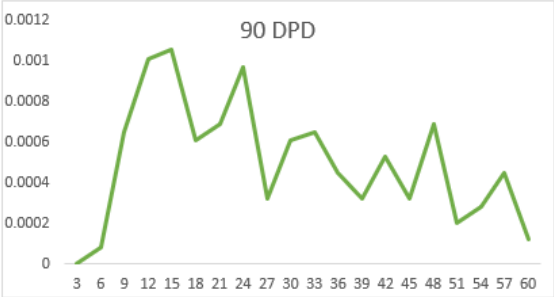
Sharp dip in Roll back probability indicates absorbing state (bad definition)

8.2 Vintage Analysis

Loan Id	30DPD	60DPD	90DPD																								
100007365142						3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60		
100007386460					24640	0	6	189	200	148	134	131	96	68	56	39	40	28	32	20	12	10	15	10	6		
100011322040					24640	0	2	32	38	26	44	38	35	20	25	20	19	9	12	7	16	13	4	3	3		
100015192562					24640	0	2	16	25	26	15	17	24	8	15	16	11	8	13	8	17	5	7	11	3		
100015874399																											
100017922445																											
100020205696						0	0.02%	0.77%	0.81%	0.60%	0.54%	0.53%	0.39%	0.28%	0.23%	0.16%	0.16%	0.11%	0.13%	0.08%	0.05%	0.04%	0.06%	0.04%	0.02%		
100021703104						0	0.01%	0.13%	0.15%	0.11%	0.18%	0.15%	0.14%	0.08%	0.10%	0.08%	0.08%	0.04%	0.05%	0.03%	0.06%	0.05%	0.02%	0.01%	0.01%		
100023274028	18	23	30			0	0.01%	0.06%	0.10%	0.11%	0.06%	0.07%	0.10%	0.03%	0.06%	0.06%	0.04%	0.03%	0.05%	0.03%	0.07%	0.02%	0.03%	0.04%	0.01%		
100026859568																											
100032678432																											
100033892603	37																										
100035719682																											
100036915856																											
100048724218	11																										
100057706656	14																										
100069564325																											
100072038496																											
100076749532																											
100077104442																											
100079064875																											
100094005643																											
100094030068	16																										
100100228738	17	46																									
100111651961																											
100117450928																											
100117997393																											
100118647578	8	49																									
100119700071	12																										
100122555751																											
100122893931																											
100123116925																											
100125587957																											
100132383737																											
100135299882	33																										
100138190869																											
100148648972																											
100151690382																											
100160404887																											
100162317966																											
100166340980	19																										







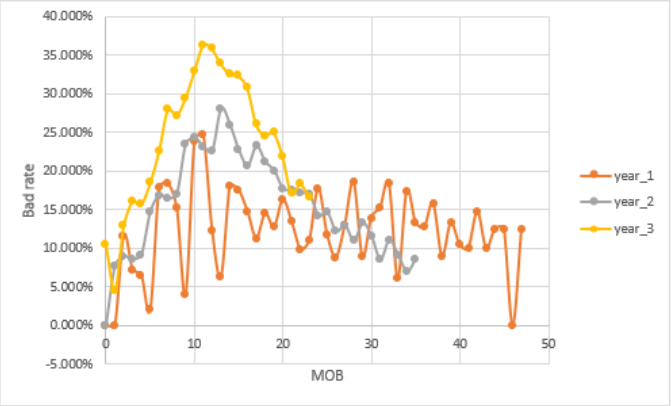
8.3 Seasoning Analysis

Seasoning Analysis - Plotting MOB vs Bad rates for different vintage years

Defaults	Vintage			
MOB	0	1	2	3
0		0	0	47
1		0	37	3
2		11	108	114
3	0	7	158	281
4	1	2	137	306
5	0	2	308	162
6	0	23	322	313
7	0	16	333	497
8	1	5	273	518
9	0	7	484	251
10	1	33	457	458
11	0	19	453	613
12	2	7	354	630
13	1	13	536	264
14	2	30	437	396
15	0	21	391	443
16	1	10	281	442
17	3	24	372	168
18	2	23	296	267
19	2	15	292	294
20	1	13	201	328
21	5	28	235	139
22	2	14	201	156
23	3	14	207	139
24	1	14	218	
25	6	22	155	
26	3	11	131	
27	4	15	113	
28	1	13	86	
29	5	14	67	
30	4	15	47	
31	2	16	34	
32	2	11	33	
33	3	8	11	
34	7	17	9	
35	1	12	11	
36	4	17		
37	5	15		
38	8	7		
39	3	9		
40	7	5		
41	7	3		
42	8	5		

Total	Vintage			
MOB	0	1	2	3
0		16	66	450
1		4	483	66
2		95	1207	886
3	7	97	1848	1749
4	15	31	1496	1948
5	18	100	2100	876
6	21	129	1913	1386
7	18	87	2027	1775
8	11	33	1607	1903
9	16	175	2063	851
10	14	138	1881	1390
11	8	77	1956	1687
12	7	57	1566	1756
13	14	208	1911	778
14	11	167	1680	1215
15	7	120	1719	1369
16	6	68	1360	1435
17	7	213	1602	644
18	9	159	1398	1085
19	7	118	1457	1177
20	1	80	1137	1498
21	17	209	1340	814
22	11	143	1173	849
23	29	127	1223	837
24	61	79	1534	
25	37	187	1060	
26	23	125	1077	
27	45	116	873	
28	70	70	785	
29	46	158	506	
30	44	108	407	
31	54	105	400	
32	81	60	298	
33	51	133	121	
34	54	98	129	
35	60	90	128	
36	87	134		
37	52	95		
38	64	78		
39	61	68		
40	82	48		
41	50	30		
42	64	34		

Bad rate	Vintage			
MOB	0	1	2	3
0		0.000%	0.000%	10.444%
1		0.000%	7.660%	4.545%
2		11.579%	8.948%	12.867%
3	0.000%	7.216%	8.550%	16.066%
4	6.667%	6.452%	9.158%	15.708%
5	0.000%	2.000%	14.667%	18.493%
6	0.000%	17.829%	16.832%	22.583%
7	0.000%	18.391%	16.428%	28.000%
8	9.091%	15.152%	16.988%	27.220%
9	0.000%	4.000%	23.461%	29.495%
10	7.143%	23.913%	24.296%	32.950%
11	0.000%	24.675%	23.160%	36.337%
12	28.571%	12.281%	22.605%	35.877%
13	7.143%	6.250%	28.048%	33.933%
14	18.182%	17.964%	26.012%	32.593%
15	0.000%	17.500%	22.746%	32.359%
16	16.667%	14.706%	20.662%	30.801%
17	42.857%	11.268%	23.221%	26.087%
18	22.222%	14.465%	21.173%	24.608%
19	28.571%	12.712%	20.041%	24.979%
20	100.000%	16.250%	17.678%	21.896%
21	29.412%	13.397%	17.537%	17.076%
22	18.182%	9.790%	17.136%	18.375%
23	10.345%	11.024%	16.926%	16.607%
24	1.639%	17.722%	14.211%	
25	16.216%	11.765%	14.623%	
26	13.043%	8.800%	12.163%	
27	8.889%	12.931%	12.944%	
28	1.429%	18.571%	10.955%	
29	10.870%	8.861%	13.241%	
30	9.091%	13.889%	11.548%	
31	3.704%	15.238%	8.500%	
32	2.469%	18.333%	11.074%	
33	5.882%	6.015%	9.091%	
34	12.963%	17.347%	6.977%	
35	1.667%	13.333%	8.594%	
36	4.598%	12.687%		
37	9.615%	15.789%		
38	12.500%	8.974%		
39	4.918%	13.235%		
40	8.537%	10.417%		
41	14.000%	10.000%		
42	12.500%	14.706%		



9.1 Data Preparation Under Instant Cure & Probationary Period Method

PEAKS2TAILS

Data Preparation

To build a PD base, we need an account to be non-defaulted /up-to-date/ Current status in the beginning of the year and default at least once within the year.

Let's consider the following account. The time points correspond to month beginnings.

up-to-date Default Cure Write-off

	Year 1												Year 2												Year 3												Year 4											
months	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
Events																																																

Under Instant Cure method

✓ for Y1 : Account is up-to-date in the beginning and has defaulted in the year.
So, it will enter in PD base for year 1.

✓ for Y2 : Since, account cures instantly, it will be in up-to-date status in the beginning of year 2.
The Account does default in year 2 as well.
So, it will enter in PD base for year 2.

✗ for Y3 : Since the account does not cure in year 2, the status of the account in the beginning of year 3 will be Defaulted.
So, it will not enter in the PD base for year 3.

✗ for Y4 : Since, the account is already written off in year 3, it won't enter in PD base for year 4.

Under Probationary period method

✓ for Y1 : Account is up-to-date in the beginning and has defaulted in the year.
So, it will enter in PD base for year 1.

✗ for Y2 : Since, account cures on month 10, it will be under probation till month 3 of Year2 and will be in Defaulted status in this per
Therefore, the Account Status in the beginning of year 2 will be Defaulted.
So, it will not enter in PD base for year 2.

✗ for Y3 : Since the account does not cure in year 2, the status of the account in the beginning of year 3 will be Defaulted.
So, it will not enter in the PD base for year 3.

✗ for Y4 : Since, the account is already written off in year 3, it won't enter in PD base for year 4.

9.2 Behavioural Scorecards

PEAKS2TAILS

Calculate status of each ID for each snapshot.

							Default Rates	1.099%		9.202%		17.763%		22.289%		6.475%		5.704%		11.319%		17.542%	
								1		2		3		4		5		6		7		8	
		min	1	12				snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end	snapshot	Perf_end
		max	49	60				1	13	5	17	9	21	13	25	17	29	21	33	25	37	29	41
ID	Origin_month	# Obs	first_time obs	last_time obs	last status_time	first status_time		status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end	status_snapshot	status_Perf_end
1	-7	24	25	48	1	0									0		0		0	0	0	0	0
4	-2	35	25	60	0	0									0		0		0	0	0	0	0
6	19	31	25	56	1	0									0		0		0	0	0	0	0
9	18	13	25	37	1	0									0		0		0	0	1	0	1
26	18	33	25	58	2	0									0		0		0	0	0	0	0
28	18	35	25	60	0	0									0		0		0	0	0	0	0
31	19	35	25	60	0	0									0		0		0	0	0	0	0
56	-15	35	25	60	0	0									0		0		0	0	0	0	0
57	-14	35	25	60	0	0									0		0		0	0	0	0	0
58	-14	35	25	60	0	0									0		0		0	0	0	0	0
61	-13	35	25	60	0	0									0		0		0	0	0	0	0
65	25	27	33	60	0	0													0		0		0
66	25	27	33	60	0	0													0		0		0
69	25	27	33	60	0	0													0		0		0
80	25	27	33	60	0	0													0		0		0
82	25	19	33	52	1	0													0		0		0
88	21	24	25	48	1	0									0		0		0	0	0	0	0
90	25	12	33	44	2	0													0		0		0
100	21	16	25	40	1	0									0		0		0	0	0	0	1
104	21	35	25	60	0	0									0		0		0	0	0	0	0
113	21	35	25	60	0	0									0		0		0	0	0	0	0
114	22	35	25	60	0	0									0		0		0	0	0	0	0
117	21	15	25	39	1	0									0		0		0	0	0	0	1
126	21	35	25	60	0	0									0		0		0	0	0	0	0
133	21	16	25	40	1	0									0		0		0	0	0	0	1
134	21	35	25	60	0	0									0		0		0	0	0	0	0
135	21	35	25	60	0	0									0		0		0	0	0	0	0
136	22	35	25	60	0	0									0		0		0	0	0	0	0
140	22	17	25	41	1	0									0		0		0	0	0	0	1
146	22	35	25	60	0	0									0		0		0	0	0	0	0
153	22	35	25	60	0	0									0		0		0	0	0	0	0
157	22	35	25	60	0	0									0		0		0	0	0	0	0
158	22	30	25	55	2	0									0		0		0	0	0	0	0
160	22	17	26	42	1	0											0		0		0	0	0
164	22	23	26	48	1	0											0		0		0	0	0
165	22	16	26	41	1	0											0		0		0	0	1
166	22	34	26	60	0	0											0		0		0	0	0
168	23	12	26	37	2	0											0		0		2	0	2
170	23	21	26	46	1	0											0		0		0	0	0
172	23	34	26	60	1	0											0		0		0	0	0
175	23	34	26	60	0	0											0		0		0	0	0
179	23	34	26	60	0	0											0		0		0	0	0
181	23	30	26	56	1	0											0		0		0	0	0
183	23	18	26	43	1	0											0		0		0	0	0
186	23	24	26	49	2	0											0		0		0	0	0
194	24	34	26	60	0	0											0		0		0	0	0
196	-3	30	25	55	1	0									0		0		0	0	0	0	0
197	-3	35	25	60	0	0									0		0		0	0	0	0	0
198	-2	35	25	60	0	0									0		0		0	0	0	0	0
200	-4	12	25	36	2	0									0		0		0	0	2	0	2
201	-4	35	25	60	0	0									0		0		0	0	0	0	0

9.3 Behavioural Scorecard Logistic Regression

PEAKS2TAILS

PD = P (Y = 1)

ln(PD / (1 - PD)) = a + b x LTV_WOE + c x FICO_WOE

Max	Sum of ln (Likelihood)	-6411.277

*parameters are estimated using EXCEL Solver

	a	b	c
Training set	-1.357674857	-0.9702574	-0.960193971

SL No.	Y = status_Perf_end	LTV_orig	FICO_orig	Constant	LTV_WOE	FICO_WOE	PD	Yln(PD)+(1-Y)ln(1-PD)
1	1	80	556	1	-0.2490857	-0.180721973	0.280402	-1.271531723
2	1	65	617	1	0.53337722	-0.218563917	0.159049	-1.838545662
3	1	80	556	1	-0.2490857	-0.180721973	0.280402	-1.271531723
4	0	67.7	616	1	0.53337722	-0.218563917	0.159049	-0.173221371
5	0	85	540	1	-0.2490857	-0.180721973	0.280402	-0.329062271
6	0	85	620	1	-0.2490857	-0.218563917	0.287792	-0.339384746
7	0	85.5	654	1	-0.2490857	-0.218563917	0.287792	-0.339384746
8	0	90	508	1	-0.0204793	-0.180721973	0.237891	-0.271665812
9	0	56.7	572	1	0.85311991	-0.180721973	0.11796	-0.125517781
10	0	90	547	1	-0.0204793	-0.180721973	0.237891	-0.271665812
11	0	90	542	1	-0.0204793	-0.180721973	0.237891	-0.271665812
12	0	80	547	1	-0.2490857	-0.180721973	0.280402	-0.329062271
13	0	90	586	1	-0.0204793	-0.180721973	0.237891	-0.271665812
14	0	75	718	1	0.10930545	0.376161879	0.138845	-0.149480582
15	1	85	506	1	-0.2490857	-0.180721973	0.280402	-1.271531723
16	0	80	623	1	-0.2490857	-0.218563917	0.287792	-0.339384746
17	0	85	578	1	-0.2490857	-0.180721973	0.280402	-0.329062271
18	0	53.8	641	1	0.85311991	-0.218563917	0.121793	-0.129873249
19	0	85	516	1	-0.2490857	-0.180721973	0.280402	-0.329062271
20	1	90	589	1	-0.0204793	-0.180721973	0.237891	-1.435942299
21	0	80	514	1	-0.2490857	-0.180721973	0.280402	-0.329062271
22	0	80	587	1	-0.2490857	-0.180721973	0.280402	-0.329062271
23	0	73.2	525	1	0.10930545	-0.180721973	0.215819	-0.243115125
24	0	80	533	1	-0.2490857	-0.180721973	0.280402	-0.329062271
25	0	73.8	590	1	0.10930545	-0.180721973	0.215819	-0.243115125
26	0	75	614	1	0.10930545	-0.218563917	0.222032	-0.251069522
27	1	80	556	1	-0.2490857	-0.180721973	0.280402	-1.271531723
28	1	85	571	1	-0.2490857	-0.180721973	0.280402	-1.271531723
29	1	85.5	654	1	-0.2490857	-0.218563917	0.287792	-1.245518592
30	0	80	555	1	-0.2490857	-0.180721973	0.280402	-0.329062271
31	0	90	541	1	-0.0204793	-0.180721973	0.237891	-0.271665812
32	0	79.5	648	1	0.10930545	-0.218563917	0.222032	-0.251069522
33	0	90	542	1	-0.0204793	-0.180721973	0.237891	-0.271665812
34	0	80	547	1	-0.2490857	-0.180721973	0.280402	-0.329062271
35	0	90	586	1	-0.0204793	-0.180721973	0.237891	-0.271665812

9.4 Behavioural Scorecards WOE & IV

Training set | obs 12980

Windo	ID	snapsh	Perf_e	status_Perf_e	MOB	LTV_o	FICO_o
2	7836	5	17	1	13	80	556
3	7824	9	21	1	12	65	617
3	7836	9	21	1	17	80	556
3	18054	9	21	0	13	67.7	616
3	18057	9	21	0	14	85	540
3	18062	9	21	0	13	85	620
3	18067	9	21	0	13	85.5	654
3	18075	9	21	0	12	90	508
3	18081	9	21	0	15	56.7	572
3	18095	9	21	0	14	90	547
3	18103	9	21	0	13	90	542
3	18108	9	21	0	14	80	547
3	18114	9	21	0	12	90	586
3	18115	9	21	0	14	75	718
3	18123	9	21	1	12	85	506
3	18125	9	21	0	14	80	623
3	18141	9	21	0	13	85	578
3	18142	9	21	0	14	53.8	641
3	18144	9	21	0	12	85	516
3	18148	9	21	1	14	90	589
3	18149	9	21	0	14	80	514
3	18152	9	21	0	14	80	587
3	18154	9	21	0	12	73.2	525
3	18156	9	21	0	12	80	533
3	18161	9	21	0	13	73.8	590
3	40487	9	21	0	13	75	614
4	7836	13	25	1	21	80	556
4	7854	13	25	1	18	85	571
4	18067	13	25	1	17	85.5	654
4	18084	13	25	0	15	80	555
4	18086	13	25	0	15	90	541
4	18088	13	25	0	15	79.5	648
4	18103	13	25	0	17	90	542
4	18108	13	25	0	18	80	547
4	18114	13	25	0	16	90	586
4	18115	13	25	0	18	75	718
4	18118	13	25	0	15	90	544
4	18123	13	25	1	16	85	506
4	18144	13	25	0	16	85	516

LTV_orig		
	min	50.1
	max	145.4

LB	UB	Defaults	Non-Defaults	%Default	%Non-Default	WOE	IV
0	60	76	693	2.86%	6.71%	85.31%	0.03
60	70	191	1265	7.19%	12.25%	53.34%	0.03
70	80	530	2297	19.95%	22.25%	10.93%	0.00
80	90	1365	4134	51.37%	40.05%	-24.91%	0.03
90	100	403	1534	15.17%	14.86%	-2.05%	0.00
100	150	92	400	3.46%	3.87%	11.25%	0.00
	Total	2657	10323	100.00%	100.00%		0.09

*Data transformation in Sheet3

FICO_orig		
	min	413
	max	828

LB	UB	Defaults	Non-Defaults	%Default	%Non-Default	WOE	IV
400	500	41	127	1.54%	1.23%	-22.66%	0.00
500	600	490	1589	18.44%	15.39%	-18.07%	0.01
600	700	1364	4259	51.34%	41.26%	-21.86%	0.02
700	800	746	4222	28.08%	40.90%	37.62%	0.05
800	900	16	126	0.60%	1.22%	70.65%	0.00
	Total	2657	10323	100.00%	100.00%		0.08

Information Value (IV) Vs Predictive Power

IV	Predictive Power
< 0.02	Not Useful
0.02 - 0.1	Weak Predictor
0.1 - 0.3	Medium Predictor
0.3 - 0.5	Strong Predictor
> 0.5	Suspicious

9.5 Out Of Sample Validation

Validating Model Performance on 'in-time Out-of-sample validation set'

	a	b	c
	-1.357674857	-0.9702574	-0.960193971

SL No.	Y = status_Perf_end	LTV_orig	FICO_orig	Constant	LTV_WOE	FICO_WOE	PD	Predicted
1	0	90	662	1	-0.0204793	-0.218563917	0.244541	1
2	0	84.6	567	1	-0.2490857	-0.180721973	0.280402	1
3	1	85	571	1	-0.2490857	-0.180721973	0.280402	1
4	0	90	553	1	-0.0204793	-0.180721973	0.237891	0
5	0	90	627	1	-0.0204793	-0.218563917	0.244541	1
6	0	80	558	1	-0.2490857	-0.180721973	0.280402	1
7	0	80	644	1	-0.2490857	-0.218563917	0.287792	1
8	0	62.9	662	1	0.53337722	-0.218563917	0.159049	0
9	0	67.7	616	1	0.53337722	-0.218563917	0.159049	0
10	0	85	540	1	-0.2490857	-0.180721973	0.280402	1
11	0	90	508	1	-0.0204793	-0.180721973	0.237891	0
12	0	90	547	1	-0.0204793	-0.180721973	0.237891	0
13	1	80	558	1	-0.2490857	-0.180721973	0.280402	1
14	0	80	644	1	-0.2490857	-0.218563917	0.287792	1
15	0	53.8	641	1	0.85311991	-0.218563917	0.121793	0
16	1	90	589	1	-0.0204793	-0.180721973	0.237891	0
17	0	66.7	490	1	0.53337722	-0.226561729	0.160078	0
18	0	55.9	564	1	0.85311991	-0.180721973	0.11796	0
19	1	75	580	1	0.10930545	-0.180721973	0.215819	0
20	1	72	652	1	0.10930545	-0.218563917	0.222032	0
21	1	80	545	1	-0.2490857	-0.180721973	0.280402	1
22	0	90	544	1	-0.0204793	-0.180721973	0.237891	0
23	0	83.6	597	1	-0.2490857	-0.180721973	0.280402	1
24	0	75	560	1	0.10930545	-0.180721973	0.215819	0
25	0	85	626	1	-0.2490857	-0.218563917	0.287792	1
26	0	65	551	1	0.53337722	-0.180721973	0.154249	0
27	0	85	544	1	-0.2490857	-0.180721973	0.280402	1
28	1	94.9	475	1	-0.0204793	-0.226561729	0.245963	1
29	0	93.5	536	1	-0.0204793	-0.180721973	0.237891	0
30	0	97.1	705	1	-0.0204793	0.376161879	0.154597	0
31	1	85	615	1	-0.2490857	-0.218563917	0.287792	1
32	0	80	558	1	-0.2490857	-0.180721973	0.280402	1
33	1	95	601	1	-0.0204793	-0.218563917	0.244541	1
34	0	52	797	1	0.85311991	0.376161879	0.072655	0
35	0	90.3	580	1	-0.0204793	-0.180721973	0.237891	0

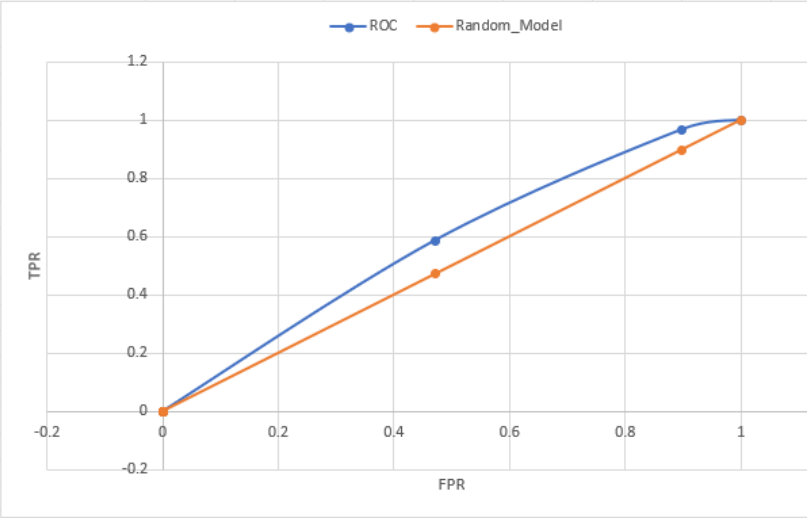
PD Cut-off 0.24

Actual	Predicted 0	Predicted 1
0	2400	1022
1	550	355
Total	2950	1377

Sensitivity (TPR)	0.3923
Specificity (TNR)	0.7013
FPR	0.2987
Accuracy	0.6367

cut-off	TPR	FPR	Random Model
0	1	1	1
0.1	0.9657	0.8966	0.8966
0.2	0.5867	0.4714	0.4714
0.3	0	0	0
0.4	0	0	0
0.5	0	0	0
0.6	0	0	0
0.7	0	0	0
0.8	0	0	0
0.9	0	0	0
1	0	0	0

AUC	0.57
Gini	0.1399



9.6 Out Of Time Validation

Validating Model Performance on 'Out-of-time validation set'

	a	b	c
	-1.357674857	-0.9702574	-0.960193971

SL No.	Y = status_Perf_end	LTV_orig	FICO_orig	Constant	LTV_WOE	FICO_WOE	PD	Predicted
1	1	69.4	715	1	0.53337722	0.376161879	0.096531	0
2	0	81.8	587	1	-0.2490857	-0.180721973	0.280402	1
3	0	80	670	1	-0.2490857	-0.218563917	0.287792	1
4	0	75	630	1	0.10930545	-0.218563917	0.222032	0
5	0	85	575	1	-0.2490857	-0.180721973	0.280402	1
6	0	95	639	1	-0.0204793	-0.218563917	0.244541	1
7	0	52.5	664	1	0.85311991	-0.218563917	0.121793	0
8	0	54.6	669	1	0.85311991	-0.218563917	0.121793	0
9	0	56.9	618	1	0.85311991	-0.218563917	0.121793	0
10	0	64.3	652	1	0.53337722	-0.218563917	0.159049	0
11	0	80	606	1	-0.2490857	-0.218563917	0.287792	1
12	0	70	514	1	0.10930545	-0.180721973	0.215819	0
13	0	75	540	1	0.10930545	-0.180721973	0.215819	0
14	0	80	544	1	-0.2490857	-0.180721973	0.280402	1
15	1	60.6	552	1	0.53337722	-0.180721973	0.154249	0
16	1	90	609	1	-0.0204793	-0.218563917	0.244541	1
17	0	73	579	1	0.10930545	-0.180721973	0.215819	0
18	0	80	636	1	-0.2490857	-0.218563917	0.287792	1
19	0	80	533	1	-0.2490857	-0.180721973	0.280402	1
20	0	77.4	519	1	0.10930545	-0.180721973	0.215819	0
21	0	88.2	605	1	-0.2490857	-0.218563917	0.287792	1
22	0	80	646	1	-0.2490857	-0.218563917	0.287792	1
23	0	90	672	1	-0.0204793	-0.218563917	0.244541	1
24	0	78.8	618	1	0.10930545	-0.218563917	0.222032	0
25	0	70	588	1	0.10930545	-0.180721973	0.215819	0
26	0	80	645	1	-0.2490857	-0.218563917	0.287792	1
27	0	80	622	1	-0.2490857	-0.218563917	0.287792	1
28	1	75.5	605	1	0.10930545	-0.218563917	0.222032	0
29	1	51.6	698	1	0.85311991	-0.218563917	0.121793	0
30	0	80	650	1	-0.2490857	-0.218563917	0.287792	1
31	1	55.1	654	1	0.85311991	-0.218563917	0.121793	0
32	0	80	589	1	-0.2490857	-0.180721973	0.280402	1
33	0	80	585	1	-0.2490857	-0.180721973	0.280402	1
34	0	85	562	1	-0.2490857	-0.180721973	0.280402	1
35	0	80	613	1	-0.2490857	-0.218563917	0.287792	1

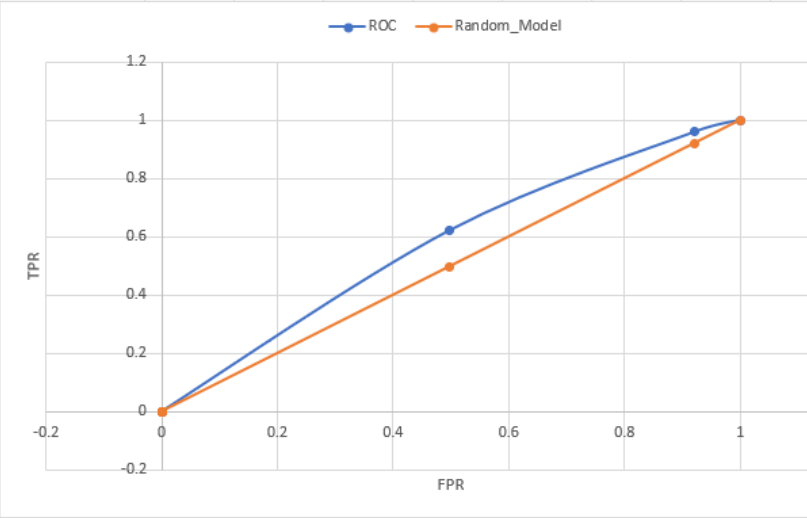
PD Cut-off 0.24

	Predicted	
Actual	0	1
0	14250	7041
1	3230	2222
Total	17480	9263

Sensitivity (TPR)	0.4076
Specificity (TNR)	0.6693
FPR	0.3307
Accuracy	0.6159

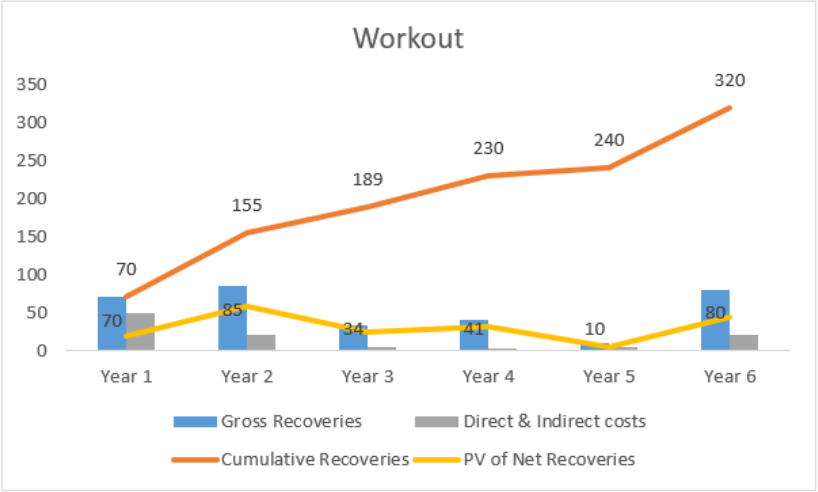
cut-off	TPR	FPR	Random Model
0	1	1	1
0.1	0.96	0.922	0.922
0.2	0.6209	0.4974	0.4974
0.3	0	0	0
0.4	0	0	0
0.5	0	0	0
0.6	0	0	0
0.7	0	0	0
0.8	0	0	0
0.9	0	0	0
1	0	0	0

AUC	0.5665
Gini	0.133



10.1 LGD Workout

	EAD	800			
		Recovery	Cum. Recovery	Costs	PV of Recovery
1	Year 1	70	70	50	19.05
2	Year 2	85	155	20	58.96
3	Year 3	34	189	5	25.05
4	Year 4	41	230	3	31.26
5	Year 5	10	240	5	3.92
6	Year 6	80	320	20	44.77
					0.23



10.2 LGD Decision Tree

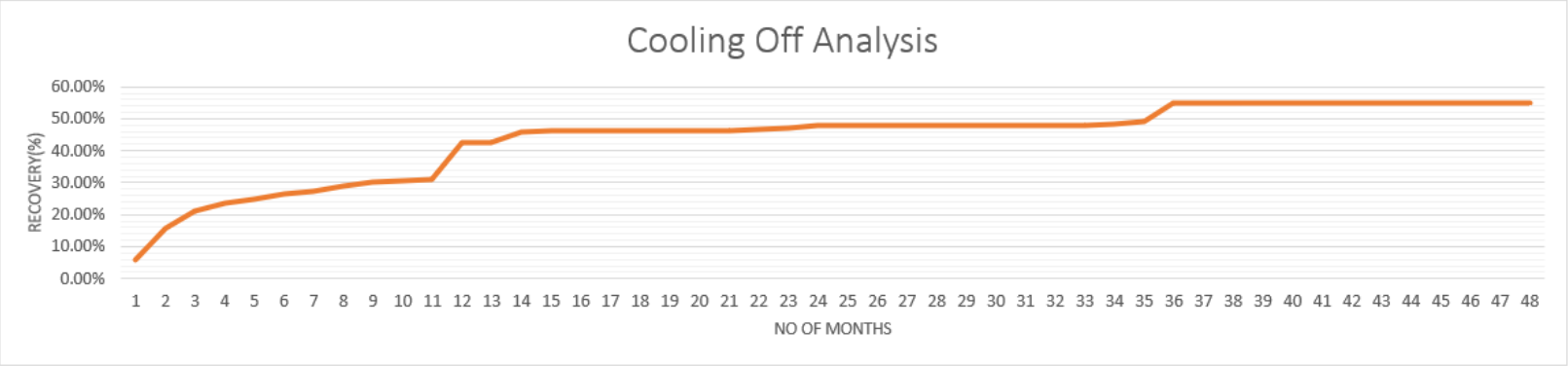
	CN	CO	CP	CQ	CR	CS	CU	CV	CW	CX	CY	CZ	DA	DB	DC	DD	DE	DF	DG
1	33	34	35	36															
2	M33	M34	M35	M36		Customer No.	Default Resoltuion	Outstanding at Default	Flag	Recovery	Collateral	Total Recovery	LGD	LGD Capped	LGD_Flag	Outstanding_Flag			
3	0	0	0	2547		1	Written Off	18408.93	1	2547.17	0	2547.17	86.16%	86.16%	15861.76	18408.93			
4	0	0	0	0		2	Written Off	5204.45	1	0.00	0	0.00	100.00%	100.00%	5204.45	5204.45			
5	0	0	0	1087		3	Written Off	16168.15	1	1086.53	0	1086.53	93.28%	93.28%	15081.62	16168.15			
6	0	0	0	52		4	Written Off	8561.78	1	51.82	0	51.82	99.39%	99.39%	8509.96	8561.78			
7	0	0	0	1240		5	Written Off	26774.57	1	1240.35	0	1240.35	95.37%	95.37%	25534.22	26774.57			
8	0	0	0	0		6	Written Off	11553.87	1	0.00	0	0.00	100.00%	100.00%	11553.87	11553.87			
9	0	1396	2275	1456		7	Written Off	25000	1	5126.83	0	5126.83	79.49%	79.49%	19873.17	25000			
10	0	0	0	2617		8	Written Off	18996.7	1	2616.62	0	2616.62	86.23%	86.23%	16380.08	18996.7			
11	0	0	0	0		9	Written Off	1323.5	1	579.32	0	579.32	56.23%	56.23%	744.18	1323.5			
12	0	0	0	203		10	Written Off	1087.38	1	550.58	0	550.58	49.37%	49.37%	536.80	1087.38			
13	0	0	0	0		11	Written Off	546.28	1	329.44	0	329.44	39.69%	39.69%	216.84	546.28			
14	0	0	0	170		12	Written Off	783.24	1	401.30	0	401.30	48.76%	48.76%	381.94	783.24			
15	0	0	0	0		13	Written Off	453.27	1	229.55	0	229.55	49.36%	49.36%	223.72	453.27			
16	0	0	0	104		14	Written Off	1032.7	1	281.78	0	281.78	72.71%	72.71%	750.92	1032.7			
17	0	0	0	76		15	Written Off	734.24	1	238.60	0	238.60	67.50%	67.50%	495.64	734.24			
18	0	0	0	52		16	Liquidated	298.45	1	174.19	0	174.19	41.63%	41.63%	124.26	298.45			
19	0	0	0	45		17	Written Off	373.65	1	120.89	0	120.89	67.65%	67.65%	252.76	373.65			
20	0	0	0	44		18	Written Off	723.34	1	141.06	0	141.06	80.50%	80.50%	582.28	723.34			
21	0	0	0	0		19	Written Off	112.83	1	72.33	0	72.33	35.90%	35.90%	40.50	112.83			
22	0	0	0	39		20	Written Off	286.19	1	107.04	0	107.04	62.60%	62.60%	179.15	286.19			
23	0	0	0	35		21	Written Off	468.33	1	93.83	0	93.83	79.97%	79.97%	374.50	468.33			
24	0	0	0	0		22	Written Off	80.08	1	44.56	0	44.56	44.35%	44.35%	35.52	80.08			
25	0	0	0	0		23	Written Off	62.07	1	43.87	0	43.87	29.32%	29.32%	18.20	62.07			
26	0	0	0	0		24	Written Off	72.01	1	38.17	0	38.17	47.00%	47.00%	33.84	72.01			
27	0	0	0	0		25	Written Off	82.85	1	33.89	0	33.89	59.10%	59.10%	48.96	82.85			
28	0	0	0	139		26	Written Off	1374.33	1	173.08	0	173.08	87.41%	87.41%	1201.25	1374.33			
29	0	0	0	12		27	Written Off	437.02	1	32.73	0	32.73	92.51%	92.51%	404.29	437.02			
30	0	0	0	8		28	Written Off	165.65	1	20.65	0	20.65	87.54%	87.54%	145.00	165.65			
31	0	0	0	0		29	Written Off	7000	1	0.00	0	0.00	100.00%	100.00%	7000.00	7000			
32	0	0	0	1129		30	Written Off	15000	1	1334.56	0	1334.56	91.10%	91.10%	13665.44	15000			
33	0	0	0	0		31	Written Off	327.98	1	0.00	0	0.00	100.00%	100.00%	327.98	327.98			
34	0	0	0	0		32	Written Off	1386.67	1	0.00	0	0.00	100.00%	100.00%	1386.67	1386.67			
35	0	0	0	0		33	Written Off	400.58	1	0.00	0	0.00	100.00%	100.00%	400.58	400.58			
36	0	0	0	0		34	Written Off	384.55	1	0.00	0	0.00	100.00%	100.00%	384.55	384.55			

Weighted Average LGD	60.76%
Recovery	39.24%
Cost of Recovery	2%
LGD	62.76%

10.3 LGD Cooling Off Period

User Input

Direct Cost of Recovery	2%
Indirect Cost of Recovery	0%
Total Cost of Recovery	2%



10.4 LGD Bimodal

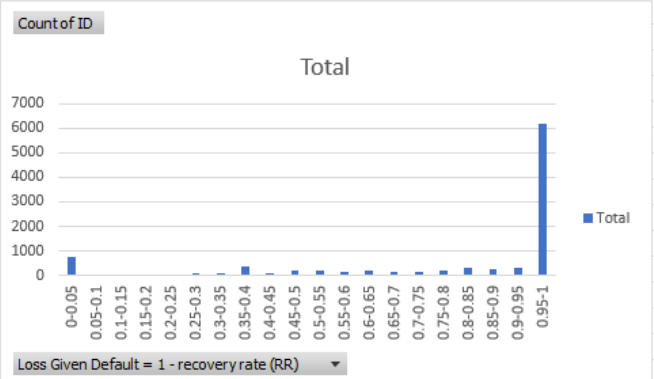
Label_"Balance capital > 500"	(All)

Row Labels	Count of ID	Average of Capital Balance	Average of Recovery rate	Average of Loss Given Default = 1 - recovery rate (RR)
default (not written off)	179	80.63256983	5%	95%
keep in default	902	628.0258315	41%	59%
out of default	163	1061.997546	67%	33%
refinancing	97	2458.327526	39%	61%
written off	8426	1833.631409	16%	84%
Grand Total	**9767**	**1683.490749**	**0.191702602**	**0.808297398**

Label_"Balance capital > 500"	(All)
Final Status	(All)
Sum_DCF	(All)

Row Labels	Count of ID	Average of Recovery rate	ss Given Default = 1 - recovery rate (RR)
0-1000	5327	0.245258733	0.754741267
1000-2000	2169	0.129625737	0.870374263
2000-3000	919	0.137978842	0.862021158
3000-4000	443	0.135673455	0.864326545
4000-5000	253	0.106524095	0.893475905
5000-6000	166	0.11548878	0.88451122
6000-7000	101	0.124744407	0.875255593
7000-8000	79	0.10422864	0.89577136
>8000	310	0.099506254	0.900493746
Grand Total	**9767**	**0.191702602**	**0.808297398**

Row Labels	Count of ID
0-0.05	745
0.05-0.1	32
0.1-0.15	35
0.15-0.2	49
0.2-0.25	53
0.25-0.3	70
0.3-0.35	83
0.35-0.4	358
0.4-0.45	117
0.45-0.5	218
0.5-0.55	216
0.55-0.6	132
0.6-0.65	174
0.65-0.7	148
0.7-0.75	156



10.5 LGD Tobit

PEAKS2TAILS

Tobit Model

b0	b1	b2	c0	c1	c2
-0.827	1.411755	-0.00883	-27.1387	-7.2091	1.184872

Maximize | total LL | -145.042238

$$Y = \begin{cases} 0 & for\ Y^* \le 0 \\ X'\beta + \varepsilon & for\ 1 > Y^* > 0, where\ the\ error\ term\ \varepsilon \sim Normal(0, \sigma^2) \\ 1 & for\ Y^* \ge 1 \end{cases}$$

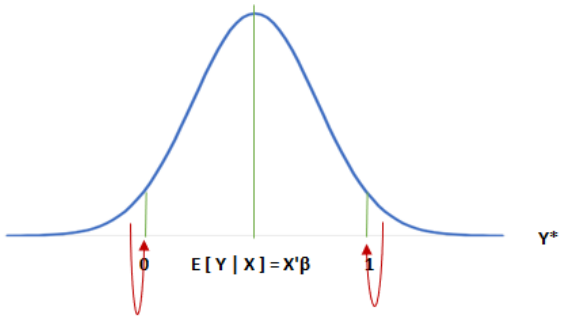
Idea is to consider a latent variable Y* which follows Normal distribution
But then Y* can take all values in Real Number line if it's normal

Since Y is bounded [0 , 1]
All the mass of Y* < 0 collapses onto value Y = 0
All the mass of Y* > 1 collapses onto value Y = 1

So we can model Y as normal random only that
Y = 0 and Y = 1 will behave as point masses (discrete)

Likelihood of Y therefore is given as

$$L(Y) = \begin{cases} CDF(Y^*=0) & , Y = 0 \\ PDF(Y^*) & , Y = (0,1) \\ 1 - CDF(Y^*=1) & , Y = 1 \end{cases}$$



To address heteroscedasticity we need to have another model for variance

$$E(\varepsilon^2) = \sigma^2 \times (1 + EXP(Z'G))$$

Let b0, b1, b2 be the parameters for Y*
c0, c1, c2 be the parameters for variance of error term

(y)	(x1)	(x2)	Y*				Y			
LGD	LTV	MOB	mean	SD	PDF(Y*)	CDF(Y*)	Likelihood(Y)	LL (Y)	fitted (LGD)	R^2
0.1375	0.4484	0.9928	-0.20269	1	0.376516	0.633134	0.376515949	-0.97679	0	
0.579	0.8388	0.1415	0.355925	1	0.389139	0.588262	0.389138542	-0.94382	0.355924605	44.60%
0.2655	0.5225	-0.213	-0.08747	1	0.374852	0.637937	0.374851618	-0.98123	0	
0.7821	1.2316	-0.745	0.9183	1	0.395256	0.445812	0.39525643	-0.92822	0.91829978	
0.0041	0.4	-0.639	-0.25666	1	0.385607	0.602863	0.385606619	-0.95294	0	
0	0.5	-0.923	-0.11298	1	0.396404	0.544979	0.544978556	-0.60701	0	
0.7138	0.9623	-0.639	0.537142	1	0.392765	0.570116	0.392764765	-0.93454	0.537141525	
0.1698	0.2702	1.0992	-0.45518	1	0.328171	0.733999	0.32817059	-1.11422	0	
0.4655	0.8474	0.3898	0.365839	1	0.396965	0.539705	0.396964801	-0.92391	0.36583916	
0.2513	0.75	1.0637	0.222423	1	0.398776	0.511503	0.398776442	-0.91935	0.222423131	
0.5207	0.8409	0.2124	0.358284	1	0.393713	0.564523	0.39371307	-0.93213	0.358284009	
0.5295	0.822	0.4962	0.329084	1	0.39101	0.579422	0.391010146	-0.93902	0.32908424	
0.0959	0.9351	0.0351	0.492817	1	0.368723	0.345714	0.368722749	-0.99771	0.492817477	
0.4045	0.6455	0.1769	0.082659	1	0.378804	0.626221	0.378804218	-0.97074	0.082659009	
0.5285	0.787	-0.426	0.287845	1	0.387556	0.595089	0.38755565	-0.9479	0.287845496	
0.6209	0.8808	-0.923	0.42459	1	0.391329	0.577812	0.391329496	-0.93821	0.424590027	
0	0.2565	4.1851	-0.50181	1	0.351746	0.692099	0.692099283	-0.36803	0	
0.2631	0.426	0.6735	-0.23157	1	0.353003	0.689577	0.353003142	-1.04128	0	
0.2702	0.4313	0.6381	-0.2238	1	0.353119	0.689343	0.353118837	-1.04095	0	
0	0.45	-0.568	-0.1867	1	0.392049	0.574053	0.574053137	-0.55503	0	
0.0315	0.6	-0.532	0.024747	1	0.398933	0.50269	0.398933212	-0.91896	0.024747408	
0.162	0.449	-0.213	-0.1913	1	0.374804	0.638071	0.374804335	-0.98135	0	
0.0144	0.1	0.709	-0.69209	1	0.310826	0.760067	0.310826157	-1.16852	0	
0.4266	0.5761	0.1769	-0.01531	1	0.361828	0.670728	0.36182789	-1.01659	0	
0.3265	0.6088	0.0351	0.032113	1	0.382029	0.615754	0.382029062	-0.96226	0.03211306	
0.4537	0.6036	-1.135	0.035187	1	0.365494	0.662207	0.365493566	-1.00651	0.035187302	
0.0008	0.1	-0.71	-0.67956	1	0.316517	0.75186	0.316516561	-1.15038	0	
0	0.4	0.7799	-0.26919	1	0.384747	0.606107	0.606106922	-0.5007	0	
0.225	0.9973	-0.781	0.587829	1	0.373526	0.358359	0.373525807	-0.98477	0.587828865	
0	0.115	2.7308	-0.68873	1	0.314707	0.754504	0.754504098	-0.28169	0	
0	0.2987	0.1415	-0.40652	1	0.367302	0.657821	0.657821398	-0.41882	0	
0.3115	0.5508	0.3188	-0.05223	1	0.373409	0.641962	0.373409204	-0.98508	0	
0	0.2755	0.5316	-0.44271	1	0.361702	0.671013	0.671012705	-0.39897	0	
0	0.2556	0.1415	-0.46746	1	0.35765	0.679915	0.679915457	-0.38579	0	
0.4216	0.6465	0.6381	0.080024	1	0.376335	0.633665	0.37633497	-0.97728	0.080023881	
0	0.2328	-0.887	-0.49046	1	0.353733	0.688096	0.6880956	-0.37383	0	

10.6 LGD NLS Regression

NLS regression model

$$Y = \frac{1}{1 + EXP(-X`\beta)} + \varepsilon, where\ the\ error\ term\ \varepsilon \sim Normal(0, \sigma^2)$$

It's non linear because Y is not a linear function of X

The conditional Mean of Y i.e. **E[Y|X]**

$$E[Y|X] = \frac{1}{1 + EXP(-X`\beta)}$$

This is the sigmoid function which returns values only in the range [0,1]
Thus ensuring our fitted Y will always be a fraction

To address heteroscedasticity we need to have another model for variance

$$E(\varepsilon^2) = \sigma^2 \times (1 + EXP(Z`G))$$

Let b0, b1, b2 be the parameters for Y*
c0, c1, c2 be the parameters for variance of error term

σ is taken as 0.1

b0	b1	b2	c0	c1	c2
-3.93324	4.518042	-0.04178	-3.40887	5.29388	-0.145807216

Maximize total LL 104.7014

(y)	(x1)	(x2)							R^2
LGD	LTV	MOB	mean	SD	Likelihood(Y)	LL (Y)	fitted (LGD)		
0.1375	0.4484	0.9928	0.124699	0.114341	3.467330017	1.243385	0.124699017		
0.579	0.8388	0.1415	0.462714	0.193607	1.720491289	0.54261	0.462714164		54.99%
0.2655	0.5225	-0.213	0.173129	0.124195	2.436333938	0.890494	0.173128753		
0.7821	1.2316	-0.745	0.840554	0.510124	0.776924184	-0.25241	0.840553522		
0.0041	0.4	-0.639	0.109162	0.114094	2.28833081	0.827823	0.109162079		
0	0.5	-0.923	0.163055	0.123854	1.354064342	0.303111	0.163055258		
0.7138	0.9623	-0.639	0.608519	0.263077	1.399728708	0.336278	0.608518788		
0.1698	0.2702	1.0992	0.059627	0.105728	2.193095197	0.785314	0.059626708		
0.4655	0.8474	0.3898	0.469774	0.194261	2.053148469	0.719374	0.469773955		
0.2513	0.75	1.0637	0.35683	0.15816	2.018664254	0.702436	0.356830318		
0.5207	0.8409	0.2124	0.464353	0.193667	1.974479727	0.680305	0.464353055		
0.5295	0.822	0.4962	0.440257	0.18406	1.927089584	0.656011	0.440257043		
0.0959	0.9351	0.0351	0.572025	0.237644	0.225597647	-1.489	0.572025063		
0.4045	0.6455	0.1769	0.264165	0.1408	1.723970139	0.54463	0.26416472		
0.5285	0.787	-0.426	0.411053	0.180825	1.786676853	0.580357	0.411053243		
0.6209	0.8808	-0.923	0.52117	0.223798	1.614142452	0.478804	0.521170367		
0	0.2565	4.1851	0.049776	0.103434	3.43523518	1.234085	0.049776215		
0.2631	0.426	0.6735	0.115394	0.113399	1.506580626	0.409843	0.115394434		
0.2702	0.4313	0.6381	0.118008	0.113823	1.433876971	0.360382	0.118008063		
0	0.45	-0.568	0.132804	0.117862	1.794076573	0.58449	0.132804008		
0.0315	0.6	-0.532	0.231446	0.136253	0.997484491	-0.00252	0.231445637		
0.162	0.449	-0.213	0.130574	0.11694	3.290437323	1.19102	0.130574129		
0.0144	0.1	0.709	0.028999	0.102501	3.852959097	1.348841	0.028999292		
0.4266	0.5761	0.1769	0.207843	0.12963	0.74088944	-0.2999	0.207843367		
0.3265	0.6088	0.0351	0.234284	0.135127	2.339516141	0.849944	0.234284336		
0.4537	0.6036	-1.135	0.238923	0.139762	0.876617803	-0.13168	0.238922871		
0.0008	0.1	-0.71	0.030716	0.103067	3.710926538	1.311282	0.030715898		
0	0.4	0.7799	0.10353	0.111596	2.324724198	0.843601	0.103529651		
0.225	0.9973	-0.781	0.646851	0.287677	0.473173544	-0.74829	0.646850896		
0	0.115	2.7308	0.028535	0.102021	3.760378871	1.32452	0.028534681		
0	0.2987	0.1415	0.069818	0.107589	3.003990328	1.099942	0.069817969		
0.3115	0.5508	0.3188	0.188768	0.125819	1.970649626	0.678363	0.188768196		
0	0.2755	0.5316	0.062352	0.106378	3.158315165	1.150039	0.062352035		
0	0.2556	0.1415	0.058167	0.106083	3.235780081	1.17427	0.058167077		
0.4216	0.6465	0.6381	0.261322	0.138691	1.475222902	0.388809	0.261322176		
0	0.2328	-0.887	0.054983	0.106261	3.283965371	1.189052	0.054982767		

10.7 LGD Fractional Logit

Fractional Logit

$$F(Y) = G(X`\beta)^{Y} \times \left(1 - G(X`\beta)\right)^{1-Y} \; for \; 1 \geq Y \geq 0$$

We do not model an error term and so we don't need any distribution for the same
Rather we specify logistic likelihood for Y directly

G(X'β) is the conditional Mean of Y i.e. **E[Y|X]**

$$E[Y|X] = \frac{1}{1 + EXP(-X`\beta)}$$

The formulation is identical to a binary logistic regression framework
The only difference is in binary logistic regression Y is Bernouli { 0, 1 } - discrete values
Here, Y is a continuous fraction [0 , 1] , hence the name **fractional logit**

Also in binary logistic regression, only one part of the likelihood function F(Y) is active
depending on if Y = 0 or Y = 1

In fractional logit, both parts will be active as long as Y is a +ve fraction

b0	b1	b2
-3.71936	4.014858	-0.07811

Maximize | total LL | -77.5933

(y) LGD	(x1) LTV	(x2) MOB	G (X'β)	ln (F(y))	fitted (LGD)
0.1375	0.4484	0.9928	0.119571402	-0.40182	0.119571402
0.579	0.8388	0.1415	0.410293493	-0.73816	0.410293493
0.2655	0.5225	-0.213	0.167296734	-0.60915	0.167296734
0.7821	1.2316	-0.745	0.783058175	-0.5243	0.783058175
0.0041	0.4	-0.639	0.1126936	-0.12803	0.1126936
0	0.5	-0.923	0.162482955	-0.17731	0.162482955
0.7138	0.9623	-0.639	0.548340935	-0.65637	0.548340935
0.1698	0.2702	1.0992	0.061791434	-0.52559	0.061791434
0.4655	0.8474	0.3898	0.41393606	-0.6962	0.41393606
0.2513	0.75	1.0637	0.311890098	-0.57263	0.311890098
0.5207	0.8409	0.2124	0.411007342	-0.7167	0.411007342
0.5295	0.822	0.4962	0.387487844	-0.73264	0.387487844
0.0959	0.9351	0.0351	0.50804802	-0.70629	0.50804802
0.4045	0.6455	0.1769	0.241996178	-0.73893	0.241996178
0.5285	0.787	-0.426	0.371385221	-0.74237	0.371385221
0.6209	0.8808	-0.923	0.472260494	-0.70812	0.472260494
0	0.2565	4.1851	0.046691364	-0.04782	0.046691364
0.2631	0.426	0.6735	0.112873465	-0.66217	0.112873465
0.2702	0.4313	0.6381	0.115297585	-0.67308	0.115297585
0	0.45	-0.568	0.133736735	-0.14357	0.133736735
0.0315	0.6	-0.532	0.219452917	-0.28772	0.219452917
0.162	0.449	-0.213	0.130088073	-0.44721	0.130088073
0.0144	0.1	0.709	0.033141967	-0.08238	0.033141967
0.4266	0.5761	0.1769	0.194600805	-0.82237	0.194600805
0.3265	0.6088	0.0351	0.21789214	-0.66298	0.21789214
0.4537	0.6036	-1.135	0.230195919	-0.8093	0.230195919
0.0008	0.1	-0.71	0.036882638	-0.04016	0.036882638
0	0.4	0.7799	0.102079035	-0.10767	0.102079035
0.225	0.9973	-0.781	0.5855596	-0.80306	0.5855596
0	0.115	2.7308	0.030152265	-0.03062	0.030152265
0	0.2987	0.1415	0.0737089	-0.07657	0.0737089
0.3115	0.5508	0.3188	0.177571214	-0.67296	0.177571214
0	0.2755	0.5316	0.065702555	-0.06796	0.065702555
0	0.2556	0.1415	0.062716292	-0.06477	0.062716292
0.4216	0.6465	0.6381	0.236186441	-0.76426	0.236186441
0	0.2328	-0.887	0.062079944	-0.06409	0.062079944

R^2
54.93%

10.8 LGD Fractional & Beta

PEAKS2TAILS

Two part Model (Logit + Beta)

We will employ a binary logit model to get the likelihood for LGD = 0 case
For LGD ≠ 0 case, we can emply either Beta Regression or Simplex Regression

	b0	b1	b2
Logit parameters	-1.5306	4.881516	0.026797
Beta (mu) parameters	-2.92551	3.251345	-0.03085
Beta (φ) parameters	2.717648	-0.80074	0.48286

Maximize LL -1.23023397

(y)	(x1)	(x2)											
LGD	LTV	MOB	x_a	x_b	x_c	mu_xa	mu_xb	(φ)	w	t	L	LL	fitted (LGD)
0.1375	0.448436	0.9928	0.685048	-1.49812	2.837931	0.66486448	0.182707	17.0803872	3.120699354	13.95968785	3.033882605	1.109843187	0.121475124
0.579	0.838799	0.1415	2.5678	-0.20265	2.114299	0.9287603	0.449511	8.28377969	3.723651761	4.560127929	1.594878952	0.466797841	0.417488159
0.2655	0.522509	-0.213	1.01432	-1.22007	2.196294	0.73386474	0.227924	8.991627626	2.049408049	6.942219576	1.749177545	0.559145703	0.167265412
0.7821	1.231607	-0.745	4.461538	1.101869	1.371583	0.98858716	0.75061	3.941584473	2.958593359	0.982991114	1.797639077	0.58647418	0.742043569
0.0041	0.4	-0.639	0.404887	-1.60526	2.088865	0.59986117	0.167248	8.075746903	1.350655455	6.725091449	1.298479471	0.261193942	0.1003258
0	0.5	-0.923	0.885434	-1.27137	1.871774	0.70794709	0.219024	6.499816794	1.423612794	5.076204	0.292052908	-1.2308203	0.155057069
0.7138	0.962283	-0.639	3.149681	0.222921	1.638623	0.95889615	0.555501	5.148073995	2.859758605	2.28831539	1.503484435	0.407785371	0.532667464
0.1698	0.270247	1.0992	-0.18193	-2.08075	3.031995	0.4546426	0.110982	20.73856385	2.301602576	18.43696128	1.33732555	0.290671761	0.050457042
0.4655	0.847374	0.3898	2.616313	-0.18243	2.227323	0.93190413	0.45452	9.275004677	4.215671148	5.059333529	2.20007839	0.788492991	0.423568665
0.2513	0.75	1.0637	2.159041	-0.51982	2.630711	0.89651057	0.372895	13.88363074	5.177141972	8.706488763	2.036223275	0.711096756	0.334304662
0.5207	0.840914	0.2124	2.580025	-0.19796	2.14686	0.92956488	0.450671	8.557946963	3.856820909	4.701126054	1.930542879	0.657801248	0.418928195
0.5295	0.822004	0.4962	2.495322	-0.26819	2.299019	0.92381325	0.43335	9.96440398	4.318077913	5.646326068	1.875218797	0.628725344	0.40033479
0.0959	0.9351	0.0351	3.035045	0.113746	1.985806	0.95413247	0.528406	7.28491358	3.84939101	3.43552257	0.075009076	-2.59014616	0.504169185
0.4045	0.645456	0.1769	1.624945	-0.83236	2.286244	0.835476	0.303145	9.837915307	2.982318506	6.855596801	1.555184456	0.44159416	0.253270683
0.5285	0.787028	-0.426	2.299872	-0.35346	1.88172	0.9088664	0.412543	6.564785027	2.708258555	3.856526473	1.477824949	0.390571378	0.374946812
0.6209	0.880785	-0.923	2.744241	-0.0333	1.566864	0.93958726	0.491675	4.791600312	2.35590962	2.435690692	1.403451946	0.338934878	0.461971476
0	0.256512	4.1851	-0.16629	-2.22062	4.533057	0.45852397	0.097914	93.04256934	9.110158304	83.93241104	0.541476033	-0.61345647	0.044895857
0.2631	0.425979	0.6735	0.566873	-1.56128	2.701768	0.63804133	0.173463	14.90606224	2.585650399	12.32041185	1.325406748	0.281719392	0.110676569
0.2702	0.431263	0.6381	0.591717	-1.54301	2.68041	0.64375901	0.176099	14.59106897	2.569469639	12.02159933	1.28127709	0.247857307	0.113365185
0	0.45	-0.568	0.650863	-1.44488	2.083083	0.65720502	0.190791	8.029182338	1.531897705	6.497284634	0.342794985	-1.07062272	0.125388964
0.0315	0.6	-0.532	1.384041	-0.95827	1.980099	0.79963927	0.277225	7.243459002	2.008066291	5.235392711	0.706493322	-0.34744153	0.221679816
0.162	0.448964	-0.213	0.65531	-1.45919	2.255184	0.65820604	0.188591	9.537049477	1.79860384	7.738445637	2.174846638	0.776958151	0.124131882
0.0144	0.1	0.709	-1.02345	-2.62225	2.97992	0.26435602	0.06772	19.68623877	1.333159168	18.3530796	2.744356621	1.009546665	0.017902285
0.4266	0.576058	0.1769	1.28618	-1.058	2.341813	0.78349991	0.257692	10.40007704	2.680015646	7.720061398	0.916712046	-0.08696187	0.201901584
0.3265	0.608765	0.0351	1.442037	-0.94728	2.247115	0.80876994	0.279432	9.460401361	2.64353909	6.81686227	1.862651192	0.622000845	0.22599622
0.4537	0.603625	-1.135	1.38558	-0.92788	1.686034	0.79988564	0.283355	5.398030027	1.529561002	3.868469025	0.912907861	-0.09112032	0.226651922
0.0008	0.1	-0.71	-1.06147	-2.57847	2.294833	0.25702883	0.070537	9.922779211	0.699922606	9.222856604	7.855992314	2.061276593	0.01813003

Beta Model

$$F(Y) = \frac{Gamma(\omega + \tau)}{(Gamma(\omega) \times Gamma(\tau))} \times Y^{\omega-1} \times (1-Y)^{\tau-1}$$

In the above formulation, while ω is pulling the density toward 0, τ is pushing the density toward 1. Without the lo generality, ω and τ can be re-parameterized and translated into two other parameters, location parameter μ dispersion parameter φ with $\omega = \mu \times \varphi$ and $\tau = \varphi \times (1 - \mu)$, of which μ is the expected mean and φ governs variance such that

$$\sigma^2 = \frac{\mu \times (1-\mu)}{(1+\varphi)}$$

Within the framework of Generalized Linear Models (GLM), μ and φ can be modeled separately with a location n for μ and a dispersion model for φ using two different or identical sets of covariates X and Z. Since the expe mean μ is bounded by 0 and 1, a natural choice of the link function is the Logit function such that $LOG\ [\mu / (1 - X'\beta$. With the strictly positive nature of φ, the Log function seems appropriate such that $LOG\ (\varphi) = Z'\gamma$.

R^2
56.60%

10.9 LGD Logit & Simplex

Two part Model (Logit + Simplex)

We will employ a binary logit model to get the likelihood for LGD = 0 case
For LGD ≠ 0 case, we can emply either Beta Regression or Simplex Regression

	0.539468306	0.515262533	0.515565972
	0.472959463	0.429143827	0.168625322
	0.383199954	0.202347489	0.832226131

	b0	b1	b2
Logit parameters	-1.53025	4.880866	0.026345
Simplex (D) parameters	-2.84058	2.790531	-0.03101
Simplex (V) parameters	4.637479	-3.61859	-0.97709

Maximize LL -46.0363166

(y)	(x1)	(x2)											
LGD	LTV	MOB	x_a	x_b	x_c	mu_xa	mu_xb	s^2	v	d	L	LL	fitted (LGD)
0.1375	0.44843561	0.9928	0.684655	-1.61999	2.044761	0.66477674	0.165206	7.727313465	0.001667347	0.340850603	2.285494817	0.826582551	0.109825317
0.579	0.83879876	0.1415	2.567538	-0.50427	1.463982	0.92874292	0.376537	4.323139764	0.014483782	3.051347118	1.040395964	0.039601376	0.349706495
0.2655	0.52250857	-0.213	1.014423	-1.37589	2.955083	0.73388482	0.20167	19.20330957	0.007414917	0.805554791	0.759779766	-0.27472667	0.148002529
0.7821	1.231607	-0.745	4.461421	0.61937	0.909013	0.98858584	0.650075	2.481870579	0.004951926	1.974756207	2.389844827	0.871228438	0.642655227
0.0041	0.4	-0.639	0.405261	-1.70455	3.814283	0.59995109	0.153871	45.34422374	6.80767E-08	324.0929449	3.821457869	1.340631991	0.092315287
0	0.5	-0.923	0.885872	-1.4167	3.729685	0.70803761	0.195179	41.66599691	0		0.29196239	-1.23113029	0.138194149
0.7138	0.9622832	-0.639	3.149691	-0.13549	1.779611	0.95889653	0.46618	5.927552731	0.008525331	4.846983923	1.130645071	0.122788329	0.447018612
0.1698	0.27024722	1.0992	-0.18226	-2.12053	2.585579	0.45456175	0.107117	13.27096599	0.002800129	3.044455246	0.838778888	-0.17580815	0.048691483
0.4655	0.84737386	0.3898	2.615933	-0.48804	1.190348	0.93187998	0.380355	3.288226805	0.015403273	0.52491444	1.525170741	0.422106365	0.354445261
0.2513	0.75	1.0637	2.158419	-0.78066	0.884209	0.89645288	0.314177	2.421067477	0.006658297	0.453215378	2.565094061	0.941995149	0.281644577
0.5207	0.84091352	0.2124	2.579728	-0.50057	1.387014	0.92954549	0.377407	4.002880242	0.015544564	1.49077462	1.234053485	0.210304267	0.350816785
0.5295	0.82200419	0.4962	2.49491	-0.56214	1.178178	0.92378424	0.363053	3.248449626	0.015462395	2.079588846	1.193963586	0.177278517	0.335382969
0.0959	0.9351	0.0351	3.034768	-0.23224	1.21948	0.95412033	0.4422	3.385428329	0.000651786	22.73393215	0.282136957	-1.26536266	0.421911626
0.4045	0.64545599	0.1769	1.624792	-1.0449	2.128952	0.83545494	0.260206	8.40605393	0.013977244	2.333224502	0.846357438	-0.1668135	0.21739008
0.5285	0.78702774	-0.426	2.299899	-0.63114	2.205843	0.90886866	0.347252	9.077898798	0.015473197	2.565902785	0.839945279	-0.17441853	0.315606298
0.6209	0.88078466	-0.923	2.744431	-0.35411	2.351782	0.93959808	0.412386	10.50427682	0.013041865	3.145264402	0.871925733	-0.13705103	0.387477019
0	0.25651197	4.1851	-0.168	-2.25455	-0.37994	0.45809923	0.094958	0.683904598	0		0.541900775	-0.61267237	0.043500031
0.2631	0.4259793	0.6735	0.566638	-1.67276	2.43794	0.63798707	0.158057	11.44943585	0.007286593	3.212666396	0.765838112	-0.26678447	0.100838431
0.2702	0.4312634	0.6381	0.591495	-1.65691	2.453477	0.64370801	0.160177	11.62871107	0.007667208	3.391799454	0.743324135	-0.29662308	0.103107371
0	0.45	-0.568	0.651174	-1.56723	3.564038	0.65727488	0.172612	35.30547457	0		0.342725125	-1.07082654	0.113453498
0.0315	0.6	-0.532	1.384238	-1.14975	2.986592	0.79967078	0.240535	19.8180301	2.83681E-05	42.936919	4.554217033	1.516053624	0.192348889
0.162	0.44896376	-0.213	0.65546	-1.58112	3.221211	0.65823991	0.170637	25.05844816	0.002502315	0.027372516	1.048115166	0.046993471	0.112320162
0.0144	0.1	0.709	-1.02349	-2.58351	3.582868	0.26434837	0.070207	35.97656032	2.87648E-06	51.33624027	5.079149662	1.625143858	0.018559166
0.4266	0.57605845	0.1769	1.286072	-1.23856	2.380073	0.78348157	0.224687	10.80569423	0.014636709	5.492592027	0.609556114	-0.49502427	0.176038464
0.3265	0.60876535	0.0351	1.441972	-1.14289	2.400351	0.80875984	0.241791	11.02704776	0.010631147	0.9700644	0.901793754	-0.10336944	0.195550646
0.4537	0.60362525	-1.135	1.386046	-1.12093	3.562654	0.79996033	0.245838	35.25665507	0.015226153	5.070386465	0.405353694	-0.90299527	0.196660831
0.0008	0.1	0.71	1.06087	2.53951	4.969175	0.2571437	0.073134	143.9080576	4.91871E-10	1442.941548	2.563416998	0.941341133	0.018805977

Simplex Model

$$F(Y) = \{2 \times \pi \times \sigma^2 \times V(Y)\}^{-0.5} \times EXP\left\{\frac{-1}{2 \times \sigma^2} \times D(Y)\right\}$$

The variance function ***V(Y)*** and the deviance function ***D(Y)*** vary by distributional assumptions. For the Simplex distribution,

$$V(Y) = Y^3 \times (1 - Y)^3$$

$$D(Y) = \frac{(Y - \mu)^2}{Y \times (1 - Y) \times \mu^2 \times (1 - \mu)^2}$$

R^2
56.55%

10.10 RR Extrapolation- Chain Ladder

PEAKS2TAILS

Step 5: Extrapolation of Recovery Rates

Use CLM & run-off trinagle

Calculate the projected recovery rates (RR) for subsequent years of recovery using the chain ladder approach for year of default, YD and year of recovery, $y1$.

(i) Calculate the ratio of recovery rates., estimate CLM factor

(ii) Multiply (i) by the recovery rate in the previous recovery year ($RRYD,y0$) to compute the extrapolated recovery rate($RRYD,y1$) .

$$(i)\ Ratio_{y1} = \frac{Average(RR_{YD_1,}RR_{YD_2,}y1}{Average(RR_{YD_1,}RR_{YD_2,}y0} \quad (ii)\ RR_{YD,y1} = \frac{(RR_{YD_1,}RR_{YD_2,}y1}{(RRYD_{_1,}RR_{YD_2,}y0} * RR_{YD,y0}$$

	7	8	9	10	11	12	13	14	15	16		
Chart 4			**Recovery Rate**									
Year of Default	**1st Year**	**2nd Year**	**3rd Year**	**4th Year**	**5th Year**	**6th Year**	**7th Year**	**8th Year**	**9th Year**	**10th Year**	**Sum**	**LGD**
2008					2.98%	2.72%	3.06%	2.90%	2.82%	0.19%		
2009				3.14%	3.69%	2.54%	1.97%	2.07%	0.20%			
2010			3.24%	3.09%	2.90%	1.17%	1.07%	0.13%				
2011		6.38%	5.97%	4.32%	4.52%	3.42%	0.14%					
2012	10.02%	11.21%	7.80%	3.91%	2.54%	0.24%	0.06%	0.04%	0.03%	0.00%	35.86%	64.14%
2013	10.26%	10.29%	6.25%	1.66%	0.31%	0.16%	0.04%	0.03%	0.02%	0.00%	29.04%	70.96%
2014	12.15%	7.24%	5.51%	0.59%	0.30%	0.16%	0.04%	0.03%	0.02%	0.00%	26.04%	73.96%
2015	8.55%	4.32%	0.81%	0.16%	0.08%	0.04%	0.01%	0.01%	0.00%	0.00%	13.99%	86.01%
2016	21.38%	2.77%	1.52%	0.29%	0.15%	0.08%	0.02%	0.01%	0.01%	0.00%	26.23%	73.77%
2017	28.30%	6.71%	3.67%	0.70%	0.36%	0.19%	0.05%	0.04%	0.02%	0.00%	40.03%	59.97%
CLM-factor		23.7%	54.7%	19.1%	51.2%	51.8%	26.2%	72.2%	60.7%	6.6%		
								6-year avgerage		LGD		71.47%

Note: 5 year-average can also be used for CLM factor

Note: 10 years recovery rate is generated

11.1 CCF Calculation

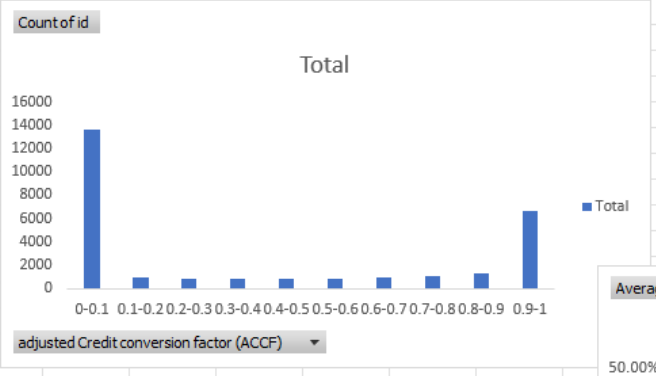
PEAKS2TAILS

	A	B	C	D	E	H	I	J	L	M	N	O	P
1	loan no	Industry	Nature of facility	Type of Guarantee	Date of npa/restructed failure classification	Bal O/s at default month or at nearest subsequent quarter In ORIGINAL CURRENCY Million	As at	preceding 1 year or thereabout ORIGINAL CURRENCY Million	Bal. O/s 1 year or thereabout preceding default IN ORIGINAL CURRENCY Million	Censored UGD_{NFB}	UGD_{NFBC}	UGD_{NFBC} Censored	UGD_{NFB}-Avg UGD_{NF}
2						$NFBO_d$		L_{d-1}	$NFBO_{d-1}$	=Min($NFBO_d/NFBO_{d-1}$,0)	=($NFBO_d$ - $NFBO_{d-1}$)/(L_{d-1} - $NFBO_{d-1}$)	Max(O, UGD_{NFBC})	
3	HOGTCINR00000611	Textiles and Garments	AIMS UNDER GUARANT	Financial Guarantee	Sep/16	3,550.50	1-Sep-15	4459.95	4459.95	79.61%		0.0%	-11.5%
4	HOGTIINR00000213	Films & Entertainment	ARANTEE INVOKED LOA	Financial Guarantee	May/17	2.55	1-May-16	2.69	2.69	94.82%		0.0%	3.7%
5	HOGTCINR00000073	S AND PHARMACEUT	AIMS UNDER GUARANT	Financial Guarantee	Mar/14	309.84	1-Sep-13	313.03	313.03	98.98%		0.0%	7.8%
6	HOGTIINR00000037	ETALS AND METAL P	ARANTEE INVOKED LOA	Financial Guarantee	Jun/15	125.29	1-Oct-14	125.80	103.60	100.00%	0.976967432	97.7%	8.9%
7	HOGTIUSD00000107	SHIPPING SERVICES	ARANTEE INVOKED LOA	Financial Guarantee	Oct/19	8.36	1-Oct-18	10.00	10.00	83.60%		0.0%	-7.5%
8	HOGTCINR00000569	uto & auto components	AIMS UNDER GUARANT	Financial Guarantee	Jun/17	37.00	1-Oct-15	37.00	37.00	100.00%		0.0%	8.9%
9	HOGTCINR00000578	uto & auto components	AIMS UNDER GUARANT	Financial Guarantee	Jun/17	37.99	1-Nov-15	38.00	38.00	99.98%		0.0%	8.8%
10	HOGTCINR00000709	Gems and jewellery	AIMS UNDER GUARANT	Financial Guarantee	Mar/18	226.19	1-Mar-17	250.00	250.00	90.48%		0.0%	-0.7%
11	HOGTCUSD00000231	Mining and Minerals	AIMS UNDER GUARANT	Financial Guarantee	Mar/18	58.86	1-Jan-17	58.86	58.86	100.00%		0.0%	8.9%
12	HOGTCUSD00000204	Oil and Gas	AIMS UNDER GUARANT	Financial Guarantee	Sep/16	27.19	1-Sep-15	27.83	27.83	97.70%		0.0%	6.6%
13	HOGTCINR00000046	MISCELLANEOUS	AIMS UNDER GUARANT	Financial Guarantee	Sep/13	658.75	1-Nov-12	1144.99	1144.99	57.53%		0.0%	-33.6%
14	HOGTIINR00000091	Ship building	GUARANTEE INVOKED LOANS	Financial Guarantee	Mar/18	1,652.69	1-Mar-17	3721.14	1466.31	100.00%	0.082660438	8.3%	8.9%
15	HOGTCEUR00000019	EPC Services	AIMS UNDER GUARANT	Financial Guarantee	Jun/14	26.54	1-Jun-13	27.12	26.24	100.00%	0.341326979	34.1%	8.9%
16	HOGTCINR00000754	Oil and Gas	AIMS UNDER GUARANT	Performance Guarantee	Nov/17	0.33	1-Nov-16	0.33	0.33	100.00%		0.0%	100.0%
17													

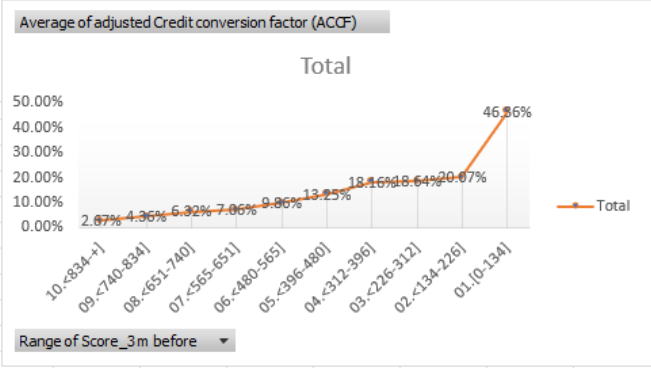
18	UGD for Financial Guarantees	Estimated UGD_{NFB}	Estimated UGD_{NFBC}
19	Weighted Avg. UGD	91.15%	2.81%
20	SD	11.71%	10.95%
21	n	13	13
22	SE	3.25%	3.04%
23	**95% CL UGD Upper Limit**	**96.49%**	**7.80%**

11.2 EAD Bimodal

Row Labels	Count of id
0-0.1	13608
0.1-0.2	921
0.2-0.3	831
0.3-0.4	898
0.4-0.5	871
0.5-0.6	883
0.6-0.7	940
0.7-0.8	1071
0.8-0.9	1275
0.9-1	6701
Grand Total	**27999**



Row Labels	Average of adjust	Count of id
0-0.1	0.00486142	48.60%
0.1-0.2	0.149125474	3.29%
0.2-0.3	0.249409295	2.97%
0.3-0.4	0.349238526	3.21%
0.4-0.5	0.449263394	3.11%
0.5-0.6	0.551025809	3.15%
0.6-0.7	0.649473187	3.36%
0.7-0.8	0.750725359	3.83%
0.8-0.9	0.852299795	4.55%
0.9-1	0.98691835	23.93%
Grand Total	**0.382756243**	**100.00%**



Row Labels	Average of adjusted Credit conversion factor (ACCF)
10.<834-+]	2.67%
09.<740-834]	4.36%
08.<651-740]	6.32%
07.<565-651]	7.06%
06.<480-565]	9.86%
05.<396-480]	13.25%
04.<312-396]	18.16%
03.<226-312]	18.64%
02.<134-226]	20.07%
01.[0-134]	46.36%
Grand Total	**0.382756243**

11.3 CCF- ULF

$EAD = B(t_0) + ULF \times [\, L(t_0) - B(t_0) \,]$

PD-Weighted ULF
0.762715267

Facility Ref	$B(t_0)$	$B(t_D)$	$L(t_0)$	Undrawn Amount	*Predicted EAD*	error	$error^2$
1	3707	6461	7062	3355	6266	195	38060.22
2	4372	7809	8356	3984	7411	398	158676.65
3	4911	7149	7903	2992	7193	-44	1939.88
4	9461	12944	14288	4827	13143	-199	39452.52
5	4001	6955	7716	3715	6834	121	14523.33
6	5565	7423	7970	2405	7399	24	560.26
7	7058	13307	13359	6301	11864	1443	2082627.37
8	4644	6246	6605	1961	6140	106	11302.96
9	9813	17866	18792	8979	16661	1205	1451012.05
10	5301	9058	9765	4464	8706	352	124072.35
11	3254	4270	4454	1200	4169	101	10148.89
12	3192	5797	5827	2635	5202	595	354316.93
13	1219	1195	1252	33	1244	-49	2417.65
14	6639	10022	10810	4171	9820	202	40688.79
15	5203	7003	7454	2251	6920	83	6910.25
16	6176	10945	11917	5741	10555	390	152296.35
17	1604	2850	3142	1538	2777	73	5320.82
18	1256	1597	1676	420	1576	21	426.82
19	8394	13884	14283	5889	12886	998	996742.24
20	4805	5674	6222	1417	5886	-212	44845.49
21	3143	5369	5641	2498	5048	321	102872.39
22	7466	14380	14784	7318	13048	1332	1775422.13
23	4412	5608	5995	1583	5619	-11	129.46
24	8250	10465	11621	3371	10821	-356	126816.59
25	6978	11941	12446	5468	11149	792	628013.33
26	1533	2752	2766	1233	2473	279	77602.40
27	9520	12702	14012	4492	12946	-244	59593.10
28	1010	1812	1855	845	1654	158	24808.01
29	7180	11828	13019	5839	11633	195	37832.41
30	6986	12438	13335	6349	11828	610	371515.57
31	2018	2225	2325	307	2252	-27	737.32
32	5870	10090	10647	4777	9513	577	332362.82

MSE
520.049

11.4 CCF- LF

PEAKS2TAILS

PD-Weighted LF

0.950258588

EAD = LF x L(t_0)

Facility Ref	B(t_0)	B(t_D)	L(t_0)	Undrawn Amount	*Predicted EAD*	error	error^2
1	3707	6461	7062	3355	6710.73	-249.73	62363.15
2	4372	7809	8356	3984	7940.36	-131.36	17255.65
3	4911	7149	7903	2992	7509.89	-360.89	130244.21
4	9461	12944	14288	4827	13577.29	-633.29	401062.19
5	4001	6955	7716	3715	7332.20	-377.20	142276.27
6	5565	7423	7970	2405	7573.56	-150.56	22668.60
7	7058	13307	13359	6301	12694.50	612.50	375150.76
8	4644	6246	6605	1961	6276.46	-30.46	927.69
9	9813	17866	18792	8979	17857.26	8.74	76.40
10	5301	9058	9765	4464	9279.28	-221.28	48962.68
11	3254	4270	4454	1200	4232.45	37.55	1409.87
12	3192	5797	5827	2635	5537.16	259.84	67518.49
13	1219	1195	1252	33	1189.72	5.28	27.84
14	6639	10022	10810	4171	10272.30	-250.30	62647.76
15	5203	7003	7454	2251	7083.23	-80.23	6436.45
16	6176	10945	11917	5741	11324.23	-379.23	143816.60
17	1604	2850	3142	1538	2985.71	-135.71	18417.88
18	1256	1597	1676	420	1592.63	4.37	19.07
19	8394	13884	14283	5889	13572.54	311.46	97005.20
20	4805	5674	6222	1417	5912.51	-238.51	56886.51
21	3143	5369	5641	2498	5360.41	8.59	73.81
22	7466	14380	14784	7318	14048.62	331.38	109810.74
23	4412	5608	5995	1583	5696.80	-88.80	7885.48
24	8250	10465	11621	3371	11042.96	-577.96	334032.05
25	6978	11941	12446	5468	11826.92	114.08	13014.61
26	1533	2752	2766	1233	2628.42	123.58	15273.19
27	9520	12702	14012	4492	13315.02	-613.02	375797.61
28	1010	1812	1855	845	1762.73	49.27	2427.56
29	7180	11828	13019	5839	12371.42	-543.42	295301.56
30	6986	12438	13335	6349	12671.70	-233.70	54614.88
31	2018	2225	2325	307	2209.35	15.65	244.88

MSE
278.408

11.5 CCF- BF

PEAKS2TAILS

PD-Weighted BF

1.425664664

EAD = BF x B(t_0)

Facility Ref	B(t_0)	B(t_D)	L(t_0)	Undrawn Amount	*Predicted EAD*	error	error^2
1	3707	6461	7062	3355	5284.94	1176.06	1383119.69
2	4372	7809	8356	3984	6233.01	1575.99	2483757.37
3	4911	7149	7903	2992	7001.44	147.56	21774.20
4	9461	12944	14288	4827	13488.21	-544.21	296168.21
5	4001	6955	7716	3715	5704.08	1250.92	1564790.04
6	5565	7423	7970	2405	7933.82	-510.82	260941.01
7	7058	13307	13359	6301	10062.34	3244.66	10527810.74
8	4644	6246	6605	1961	6620.79	-374.79	140465.07
9	9813	17866	18792	8979	13990.05	3875.95	15023008.96
10	5301	9058	9765	4464	7557.45	1500.55	2251655.15
11	3254	4270	4454	1200	4639.11	-369.11	136244.27
12	3192	5797	5827	2635	4550.72	1246.28	1553209.83
13	1219	1195	1252	33	1737.89	-542.89	294724.37
14	6639	10022	10810	4171	9464.99	557.01	310262.70
15	5203	7003	7454	2251	7417.73	-414.73	172003.67
16	6176	10945	11917	5741	8804.90	2140.10	4580006.76
17	1604	2850	3142	1538	2286.77	563.23	317232.40
18	1256	1597	1676	420	1790.63	-193.63	37494.44
19	8394	13884	14283	5889	11967.03	1916.97	3674777.09
20	4805	5674	6222	1417	6850.32	-1176.32	1383725.71
21	3143	5369	5641	2498	4480.86	888.14	788785.49
22	7466	14380	14784	7318	10644.01	3735.99	13957603.48
23	4412	5608	5995	1583	6290.03	-682.03	465168.33
24	8250	10465	11621	3371	11761.73	-1296.73	1681517.71
25	6978	11941	12446	5468	9948.29	1992.71	3970901.01
26	1533	2752	2766	1233	2185.54	566.46	320872.48
27	9520	12702	14012	4492	13572.33	-870.33	757470.13
28	1010	1812	1855	845	1439.92	372.08	138442.55
29	7180	11828	13019	5839	10236.27	1591.73	2533597.11
30	6986	12438	13335	6349	9959.69	2478.31	6142003.89
31	2018	2225	2325	307	2876.99	-651.99	425092.64

MSE
1701.13

11.6 CCF- AUF

PEAKS2TAILS

PD-Weighted BF: 0.259644526

$EAD = B(t_0) + AUF \times L(t_0)$

Facility Ref	$B(t_0)$	$B(t_D)$	$L(t_0)$	Undrawn Amount	*Predicted EAD*	error	$error^2$
1	3707	6461	7062	3355	5540.61	920.39	847118.41
2	4372	7809	8356	3984	6541.59	1267.41	1606328.98
3	4911	7149	7903	2992	6962.97	186.03	34606.91
4	9461	12944	14288	4827	13170.80	-226.80	51438.69
5	4001	6955	7716	3715	6004.42	950.58	903607.74
6	5565	7423	7970	2405	7634.37	-211.37	44675.95
7	7058	13307	13359	6301	10526.59	2780.41	7730672.99
8	4644	6246	6605	1961	6358.95	-112.95	12758.18
9	9813	17866	18792	8979	14692.24	3173.76	10072753.00
10	5301	9058	9765	4464	7836.43	1221.57	1492236.21
11	3254	4270	4454	1200	4410.46	-140.46	19728.09
12	3192	5797	5827	2635	4704.95	1092.05	1192576.15
13	1219	1195	1252	33	1544.07	-349.07	121853.32
14	6639	10022	10810	4171	9445.76	576.24	332055.62
15	5203	7003	7454	2251	7138.39	-135.39	18330.53
16	6176	10945	11917	5741	9270.18	1674.82	2805009.26
17	1604	2850	3142	1538	2419.80	430.20	185069.37
18	1256	1597	1676	420	1691.16	-94.16	8866.90
19	8394	13884	14283	5889	12102.50	1781.50	3173732.42
20	4805	5674	6222	1417	6420.51	-746.51	557274.55
21	3143	5369	5641	2498	4607.65	761.35	579646.56
22	7466	14380	14784	7318	11304.58	3075.42	9458179.47
23	4412	5608	5995	1583	5968.57	-360.57	130009.95
24	8250	10465	11621	3371	11267.33	-802.33	643731.88
25	6978	11941	12446	5468	10209.54	1731.46	2997968.39
26	1533	2752	2766	1233	2251.18	500.82	250823.92
27	9520	12702	14012	4492	13158.14	-456.14	208062.87
28	1010	1812	1855	845	1491.64	320.36	102630.15
29	7180	11828	13019	5839	10560.31	1267.69	1607032.66
30	6986	12438	13335	6349	10448.36	1989.64	3958668.32
31	2018	2225	2325	307	2621.67	-396.67	157349.88

MSE
1278.17

12.1 IFRS Low Risk

	A	B	C	D	E
1	IRB Risk Weight Function : Retail Residential Mortgage			Capital Requirement:Corporate Exposure	
2	Pool PD	1.07%		PD	0.24%
3	Pool LGD	35.00%		LGD	55%
4	EAD (Rs Crore)	5000		EAD (Rs Crore)	50
5	Effective Maturity (M)	2.5		Effective Maturity (M)	2.5
6	b	0.135		b	0.202166635
7	Maturity Adjustment (Madj)	1.254		Maturity Adjustment (Madj)	1.435234921
8	Correlation (R)	0.150		Correlation (R)	0.226583696
9	Capital Charge K (per Rs.)	0.046		K (per Rs.)	0.047037269
10	Risk weighting-Ratio(%) (K*12.5)	57.43%		Risk Weight (=K*12.5)	58.80%
11	RWA (K*12.5*EAD)	**2871.57**		RWA (K*12.5*EAD)	29.40
12	Target Risk Weight for "Low Credit Risk"	50.00%		IRB Capital Required = K*EAD	2.35
13					
14	Expected Loss Rate	0.37%		Expected Loss Rate	0.13%
15	Average Risk adjusted Return on the Portfolio	8.6263%		Average Risk adjusted Return on the Portfolio	8.8696%
16	RAROC	15.02%		RAROC	15.09%
17	Target RAROC on "Low Credit Risk"	15%		Target RAROC on "Low Credit Risk"	15%
18					
19	PIT PD	0.560%			
20		Low Risk			
21	PIT PD	2.85%			

12.2 IFRS Staging

Year	No. of Accounts	No. of Accounts NPA	Default Rate
1	16154	72	0.45%
2	16082	928	5.77%
3	16915	943	5.57%
4	17106	0	0.00%
5	17458	211	1.21%
6	14977	395	2.64%
7	15293	337	2.21%
8	16474	87	0.53%
9	16159	10	0.06%
10	14576	449	3.08%
11	15831	119	0.75%
12	16218	15	0.09%
13	15728	20	0.13%
14	16372	387	2.36%
15	16066	10	0.06%
16	16059	28	0.17%
	257468	4010	
		PD TTC PD	1.57%
		SD DR	1.91%
		SE DR	0.48%
		95% CL Upper Limit PD	2.50%
		95% CL Lower Limit PD	0.634%
		PIT PD	2.85%
		~~Low Risk~~	
		Stage 2	

POOLED PD SICR CRITERION

12.3 Absolute VS Relative Threshold

PEAKS2TAILS

Fwd. Looking CPD Matrix: as on Reporting Date

	1 year	2 year	3 year	4 year	5 year	6 year	7 year	8 year	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	N
AAA	0.01%	0.02%	0.02%	0.04%	0.09%	0.16%	0.27%	0.40%	178
AA	0.09%	0.30%	0.63%	1.10%	1.71%	2.45%	3.33%	4.34%	397
A	0.99%	2.28%	3.85%	5.65%	7.66%	9.85%	12.19%	14.66%	542
BBB	1.85%	4.27%	7.10%	10.86%	13.66%	17.21%	20.85%	24.54%	1233
BB	6.44%	12.83%	19.03%	24.95%	30.56%	35.82%	40.74%	45.31%	1905
B	12.18%	22.74%	31.87%	39.77%	46.62%	52.58%	57.76%	62.29%	1712
CCC	28.39%	46.44%	58.30%	66.39%	72.14%	76.40%	79.67%	82.27%	79

Relative	10%
Absolute	< BBB

CPD Matrix: as on Year of Origination

	1 year	2 year	3 year	4 year	5 year	6 year	7 year	8 year	
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	N
AAA	0.00%	0.01%	0.01%	0.03%	0.07%	0.12%	0.20%	0.30%	166
AA	0.06%	0.22%	0.49%	0.88%	1.40%	2.03%	2.78%	3.66%	366
A	0.94%	2.16%	3.63%	5.31%	7.17%	9.20%	11.36%	13.65%	476
BBB	1.76%	4.04%	6.72%	9.70%	12.90%	16.24%	19.66%	23.13%	1014
BB	6.12%	12.15%	17.97%	23.52%	28.77%	33.72%	38.35%	42.68%	1491
B	11.09%	20.79%	29.27%	36.69%	43.19%	48.90%	53.93%	58.36%	1311
CCC	26.32%	43.38%	54.81%	62.76%	68.51%	72.85%	76.24%	78.98%	70

Origination	AAA	5.00	0.07%	Low Risk	1.00
Reporting	AA	1.00	0.09%		
Origination	A	5.00	7.17%	Significant	2.00
Reporting	BB	2.00	12.83%		
Origination	A	6.00	9.20%	Not Significant	1.00
Reporting	BBB	4.00	10.86%		

12.4 Vasicek Model

Losses	Cumulative	Density	
0.00%	0.00%	0.000	0.00%
0.11%	0.46%	4.098	4.05%
0.21%	2.29%	8.845	2.65%
0.32%	5.24%	12.200	3.13%
0.42%	8.94%	14.326	3.44%
0.53%	13.10%	15.542	3.55%
0.63%	17.50%	16.108	3.98%
0.74%	21.99%	16.220	3.64%
0.85%	26.46%	16.018	4.18%
0.95%	30.85%	15.604	3.90%
1.06%	35.10%	15.049	3.91%
1.16%	39.19%	14.406	3.64%
1.27%	43.09%	13.712	3.40%
1.37%	46.79%	12.994	3.00%
1.48%	50.29%	12.270	3.15%
1.58%	53.60%	11.554	3.04%
1.69%	56.70%	10.856	2.61%
1.80%	59.62%	10.182	2.80%
1.90%	62.35%	9.535	2.53%
2.01%	64.91%	8.919	2.46%
2.11%	67.30%	8.335	2.50%
2.22%	69.53%	7.783	2.07%
2.32%	71.62%	7.262	2.37%
2.43%	73.56%	6.773	2.24%
2.54%	75.38%	6.314	1.87%
2.64%	77.07%	5.884	1.76%
2.75%	78.64%	5.482	1.54%
2.85%	80.11%	5.106	1.50%
2.96%	81.48%	4.755	1.26%
3.06%	82.75%	4.427	1.54%
3.17%	83.93%	4.122	1.21%
3.27%	85.04%	3.837	1.20%

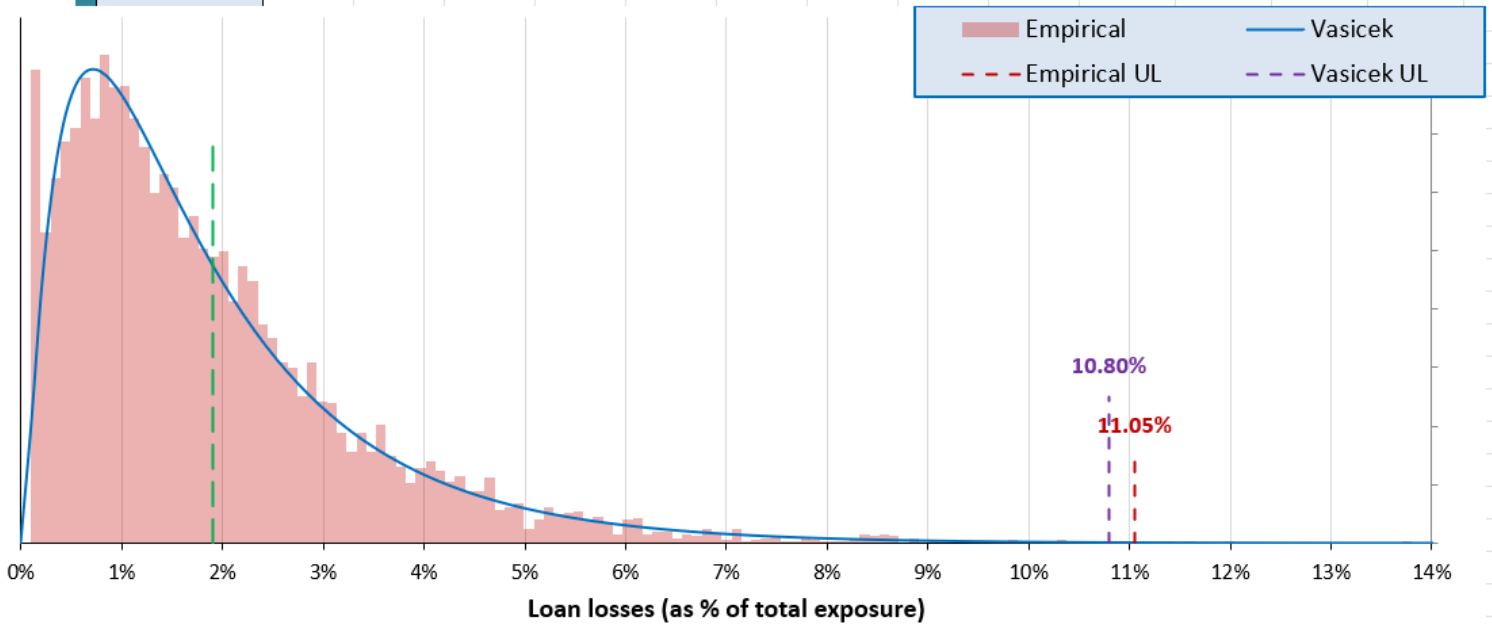
	Losses	
PD	5.0%	
LGD	38.1%	
EAD	1,10,386	
ρ	13.00%	
	Vasicek	Empirical
EL	1.89%	1.90%
99.9th worst loss	10.80%	11.05%

EL	
1.90%	0
1.90%	14

Empirical UL	
11.05%	0
11.05%	3

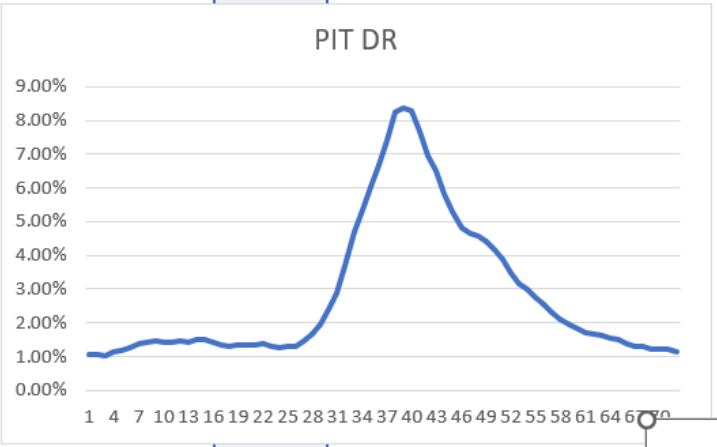
Vasicek UL	
10.80%	0
10.80%	5

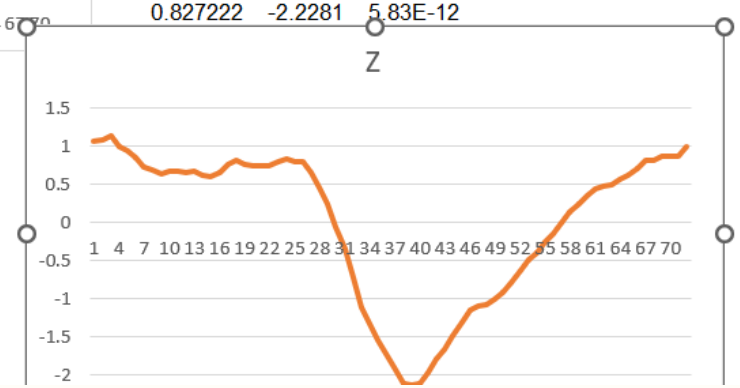
PD	0.79%	3.76%	8.60%	7.14%	9.24%	6.03%	1.82%	2.20%	7.17%	1.50%	2.48%	2.17%	0.77%	0.89%
CI	-2.41	-1.78	-1.37	-1.47	-1.33	-1.55	-2.09	-2.01	-1.46	-2.17	-1.96	-2.02	-2.42	-2.37
LGD	48.2%	31.0%	33.9%	22.3%	30.9%	59.0%	20.1%	33.8%	29.5%	41.4%	37.0%	57.7%	41.6%	42.5%
EAD	779	113	672	216	499	244	339	301	249	238	681	115	191	859
	20.08%	13.83%	12.16%	12.34%	12.12%	12.59%	16.83%	16.00%	12.33%	17.67%	15.48%	16.05%	20.16%	19.70%
Market factor	Loan 1	Loan 2	Loan 3	Loan 4	Loan 5	Loan 6	Loan 7	Loan 8	Loan 9	Loan 10	Loan 11	Loan 12	Loan 13	Loan 1
0.21	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-0.22	0	0	0	0	0	144	0	0	0	0	0	0	0	0
-0.54	0	0	0	0	0	144	0	0	0	0	0	0	0	0
-0.80	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.19	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.42	0	0	228	0	0	0	0	0	0	0	0	0	0	0
1.21	0	0	0	0	0	0	0	0	0	0	0	0	0	0



12.5 Extracting Z

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1		PIT DR				Actual							Fitted				
2	Date	0 Mortgage Loan	House Price Ind	Unemployment R	HPI Growth	PIT Score	Z					Seed Z	PIT Score				
3	1999 Q1		93.9	4.300%													
4	1999 Q2		95.8	4.300%	2.023%												
5	1999 Q3		97.7	4.200%	1.983%				Mean	-1.99373							
6	1999 Q4		99.8	4.100%	2.149%				SD	0.287332						Rho	0.077579
7	2000 Q1	1.07%	102.3	4.000%	2.505%	-2.3022513	1.073738	2.81%	Rho	0.076264		1.082889	-2.30225	1.58E-11		k	-1.90952
8	2000 Q2	1.06%	104.9	3.900%	2.542%	-2.3045206	1.081636	2.81%	k	-1.9162		1.090724	-2.30452	1.07E-12		SSE	6.31E-10
9	2000 Q3	1.01%	107.2	4.000%	2.193%	-2.3235592	1.147896	2.81%				1.156381	-2.32356	2.48E-12			
10	2000 Q4	1.13%	109.6	3.900%	2.239%	-2.2793612	0.994074	2.81%				1.003971	-2.27936	3.86E-13			
11	2001 Q1	1.18%	112.1	4.200%	2.							0.948072	-2.26315	3.01E-11			
12	2001 Q2	1.26%	114.2	4.400%	1.							0.866426	-2.23947	6.07E-13			
13	2001 Q3	1.38%	116.3	4.800%	1.							0.740166	-2.20286	2.77E-11			
14	2001 Q4	1.41%	118.3	5.500%	1.							0.708249	-2.1936	2.01E-12			
15	2002 Q1	1.47%	120.4	5.700%	1.							0.655515	-2.17831	2.21E-13			
16	2002 Q2	1.44%	123.5	5.800%	2.							0.682633	-2.18617	1.36E-12			
17	2002 Q3	1.44%	126.6	5.700%	2.							0.685316	-2.18695	7.75E-14			
18	2002 Q4	1.45%	129.2	5.900%	2.							0.671009	-2.1828	2.5E-12			
19	2003 Q1	1.44%	131.8	5.900%	2.							0.680265	-2.18548	5.02E-12			
20	2003 Q2	1.50%	134.7	6.100%	2.							0.628765	-2.17055	5.19E-13			
21	2003 Q3	1.50%	138.6	6.100%	2.							0.625004	-2.16946	1.48E-15			
22	2003 Q4	1.44%	143.1	5.800%	3.							0.678609	-2.185	1.81E-12			
23	2004 Q1	1.34%	148.1	5.700%	3.							0.782674	-2.21518	1.88E-14			
24	2004 Q2	1.29%	153.8	5.600%	3.							0.827222	-2.2281	5.83E-12			
25	2004 Q3	1.34%	159.2	5.400%	3.												
26	2004 Q4	1.36%	165.1	5.400%	3.706%	-2.2086092	0.747836	2.81%									
27	2005 Q1	1.36%	171.9	5.300%	4.119%	-2.2086442	0.747958	2.81%									
28	2005 Q2	1.37%	178.7	5.100%	3.956%	-2.20622	0.739522	2.81%									
29	2005 Q3	1.31%	184.9	5.000%	3.470%	-2.2235646	0.799886	2.81%									
30	2005 Q4	1.27%	190.2	5.000%	2.866%	-2.2354122	0.841119	2.81%									
31	2006 Q1	1.32%	193.4	4.700%	1.682%	-2.2215127	0.792745	2.81%									
32	2006 Q2	1.31%	192.7	4.600%	-0.362%	-2.2242927	0.80242	2.81%									
33	2006 Q3	1.46%	191.3	4.600%	-0.727%	-2.1812455	0.652603	2.81%									
34	2006 Q4	1.68%	190.8	4.400%	-0.261%	-2.125085	0.457148	2.81%									
35	2007 Q1	1.94%	188.6	4.500%	-1.153%	-2.0669077	0.254674	2.81%									
36	2007 Q2	2.40%	183.2	4.500%	-2.863%	-1.9767138	-0.05923	2.81%									





12.6 PIT Default Rates

Z	Unemployr	HPI Growtl	UR_lag1	HPIgr_lag1
	0.043			
	0.043	0.020234	0.043	
	0.042	0.019833	0.043	0.020234
	0.041	0.021494	0.042	0.019833
1.073738	0.04	0.02505	0.041	0.021494
1.081636	0.039	0.025415	0.04	0.02505
1.147896	0.04	0.021926	0.039	0.025415
0.994074	0.039	0.022388	0.04	0.021926
0.937634	0.042	0.02281	0.039	0.022388
0.855245	0.044	0.018733	0.042	0.02281
0.72783	0.048	0.018389	0.044	0.018733
0.695602	0.055	0.017197	0.048	0.018389
0.642372	0.057	0.017751	0.055	0.017197
0.669747	0.058	0.025748	0.057	0.017751
0.67245	0.057	0.025101	0.058	0.025748
0.658017	0.059	0.020537	0.057	0.025101
0.667361	0.059	0.020124	0.059	0.020537
0.615376	0.061	0.022003	0.059	0.020124
0.611577	0.061	0.028953	0.061	0.022003
0.665677	0.058	0.032468	0.061	0.028953
0.770715	0.057	0.034941	0.058	0.032468
0.81567	0.056	0.038488	0.057	0.034941
0.763109	0.054	0.035111	0.056	0.038488
0.747836	0.054	0.03706	0.054	0.035111
0.747958	0.053	0.041187	0.054	0.03706
0.739522	0.051	0.039558	0.053	0.041187
0.799886	0.05	0.034695	0.051	0.039558
0.841119	0.05	0.028664	0.05	0.034695
0.792745	0.047	0.016824	0.05	0.028664
0.80242	0.046	-0.00362	0.047	0.016824
0.652603	0.046	-0.00727	0.046	-0.00362
0.457148	0.044	-0.00261	0.046	-0.00727
0.254674	0.045	-0.01153	0.044	-0.00261
-0.059227	0.045	-0.02863	0.045	-0.01153
-0.337482	0.047	-0.03002	0.045	-0.02863
-0.712942	0.048	-0.03151	0.047	-0.03002
-1.114903	0.05	-0.04067	0.048	-0.03151

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.963423
R Square	0.928183
Adjusted R	0.926102
Standard E	0.419952
Observatic	72

ANOVA

	df	*SS*	*MS*	*F*	*ignificance F*
Regressioı	2	157.274	78.63698	445.8907	3.47E-40
Residual	69	12.16879	0.176359		
Total	71	169.4427			

	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Ipper 95.0%*
Intercept	3.463185	0.188717	18.35119	2.99E-28	3.086704	3.839665	3.086704	3.839665
X Variable	-58.7401	2.884484	-20.3642	6.85E-31	-64.4945	-52.9857	-64.4945	-52.9857
X Variable	41.18584	2.373193	17.35461	7.15E-27	36.45145	45.92023	36.45145	45.92023

Date	Mortgage	House Pric	Unemployment Rate						
2018 Q1	1.13%	199.2	4.100%	1.58%	4.10%	1.66%	1.73807	0.63%	
2018 Q2	1.17%	201.5	3.900%	1.155%	4.10%	1.58%	1.705918	0.65%	
2018 Q3	1.17%	203.3	3.800%	0.893%	3.90%	1.15%	1.647861	0.68%	
2018 Q4	1.18%	204.8	3.800%	0.738%	3.80%	0.89%	1.598975	0.71%	0.67%
2019 Q1	1.09%	206.3	3.900%	0.732%	3.80%	0.74%	1.534942	0.75%	
2019 Q2	1.03%	208.1	3.600%	0.873%	3.90%	0.73%	1.473976	0.79%	
2019 Q3	0.99%	210.2	3.600%	1.009%	3.60%	0.87%	1.707895	0.65%	
2019 Q4	1.10%	211.5	3.500%	0.618%	3.60%	1.01%	1.764161	0.62%	0.70%

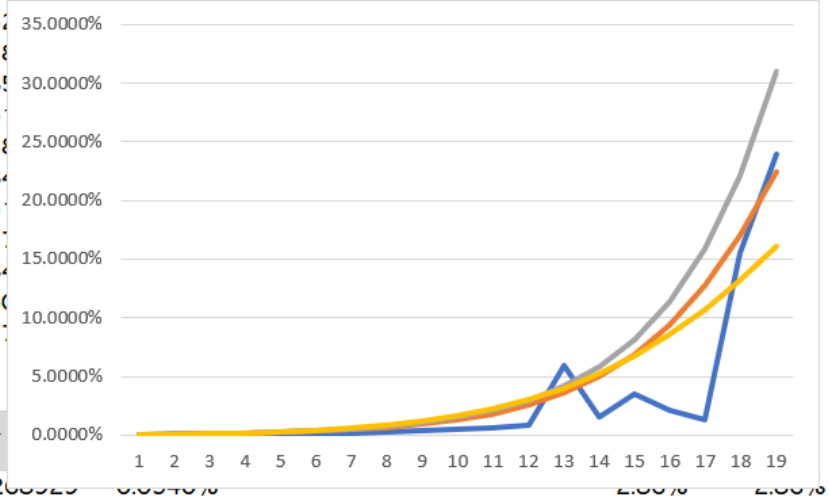
12.7 PD Curve Smoothening

19	7	-	24.0000%	7	-1.1527	-1.240949	22.4271%

Method 2

	Rating Scale		PD	# of customers	Log	Y hat	Calibrated PD
1	1		0.0300%	3	-8.1117	-720.34%	0.07%
2	2	+	0.0385%	4	-7.8623	-686.83%	0.10%
3	2		0.0555%	9	-7.4965	-653.33%	0.15%
4	2	-	0.0750%	13	-7.1954	-619.82%	0.20%
5	3	+	0.1010%	18	-6.8978	-586.32%	0.28%
6	3		0.1400%	19	-6.5713	-552.81%	0.40%
7	3	-	0.1750%	16	-6.3481	-519.31%	0.56%
8	4	+	0.2450%	19	-6.0117	-485.80%	0.78%
9	4		0.4100%	19	-5.4968	-45	
10	4	-	0.4550%	12	-5.3926	-41	
11	5	+	0.6150%	9	-5.0913	-38	
12	5		0.8400%	7	-4.7795	-35	
13	5	-	5.9000%	79	-2.8302	-31	
14	6	+	1.5220%	6	-4.1851	-28	
15	6		3.5500%	4	-3.3382	-25	
16	6	-	2.1000%	7	-3.8632	-21	
17	7	+	1.3000%	4	-4.3428	-18	
18	7		15.5000%	1	-1.8643	-15	
19	7	-	24.0000%	7	-1.4271	-11	

	3.50%
	3.50%
	0.00%
beta	0.34
a	-7.54



Method 3

	Rating Scale		PD	# of customers	Probit	Y
1	1		0.0300%	3	-3.4316	-3.2
2	2	+	0.0385%	4	-3.3634	-3.142321
3	2		0.0555%	9	-3.2611	-3.015713
			750%	13	-3.1747	-2.889105
			010%	18	-3.0873	-2.762496

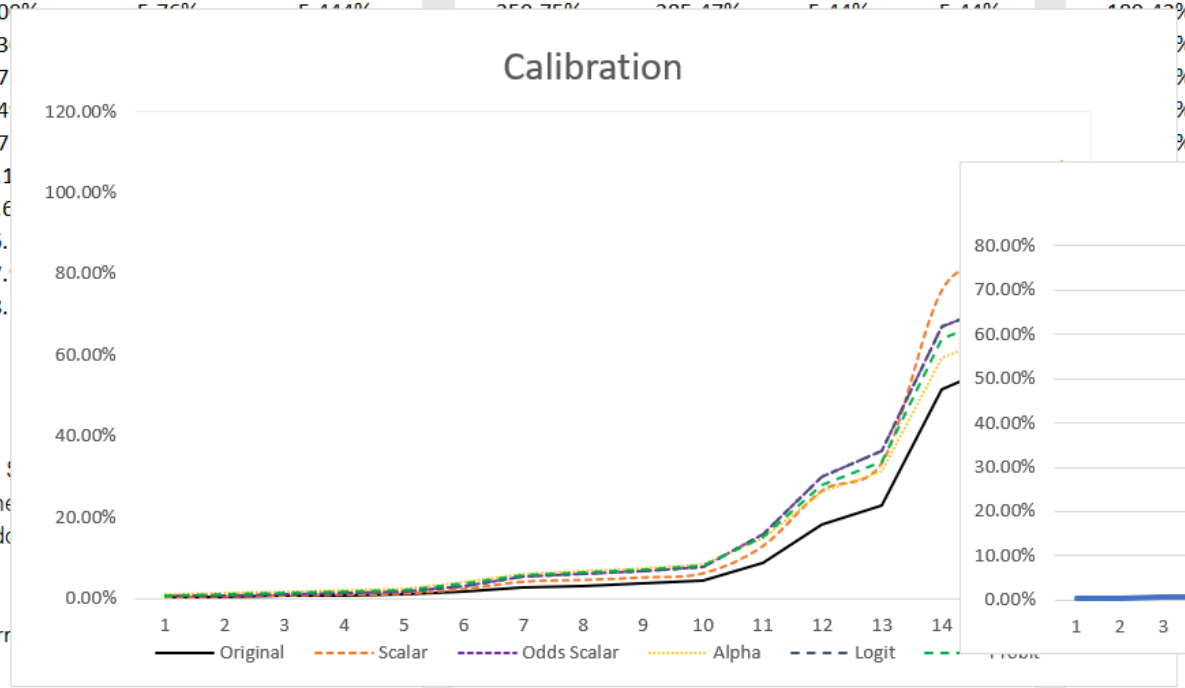
	Calibrated PD
	0.0838%
	0.1282%
	0.1932%
	0.2868%

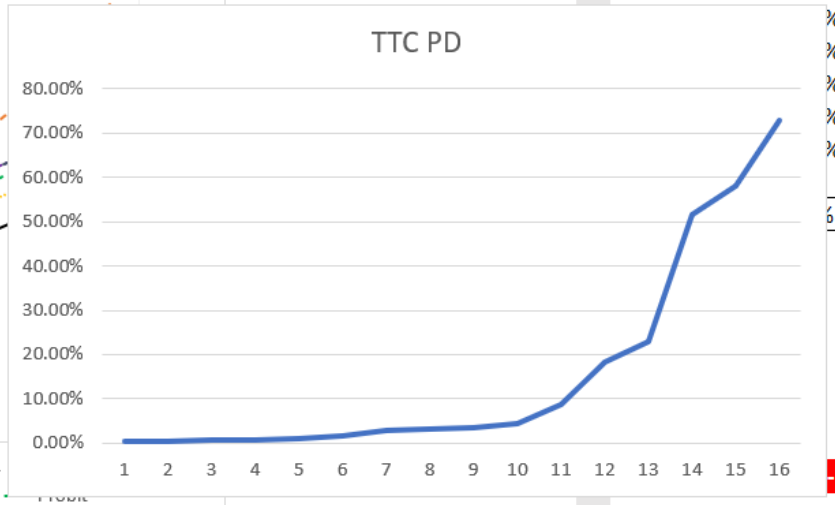
	0.00%
beta	0.1266

Chart Title

12.8 TTC To PIT Calibration

Forecasted DR	6.00%															
			Scalar	Odds Scalar			Logit Shift				Probit Shift			Alpha	
# of accounts	**MPD (TTC)**		**Scalar**	**Odds**	**Odds adjusted**	**PD**	**Log Odds**	**Log odds adj**	**Logit Space**		**Probit**	**Probit adj**	**Probit Space**		**Alpha**
76358	0.35%		0.52%	0.35%	0.67%	0.670%	-565.15%	-499.87%	0.67%	0.67%	-269.68%	-237.25%	0.88%	-5.654992	1.18%
63260	0.41%		0.61%	0.41%	0.79%	0.785%	-549.27%	-483.99%	0.78%	0.78%	-264.37%	-231.94%	1.02%	-5.496768	1.34%
56219	0.59%		0.87%	0.59%	1.14%	1.127%	-512.69%	-447.41%	1.13%	1.13%	-251.81%	-219.37%	1.41%	-5.132803	1.78%
16127	0.75%		1.11%	0.76%	1.45%	1.431%	-488.53%	-423.26%	1.43%	1.43%	-243.24%	-210.80%	1.75%	-4.892852	2.15%
4621	0.91%		1.34%	0.92%	1.76%	1.733%	-469.03%	-403.76%	1.73%	1.73%	-236.15%	-203.72%	2.08%	-4.699481	2.50%
12409	1.69%		2.50%	1.72%	3.30%	3.196%	-406.34%	-341.07%	3.20%	3.20%	-212.24%	-179.81%	3.61%	-4.080442	4.06%
9507	2.91%		4.30%	3.0								-156.98%	5.82%	-3.537017	6.22%
4692	3.19%		4.71%	3.3							%	-152.92%	6.31%	-3.445149	6.69%
6417	3.61%		5.34%	3.7							%	-147.35%	7.03%	-3.321462	7.37%
13	4.30%		6.35%	4.4							%	-139.25%	8.19%	-3.146555	8.46%
537	8.86%		13.09%	9.7							%	-102.51%	15.27%	-2.423623	14.92%
5311	18.16%		26.84%	22.1											%
10721	22.87%		33.80%	29.6											%
7852	51.52%		76.14%	106.											%
1431	57.97%		85.67%	137.											%
1389	72.84%		107.65%	268.											%
Portfolio W.A PD	4.06%		6.00%												%
Scalar	1.48			Odds											
				The											
				Total Od											
				Err											-23





12.9 Extracting Z Scores From Transition Matrix

PEAKS2TAILS

PiT Transition Matrix	S1	S2	S3
S1	95.15%	4.16%	0.68%
S2	27.67%	69.74%	2.59%
S3	3.44%	18.98%	77.58%

Av. (equilibrium) TM	S1	S2	S3
S1	90.03%	7.94%	2.03%
S2	19.01%	74.99%	6.00%
S3	1.96%	13.03%	85.02%

Bin boundaries (upper)	S1	S2	S3
S1	1000000	-1.28	-2.05
S2	1000000	0.88	-1.55
S3	1000000	2.06	1.04

Bin boundaries (lower)	S1	S2	S3
S1	-1.28	-2.05	-10000
S2	0.88	-1.55	-10000
S3	2.06	1.04	-10000

TM cond.	S1	S2	S3
S1	95.41%	3.96%	0.63%
S2	28%	69.87%	2.43%
S3	3%	19%	78%

Dev^2	S1	S2	S3
S1	0.000	0.000	0.000
S2	0.000	0.000	0.000
S3	0.000	0.000	0.000

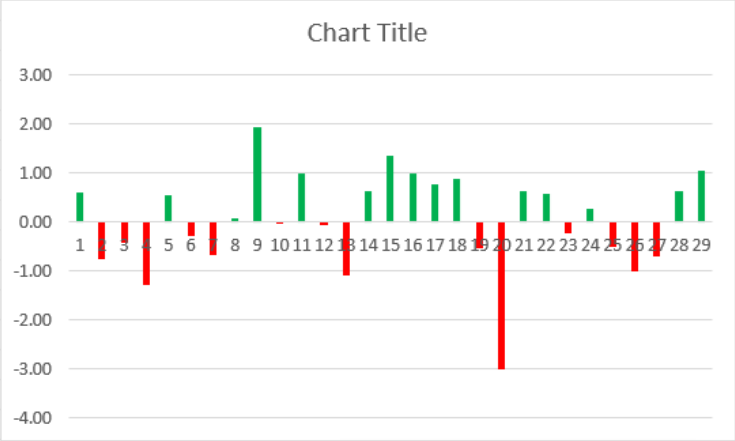
Sum Dev^2	0.000

Z	1
rho	0.1000

Compute Z-Score

Var(Z) target	Var(Z) found	Corresp. opt. rho
1	0.950	0.080

	Z
1991	0.61
1992	-0.76
1993	-0.42
1994	-1.28
1995	0.55
1996	-0.29
1997	-0.68
1998	0.06
1999	1.94
2000	-0.03
2001	0.99
2002	-0.07
2003	-1.10
2004	0.63



			1	2	3	4	5
		Variance	7.56E+26	8.33E+25	9.91E+25	2.04E+25	5.58
		Rho	0.01	0.02	0.03	0.04	0.
1	1991	Z1	1.4	1.0	-5.31E+13	0.7	
2	1992	Z2	-2.5	-1.7	-1.4	-1.2	-2.28
3	1993	Z3	-1.5	-1.0	-0.8	-0.7	
4	1994	Z4	-3.9	-2.7	-2.1	-1.12E+12	
5	1995	Z5	1.2	-2.81E+13	0.7	0.6	
6	1996	Z6	-1.2	-0.9	-1.89E+12	-0.6	-1.46
7	1997	Z7	-2.2	-1.5	-1.2	-2.17E+12	
8	1998	Z8	-2.21E+13	-0.2	-0.1	-0.1	
9	1999	Z9	3.93E+12	3.8	3.1	2.7	
10	2000	Z10	-0.4	-0.3	-0.2	-0.2	
11	2001	Z11	2.4	1.7	1.4	1.2	
12	2002	Z12	-0.5	-0.4	-0.3	-0.2	
13	2003	Z13	-3.2	-4.70E+12	-1.9	-2.64E+10	-2.76
14	2004	Z14	3.99E+13	1.0	0.8	0.8	
15	2005	Z15	3.7	2.7	2.1	1.9	
16	2006	Z16	2.6	1.9	1.08E+13	1.3	
17	2007	Z17	1.8	2.05E+13	1.1	0.9	
18	2008	Z18	2.1	3.42E+13	1.3	1.1	
19	2009	Z19	-1.8	-2.70E+12	-1.1	-6.50E+11	
20	2010	Z20	-4.97E+11	-6.0	-7.75E+11	-5.23E+11	
21	2011	Z21	1.4	1.1	0.9	-1.77E+13	
22	2012	Z22	1.3	0.9	0.7	0.7	
23	2013	Z23	-1.0	-0.7	-0.5	-0.4	-4.27
24	2014	Z24	0.4	0.3	0.3	0.2	
25	2015	Z25	-1.7	-2.86E+12	-1.0	-6.86E+11	
26	2016	Z26	-6.93E+11	-1.35E+12	-1.7	-1.5	-2.88
27	2017	Z27	-2.3	-1.7	-2.06E+10	-1.2	
28	2018	Z28	1.44E+14	1.1	0.9	-1.81E+13	
29	2019	Z29	2.9	2.1	1.7	1.5	

12.10 TTC TM To PIT TM

PEAKS2TAILS

	Z	
	baseline	adverse
2020	-0.47	-0.65
2021	-0.42	-0.84
2022	-0.38	-0.99
2023	-0.36	-0.69
2024	-0.34	-0.39
2025	-0.33	-0.24

Long-term average TM

	S1	S2	S3
S1	91%	8%	1%
S2	14%	81%	5%
S3	3%	25%	72%

Upper bin boundaries

	S1	S2	S3
S1	10000	-1.34	-2.33
S2	10000	1.08	-1.64
S3	10000	1.88	0.58

Lower bin boundaries

	S1	S2	S3
S1	-1.34	-2.33	-10000
S2	1.08	-1.64	-10000
S3	1.88	0.58	-10000

Rho

0.20

Implied ergodic state prob.

S1	S2	S3
57%	35%	8%

History + baseline [S1/2/3 part of TM yet normalized to sum to 1 per pt in time]

t-1	S1					S2					S3		
t	S1	S2	S3	MAT	WRO	S1	S2	S3	MAT	WRO	S1	S2	S3
2019	89%	9%	2%			15%	79%	6%			2%	24%	74%
2020	90%	9%	0.90%	5%	0%	7%	87%	5%	4%	0%	1%	18%	81%
2021	90%	9%	0.83%	5%	0%	8%	87%	5%	4%	0%	1%	18%	80%
2022	90%	9%	0.80%	5%	0%	8%	87%	5%	4%	0%	1%	19%	80%
2023	91%	9%	0.77%	5%	0%	8%	87%	5%	4%	0%	1%	19%	80%
2024	91%	8%	0.76%	5%	0%	8%	87%	5%	4%	0%	1%	19%	79%
2025	91%	8%	0.75%	5%	0%	8%	87%	5%	4%	0%	1%	19%	79%

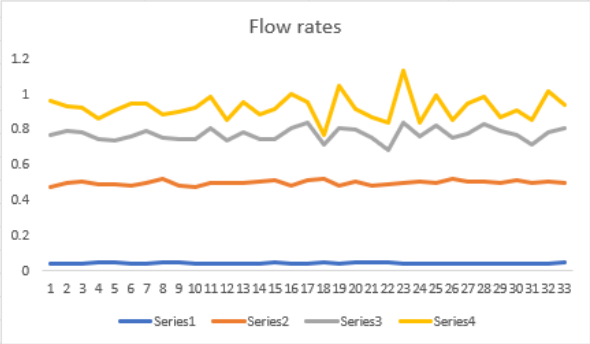
History + baseline [normalized along S1/2/3 and MAT/WRO]

2020	85%	9%	0.859%	5%	0%	7%	84%	5%	4%	0%	1%	16%	75%
2021	86%	9%	0.793%	5%	0%	8%	84%	5%	4%	0%	1%	17%	74%
2022	86%	8%	0.755%	5%	0%	8%	84%	5%	4%	0%	1%	17%	74%
2023	86%	8%	0.733%	5%	0%	8%	84%	5%	4%	0%	1%	18%	74%
2024	86%	8%	0.720%	5%	0%	8%	84%	5%	4%	0%	1%	18%	74%
2025	86%	8%	0.709%	5%	0%	8%	84%	5%	4%	0%	1%	18%	73%

13.1 Aggregate Models- Flow Rates

PEAKS2TAILS

	Balances each month												
Date	12/31/09	1/31/10	2/28/10	3/31/10	4/30/10	5/31/10	6/30/10	7/31/10	8/31/10	9/30/10	10/31/10	11/30/10	12/31/10
Bucket 0	1541917854	1525776534	1513621910	1505724580	1496416113	1484588365	1471712409	1459323380	1454053581	1438361187	1426033680	1428391315	1404093174
Bucket 1	70239075.95	66963188.9	65082614.6	66996233.35	67180628.67	66650182.66	65364612.34	61607281.61	65397183.71	65932132.84	63621076.79	61576820.52	62288851.59
Bucket 2	33932068.01	33226384.26	33478059.14	33040303.08	32639050.99	32736731.79	32028877.92	32314913.51	31929525.45				
Bucket 3	24904160.41	25996145.51	26242226.44	26350979.59	24509532.26	24098000.2	24809989.52	25402011.73	24304387.91				
Bucket 4	22988455.76	23996442.01	24223593.64	24323981.16	22624183.62	22244307.88	22901528.79	23448010.83	22434819.61				



		1/31/10	2/28/10	3/31/10	4/30/10	5/31/10	6/30/10	7/31/10	8/31/10
Bucket 0-1	Flow rates	0.043428506	0.042655404	0.044262198	0.044616811	0.044539872	0.044028779	0.041860951	0.04481336
Bucket 1-2		0.473047001	0.499947205	0.507667113	0.487177403	0.487294216	0.48055199	0.494379334	0.518275188
Bucket 2-3		0.766123229	0.789800847	0.787111925	0.741807126	0.738318041	0.75786397	0.793097148	0.752110567
Bucket 3-4		0.963551536	0.93181482	0.926902342	0.858570876	0.907577821	0.950349764	0.945103617	0.883190664

								9/30/12					
					Average Roll rates	Conditional LR		Balance					
				Bucket 0-1	0.043805977	0.015563499		1327419382.69		Avg Life (yrs)	EIR		
				Bucket 1-2	0.497811762	0.355282532		60528520.95		2	10.00%		
				Bucket 2-3	0.772511938	0.713688505		29117940.47					
				Bucket 3-4	0.923854338	0.923854338		23378765.22		Linear Amortization	4.167% per year		
				Bucket 4	1			21580398.67					

Balance Flows from Bucket 0													
	Today												
	0	1	2	3	4	5	6	7	8	9	10	11	12
Balance Forecast (%)	9/30/12	10/31/12	11/30/12	12/31/12	1/31/13	2/28/13	3/31/13	4/30/13	5/31/13	6/30/13	7/31/13	8/31/13	9/30/13
Amortization (Linear)	100%	96%	92%	88%	83%	79%	75%	71%	67%	63%	58%	54%	50%
Bucket 0-0	1.0000	0.9562	0.9143	0.8743	0.8360	0.7993	0.7643	0.7308	0.6988	0.6682	0.6389	0.6110	0.5842
Bucket 0-1		0.0438	0.0419	0.0401	0.0383	0.0366	0.0350	0.0335	0.0320	0.0306	0.0293	0.0280	0.0268
Bucket 1-2			0.0218	0.0209	0.0199	0.0191	0.0182	0.0174	0.0167	0.0159	0.0152	0.0146	0.0139
Bucket 2-3				0.0168	0.0161	0.0154	0.0147	0.0141	0.0135	0.0129	0.0123	0.0118	0.0113
Bucket 3-4 (Loss Rate)					0.0156	0.0149	0.0142	0.0136	0.0130	0.0124	0.0119	0.0114	0.0109
FEG	0.8050	1.1575	1.1171	0.9784	1.1311	1.1639	1.1546	0.8506	0.8796	0.8908	1.1778	1.1946	0.9763
PIT LR	0.0000	0.0000	0.0000	0.0000	0.0176	0.0173	0.0164	0.0116	0.0114	0.0111	0.0140	0.0136	0.0106
Charge off for Bucket 0					23367076.24	22034513.63	19991127.61	13442903.54	12658422.68	11645564.51	13948114.11	12775902.35	9397156.87
Charge Off (PV)	0.00	0.00	0.00	0.00	22604134.28	21138922.55	19020089.05	12684233.04	11845314.58	10807454.78	12837316.51	11661279.82	8506423.20

	1 Year	Lifetime
PV Charge off	140321019.5	187816978.05
Recovery	40%	40%
ECL (Bucket 0)	84192611.69	112690186.8
%	6.34%	8.49%

13.2 Aggregate Models- Snapshot/Open Pool Methods

PEAKS2TAILS

Example 1: Snapshot / Open Pool Method (slides 14-18)

Fact Pattern:

- Calculate the allowance for credit losses as of 12/31/2020
- CRE loan portfolio (pool with loans of similar risk characteristics)
  - Amortized cost basis of $10 million
  - Average life of 5 years (contractual term adjusted by prepayments and reasonably expected troubled debt restructuring)
- Totals may not sum precisely due to rounding

Current Conditions and Forecast:

- Management expects the following in 2021 and 2022:
  - Decline in real estate values
  - Rise in unemployment
- Management cannot reasonably forecast beyond 2022
- Assume 0.25% qualitative adjustment to represent both current conditions and reasonable & supportable forecasts

Slide 18

Example 1: Snapshot / Open Pool Method

(in thousands)

Year End	Amortized Cost	Net Charge-offs Associated with 2015 Snapshot Balance	Loss Rate
2015	$ 9,350		
2016	9,398	$ 31.85	0.341%
2017	10,779	31.50	0.337%
2018	11,050	14.00	0.150%
2019	10,738	8.50	0.091%
2020	10,000	2.25	0.024%
2015 Pool's cumulative charge-offs (a)		$ 88	
2015 Amort cost (b)		$ 9,350	
Inadjusted lifetime historical charge-off rate (a)/(b)			0.94%
Qualitative adjustments			0.25%
otal allowance for credit losses ratio as of 2020 (c)			1.19%
2020 Amort cost (d)		$ 10,000	
Total allowance for credit losses as of 2020 (c)x(d)		$ 119	

Not used in

Note :

Amortized Cost balance is open in the sense loans will be maturing and originating
But Charge-offs are tracked specific to the 2015 pool

13.3 Aggregate Models- Warm Method

PEAKS2TAILS

Example 2: Weighted-Average Remaining Maturity (WARM) (slides 19-25)

Fact Pattern:

- Calculate the allowance for credit losses as of 12/31/2020
- CRE loan portfolio (pool with loans of similar risk characteristics)
  - Amortized cost basis of $10 million
  - Average life of 5 years (contractual term adjusted by prepayments and reasonably expected troubled debt restructuring)
- Totals may not sum precisely due to rounding

Current Conditions and Forecast:

- Management expects the following in 2021 and 2022:
  - Decline in real estate values
  - Rise in unemployment
- Management cannot reasonably forecast beyond 2022
- Assume 0.25% qualitative adjustment to represent both current conditions and reasonable & supportable forecasts

Slide 22

Step 1: Compute annual charge-off rate (same as incurred loss info)

(in thousands)

Year	Amortized Cost	A Average Balance	B Annual Net Charge-offs	C = B / A Annual Charge-off Rate
2015	$ 9,350			
2016	9,398	$ 9,374	$ 32	0.34%
2017	10,779	10,088	33	0.35%
2018	11,050	10,914	50	0.46%
2019	10,738	10,894	42	0.38%
2020	10,000	10,369	31	0.29%
			Average annual charge-off rate	**0.37%**

Note :

Track Annual Net Charge-offs every year which is against the average balance in that year

For Step 2 , both Direct Method an WARM method lead to the same answer

13.4 Aggregate Models- Vintage Methods

PEAKS2TAILS

Slide 29

Step 1: Capture and organize historical loan charge-off data

(in thousands)

Origination		Charge-offs ($)				
Amount	Date	Period 1	Period 2	Period 3	Period 4	Period 5
$ 5,500	2015	2	19	14	8	2
$ 5,000	2016	2	35	15	8	
$ 3,500	2017	-	18	8		
$ 3,100	2018	1	14			
$ 3,100	2019	1				
$ 2,940	2020					

Slide 29

Step 1: Capture and organize historical loan charge-off data (accounts)

No. of loans

Origination		Charge-offs (accounts)				
Accounts	Date	Period 1	Period 2	Period 3	Period 4	Period 5
5500	2015	2	19	14	8	2
5000	2016	2	35	15	8	
3500	2017	-	18	8		
3100	2018	1	14			
3100	2019	1				
2940	2020					

Slide 30

Step 2: Compute loan charge-off rates

(in thousands)

Origination		Charge-offs (%)					Inception to Date Charge-offs	Total Lifetime Charge-offs
Amount	Date	Period 1	Period 2	Period 3	Period 4	Period 5		
$ 5,500	2015	0.04%	0.35%	0.25%	0.15%	0.04%	0.83%	0.83%
$ 5,000	2016	0.04%	0.70%	0.30%	0.16%		1.19%	
$ 3,500	2017	0.00%	0.50%	0.23%			0.73%	
$ 3,100	2018	0.04%	0.45%				0.49%	
$ 3,100	2019	0.02%					0.02%	
$ 2,940	2020						0.00%	

Slide 30

Step 2: Compute loan default rates

No. of loans

Origination						
Accounts	Date	Period 1	Period 2	Period 3	Period 4	Period 5
5500	2015	0.04%	0.35%	0.25%	0.15%	0.04%
5000	2016	0.04%	0.70%	0.30%	0.16%	
3500	2017	0.00%	0.50%	0.23%		
3100	2018	0.04%	0.45%			
3100	2019	0.02%				
2940	2020					

Slide 31

Step 3: Determine which historical loss period is a reasonable period on which to base the expected credit loss rate calculation

(in thousands)

Origination		Charge-offs (%)					Remaining Lifetime Charge-offs (%)	Remaining Lifetime Charge-offs ($)
Amount	Date	Period 1	Period 2	Period 3	Period 4	Period 5		
$ 5,500	2015	0.04%	0.35%	0.25%	0.15%	0.04%	N/A	N/A
$ 5,000	2016	0.04%	0.70%	0.30%	0.16%	0.04%		
$ 3,500	2017	0.00%	0.50%	0.23%	0.15%	0.04%		
$ 3,100	2018	0.04%	0.45%	0.26%	0.15%	0.04%		
$ 3,100	2019	0.02%	0.50%	0.26%	0.15%	0.04%		
$ 2,940	2020	0.03%	0.50%	0.26%	0.15%	0.04%		
Average charge-off rate		0.03%	0.50%	0.26%	0.15%	0.04%		
		2.50	50.00	26.00	15.25	4.00		

Slide 31

Step 3: Determine which historical period is a reasonable period on which to base the expected credit loss rate calculation

No. of loans

Origination						
Accounts	Date	Period 1	Period 2	Period 3	Period 4	Period 5
5500	2015	0.04%	0.35%	0.25%	0.15%	0.04%
5000	2016	0.04%	0.70%	0.30%	0.16%	0.04%
3500	2017	0.00%	0.50%	0.23%	0.15%	0.04%
3100	2018	0.04%	0.45%	0.26%	0.15%	0.04%
3100	2019	0.02%	0.50%	0.26%	0.15%	0.04%
2940	2020	0.03%	0.50%	0.26%	0.15%	0.04%
Average default rate		0.03%	0.50%	0.26%	0.15%	0.04%
		2.500	40.000	15.600	6.100	0.800

13.5 Aggregate Models- State Transition Models

Transitions	Const	Original LTV	Credit Score	Loan Age	GDP	Urate	HPI
0-1	-4.7359	0.0129	-0.0012	0.0025	46.9271	0.0488	-2.1453
1-0	0.0032	-0.0057	0.0011	-0.0066	-107.3171	-0.0261	3.379
1-2	-1.4195	-0.0045	-0.0009	0.0012	13.9664	0.0991	-4.4441
2-0	-3.113	-0.0139	-0.0042	-0.006	75.9284	0.0759	4.4648
2-1	-1.5506	0.0088	-0.0009	0.0031	4.5534	-0.0032	-10.1667
2-3	0.7757	-0.0048	-0.0016	-0.0024	-26.7858	0.0201	2.6559
3-0	-2.7207	0.0148	-0.0012	-0.0062	-57.1479	0.0824	25.1091
3-1	-0.4104	-0.0071	0.001	0.0047	-247.7389	-0.3907	22.6754
3-2	-5.0147	0.0028	0.0023	0.0057	150.1107	0.043	10.2469
3-D	1.3557	-0.0094	-0.0011	-0.0043	-104.5909	0.1031	20.7896

Balance	7200
LGD	50%

4 Month TM

	0	1	2	3	D
0	0.9695	0.022243	0.006443	0.00139	0.00042
1	0.77026	0.069718	0.068868	0.0357	0.055452
2	0.31905	0.09795	0.122146	0.07369	0.387168
3	0.20522	0.029279	0.032521	0.01871	0.714273
D	0	0	0	0	1

PD

4 month ECL

199.62721

Total ECL for 4 months

200.40834

Date	Original LTV	Credit Score	Loan Age	GDP	Urate	HPI
11/01/22	75	680	27	0.001960088	4.8	-0.000456801
12/31/22	75	680	28	0.001960088	4.9	0.001393879
1/31/23	75	680	29	0.001960088	5	0.007524459
2/28/23	75	680	30	0.001006948	5.1	0.010701303

DTM

11/01/22	0	1	2	3	D
0	0.985075274	0.014924726	0	0	0
1	0.452	0.410	0.138057946	0	0
2	0.001276453	0.194553315	0.471100975	0.333069258	0
3	0.089311976	0.075347427	0.06972959	0.248873294	0.51673771
D	0	0	0	0	1

12/31/22	0	1	2	3	D
0	0.985025061	0.014974939	0	0	0
1	0.451	0.410	0.138402	0	0
2	0.001289075	0.192052845	0.472583	0.33407522	0
3	0.09333686	0.075878276	0.071632	0.23131651	0.5278364
D	0	0	0	0	1

1/31/23	0	1	2	3	D
0	0.98511	0.01489	0	0	0
1	0.454	0.409	0.136489	0	0
2	0.001327	0.182982	0.478071	0.33762	0
3	0.1074	0.083549	0.076628	0.171496	0.560927
D	0	0	0	0	1

2/28/23	0	1	2	3	D
0	0.985744	0.014256	0	0	0
1	0.480	0.325	0.19474	0	0
2	0.001254	0.177974	0.475602	0.34517	0
3	0.12118	0.107046	0.069811	0.099277	0.602686
D	0	0	0	0	1

14.1 Wholesale Models- Building TM Using Cohort

PEAKS2TAILS

Count of 1999	Column Labels										
Row Labels	0	1	2	3	4	5	6	7	8	9	Grand Total
0	54				1	1					56
1		7									7
2			44	2							46
3	5		9	103	1						118
4	6			18	106	5	2				137
5	6			1	10	61	8	3	1		90
6	8		1	1		3	60	7	1		81
7	8						2	14	1		25
8									8		8
9	43	2	74	125	79	32	20	5	7	874	1261
Grand Total	**130**	**9**	**128**	**250**	**197**	**102**	**92**	**29**	**18**	**874**	**1829**

	1	2	3	4	5	6	7	8	0
1	90.63%	1.04%	0.00%	0.00%	1.04%	0.00%	0.00%	0.00%	7.29%
2	1.53%	85.38%	8.64%	0.14%	0.00%	0.14%	0.00%	0.00%	4.18%
3	0.14%	2.99%	86.60%	5.69%	0.35%	0.14%	0.00%	0.07%	4.03%
4	0.00%	0.00%	3.76%	85.25%	6.12%	1.02%	0.08%	0.31%	3.45%
5	0.00%	0.00%	0.66%	7.63%	71.14%	10.78%	1.66%	1.00%	7.13%
6	0.00%	0.19608%	0.39%	0.78%	7.45%	75.49%	7.84%	1.76%	6.08%
7	0.00%	0.00%	0.00%	0.00%	1.84%	7.36%	60.74%	11.04%	19.02%
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	0.00%

Count of 2000	Column Labels										
Row Labels	0	1	2	3	4	5	6	7	8	9	Grand Total
0	127			1	1			1			130
1	2	7									9
2	2		123	2	1						128
3	1	1	7	220	20		1				250
4	4			3	178	9			3		197
5	5			1	10	70	13	1	2		102
6	4					11	64	10	3		92
7	8						1	16	4		29
8									18		18
9	36	8	57	81	80	26	17	8	6	555	874
Grand Total	**189**	**16**	**187**	**308**	**290**	**116**	**96**	**36**	**36**	**555**	**1829**

	1	2	3	4	5	6	7	8	NR
1	90.63%	1.04%	0.00%	0.00%	1.04%	0.00%	0.00%	0.00%	7.29%
2	1.53%	85.38%	8.64%	0.14%	0.00%	0.14%	0.00%	0.00%	4.18%
3	0.14%	2.99%	86.60%	5.69%	0.35%	0.14%	0.00%	0.07%	4.03%
4	0.00%	0.00%	3.75%	85.08%	6.09%	1.02%	0.08%	0.31%	3.67%
5	0.00%	0.00%	0.66%	7.57%	71.38%	10.69%	1.64%	0.99%	7.07%
6	0.00%	0.19%	0.38%	0.77%	7.31%	75.38%	8.08%	1.73%	6.15%
7	0.00%	0.00%	0.00%	0.00%	1.64%	7.10%	61.20%	10.38%	19.67%
	97.75%	1.12%	0.00%	0.00%	1.12%	0.00%	0.00%	0.00%	92.71%

Count of 2001	Column Labels										
Row Labels	0	1	2	3	4	5	6	7	8	9	Grand Total
0	182		1	2	1	1	2				189
1	2	14									16
2	9	9	137	31			1				187

14.2 Wholesale Models- Building TM Using Duration Approach

PEAKS2TAILS

Total Count of transitions

	0	1	2	3	4	5	6	7	8
0	2	0	5	10	14	13	13	9	7
1	7	0	2	1	0	0	0	0	0
2	36	13	76	72	2	0	0	0	0
3	76	2	52	196	108	4	1	0	0
4	61	0	0	69	267	115	25	5	0
5	46	0	0	4	77	115	114	14	2
6	38	0	1	1	6	65	115	78	13
7	54	0	0	0	3	6	32	41	32
8	29	0	0	0	2	2	4	12	2

Total Time spent in each bucket

1719.827
138.0411
983.2877
1976.485
1772.668
813.3836
686.3123
245.8137
83.73425

	9	1	2	3	4	5	6	7	8
9	-0.041	0.000	0.003	0.006	0.008	0.008	0.008	0.005	0.004
1	0.051	-0.072	0.014	0.007	0.000	0.000	0.000	0.000	0.000
2	0.037	0.013	-0.125	0.073	0.002	0.000	0.000	0.000	0.000
3	0.038	0.001	0.026	-0.123	0.055	0.002	0.001	0.000	0.000
4	0.034	0.000	0.000	0.039	-0.155	0.065	0.014	0.003	0.000
5	0.057	0.000	0.000	0.005	0.095	-0.316	0.140	0.017	0.002
6	0.055	0.000	0.001	0.001	0.009	0.095	-0.294	0.114	0.019
7	0.220	0.000	0.000	0.000	0.012	0.024	0.130	-0.517	0.130
8	0.346	0.000	0.000	0.000	0.024	0.024	0.048	0.143	-0.585

14.3 TM Smoothening

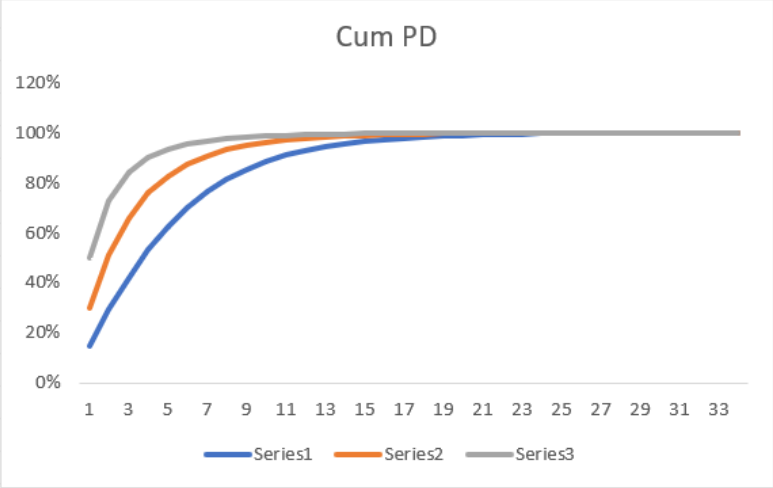
Min difference	0.0001					
	1	2	3	4	5	
1	80%	6%	8%	4%	2%	100%
2	4%	82%	5%	6%	3%	100%
3	6%	2%	84%	4%	4%	100%
4	4%	7%	5%	78%	6%	100%
5	0	0	0	0	100%	100%
	1	2	3	4	5	
1	81.82%	6.15%	6.14%	3.95%	1.95%	100%
2	4.76%	81.81%	6.15%	4.74%	2.54%	100%
3	4.75%	4.76%	81.80%	4.75%	3.95%	100%
4	4.56%	4.75%	5.56%	78.56%	6.56%	100%
5	0.00%	0.00%	0.00%	0.00%	100.00%	100%
Obj	0.0031					

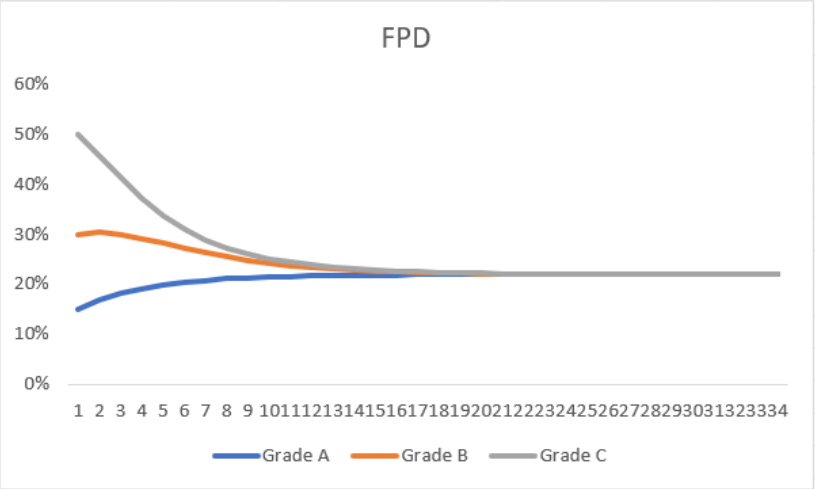
Constraints			
80%(R)	76%	>=	0%
	0%	>=	0%
	2%	>=	0%
	2%	>=	0%
80%(D)	77%	>=	0%
	0%	>=	0%
	0%	>=	0%
82%(Up)	76%	>=	0%
82%(down)	77%	>=	0%
	0%	>=	0%
82%(left)	77%	>=	0%
82%(right)	76%	>=	0%
	1%	>=	0%
	2%	>=	0%
84%(Up)	76%	>=	0%
	0%	>=	0%
84%(down)	76%	>=	0%
84%(left)	77%	>=	0%
	0%	>=	0%
84%(right)	77%	>=	0%
	1%	>=	0%
78%(Up)	74%	>=	0%
	0%	>=	0%
	1%	>=	0%
78%(left)	73%	>=	0%
	1%	>=	0%
	0%	>=	0%
78%(right)	72%	>=	0%
D2	0%	>=	0%
D3	0%	>=	0%
D4	3%	>=	0%
R1	100%	=	100%
R2	100%	=	100%

14.4 TM Central Tendency

PEAKS2TAILS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1																							
2			A	B	C	D		Cumulative PD															
3		A	75%	10%	0%	15%		15%	29%	42%	53%	62%	70%	76%	81%	85%	88%	91%	93%	94%	96%	97%	97%
4		B	5%	60%	5%	30%		30%	51%	66%	76%	83%	87%	91%	93%	95%	96%	97%	98%	98%	99%	99%	99%
5		C	1%	9%	40%	50%		50%	73%	84%	90%	93%	95%	97%	98%	98%	99%	99%	99%	99%	100%	100%	100%
6		D	0%	0%	0%	100%		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
7								FPD															
8			A	B	C	D		15%	16.76%	18.11%	19.11%	19.84%	20.38%	20.78%	21.08%	21.30%	21.46%	21.58%	21.67%	21.74%	21.80%	21.84%	21.86
9		A	57%	14%	1%	29%		30%	30.36%	29.96%	29.18%	28.23%	27.26%	26.33%	25.50%	24.78%	24.19%	23.70%	23.31%	23.00%	22.76%	22.57%	22.42
10		B	7%	37%	5%	51%		50%	45.70%	41.23%	37.11%	33.64%	30.90%	28.81%	27.21%	26.00%	25.07%	24.36%	23.80%	23.37%	23.04%	22.78%	22.59
11		C	2%	9%	16%	73%																	
12		D	0%																				
13																							
14			A																				
15		A	43%																				
16		B	7%																				
17		C	2%																				
18		D	0%																				
19																							
20			A																				
21		A	33%																				
22		B	6%																				
23		C	2%																				
24		D	0%																				
25																							
26			A																				
27		A	25%	11%	1%	62%																	
28		B	6%	10%	2%	83%																	
29		C	2%	4%	2%	93%																	
30		D	0%	0%	0%	100%																	
31																							
32			A	B	C	D																	
33		A	20%	9%	1%	70%																	





14.5 TM Mobility Index

	1	2	3	4	5	6	7	8
1	90.63%	1.04%	0.00%	0.00%	1.04%	0.00%	0.00%	0.00%
2	1.53%	85.38%	8.64%	0.14%	0.00%	0.14%	0.00%	0.00%
3	0.14%	2.99%	86.60%	5.69%	0.35%	0.14%	0.00%	0.07%
4	0.00%	0.00%	3.76%	85.25%	6.12%	1.02%	0.08%	0.31%
5	0.00%	0.00%	0.66%	7.63%	71.14%	10.78%	1.66%	1.00%
6	0.00%	0.20%	0.39%	0.78%	7.45%	75.49%	7.84%	1.76%
7	0.00%	0.00%	0.00%	0.00%	1.84%	7.36%	60.74%	11.04%
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Trace	6.55	Determinant	0.178626
Mobility	0.206823	Mobility	0.821374

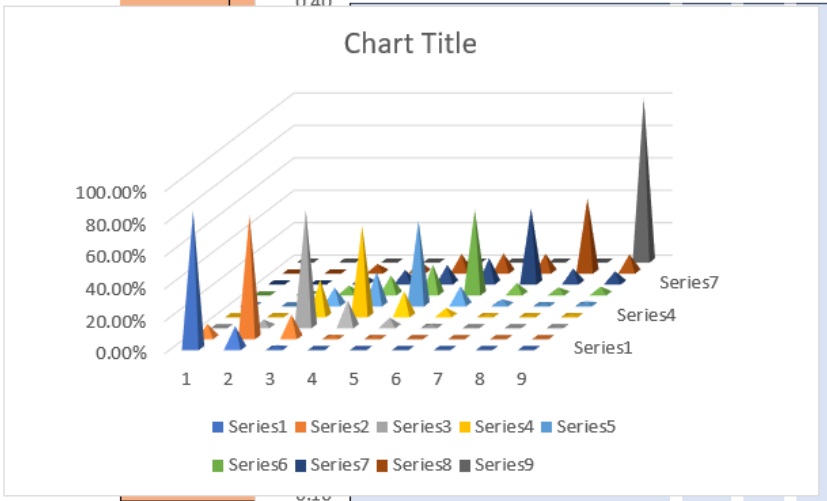
	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0	0	1	0	0	0
6	0	0	0	0	0	1	0	0
7	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	1

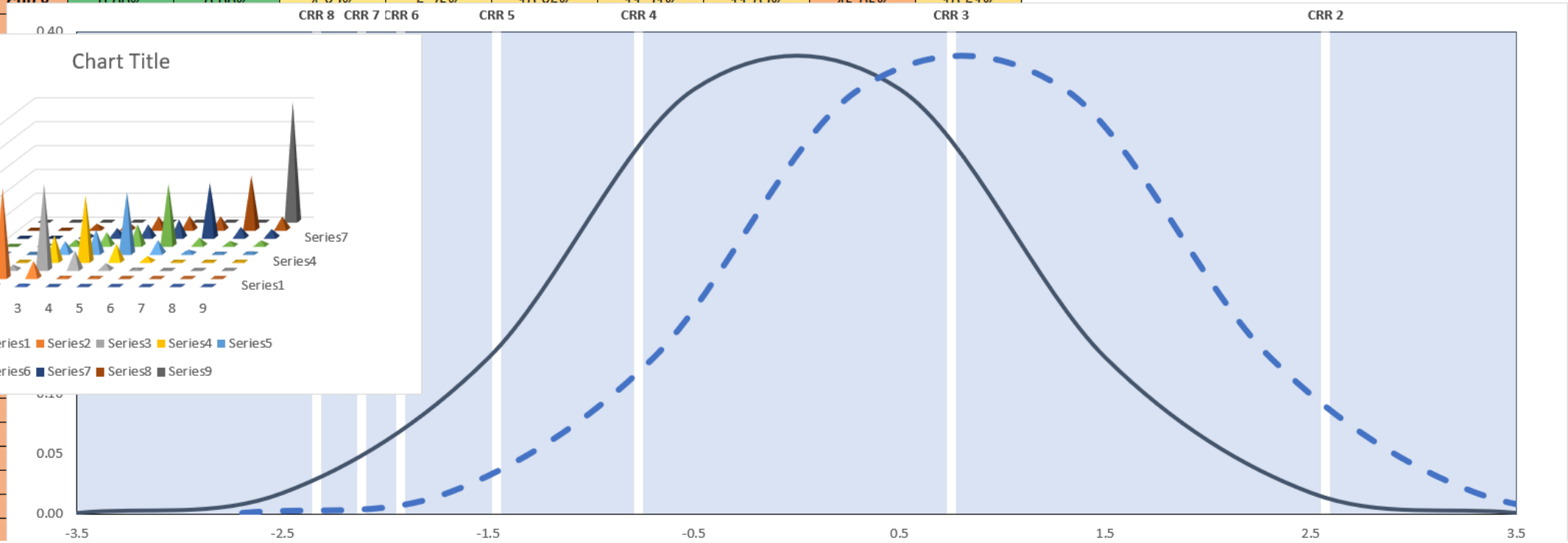
Distance	0.432626	
Euc distance	0.657743	2.383757748
Mobility	0.248604	0.148984859

14.6 TTC To PIT TM

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	84.59%	13.53%	1.76%	0.11%	0.00%	0.00%	0.00%	0.00%	0.01%
	CRR 2	8.17%	75.06%	13.97%	1.50%	0.96%	0.16%	0.06%	0.06%	0.04%
	CRR 3	0.06%	4.04%	71.73%	15.79%	5.29%	1.68%	0.39%	0.33%	0.70%
	CRR 4	0.01%	0.50%	22.10%	55.27%	14.87%	4.54%	0.99%	0.75%	0.98%
	CRR 5	0.00%	0.21%	10.38%	20.13%	51.69%	11.19%	3.13%	1.45%	1.81%
	CRR 6	0.00%	0.15%	4.83%	11.09%	17.84%	51.61%	6.48%	3.75%	4.25%
	CRR 7	0.00%	0.09%	4.82%	7.76%	10.82%	15.00%	45.81%	8.70%	7.00%
	CRR 8	0.00%	0.00%	4.82%	6.25%	10.86%	11.21%	11.02%	45.05%	10.61%

Date	q2 2017
z	0.81
Rho	0.20





14.7 TTC To PIT TM in Vasicek Space

A	B	C	D	Sum
500	300	150	50	1,000
100.00%	0.00%	0.00%	0.00%	100.00%

Rho	0.2
Z	0.36
PD	13.00%

TTC	A	B	C	D
A	75%	10%	2%	13%
B	5%	60%	5%	30%
C	1%	9%	40%	50%
D	0%	0%	0%	100%

UB	A	B	C	D
A	10000.00	-0.67	-1.04	-1.13
B	10000.00	1.64	-0.39	-0.52
C	10000.00	2.33	1.28	0.00

LB	A	B	C	D
A	-0.67	-1.04	-1.13	-10000.00
B	1.64	-0.39	-0.52	-10000.00
C	2.33	1.28	0.00	-10000.00

PIT	A	B	C	D
A	82%	8%	2%	8%
B	5%	68%	5%	22%
C	1%	10%	47%	43%
D	0%	0%	0%	100%

Weights	62.34%	12.72%	0.00%	24.94%

14.8 TTC To PIT TM in Probit Space

A	B	C	D	Sum
500	300	150	50	1,000
60.00%	20.00%	15.00%	5.00%	100.00%

Z	0.19
PD	26.30%

TTC	A	B	C	D
A	75%	10%	2%	13%
B	5%	60%	5%	30%
C	1%	9%	40%	50%
D	0%	0%	0%	100%

UB	A	B	C	D
A	10000.00	-0.67	-1.04	-1.13
B	10000.00	1.64	-0.39	-0.52
C	10000.00	2.33	1.28	0.00

LB	A	B	C	D
A	-0.67	-1.04	-1.13	-10000.00
B	1.64	-0.39	-0.52	-10000.00
C	2.33	1.28	0.00	-10000.00

PD	34.7%
	0.242

PIT	A	B	C	D
A	81%	8%	2%	9%
B	7%	64%	5%	24%
C	2%	12%	44%	42%
D	0%	0%	0%	100%

Weight	50.08%	19.73%	8.43%	21.76%

14.9 TTC To PIT TM in Logit Space

A	B	C	D	Sum
500	300	150	50	1,000
100.00%	0.00%	0.00%	0.00%	100.00%

Z	0.36
PD	13.00%

TTC	A	B	C	D
A	75%	10%	2%	13%
B	5%	60%	5%	30%
C	1%	9%	40%	50%
D	0%	0%	0%	100%

UB	A	B	C	D
A	20.72	-1.10	-1.73	-1.90
B	20.72	2.94	-0.62	-0.85
C	20.72	4.60	2.20	0.00

LB	A	B	C	D
A	-1.10	-1.73	-1.90	-20.72
B	2.94	-0.62	-0.85	-20.72
C	4.60	2.20	0.00	-20.72

PD	40.8%

PIT	A	B	C	D
A	68%	12%	3%	18%
B	4%	53%	6%	38%
C	1%	6%	34%	59%
D	0%	0%	0%	100%

Weights	62.34%	12.72%	0.00%	24.94%

14.10 Yearly TM To Quarterly TM

PEAKS2TAILS

1 year TM

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	92.59%	6.98%	0.42%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%
	CRR 2	10.54%	81.14%	7.60%	0.47%	0.22%	0.02%	0.01%	0.01%	0.00%
	CRR 3	0.04%	5.02%	81.15%	10.57%	2.39%	0.54%	0.10%	0.08%	0.11%
	CRR 4	0.00%	0.51%	29.16%	58.14%	9.55%	1.95%	0.32%	0.20%	0.18%
	CRR 5	0.00%	0.19%	13.61%	26.08%	51.24%	6.66%	1.33%	0.49%	0.40%
	CRR 6	0.00%	0.13%	6.11%	14.88%	22.63%	49.44%	3.78%	1.74%	1.28%
	CRR 7	0.00%	0.07%	6.08%	10.44%	14.21%	18.37%	43.22%	5.09%	2.52%
	CRR 8	0.00%	0.07%	6.08%	8.55%	14.34%	14.01%	12.82%	39.69%	4.44%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

mlog

Identity Matrix (I)

1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1

Generator Matrix (G)

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	-8.17%	8.07%	0.12%	-0.01%	-0.01%	0.00%	0.00%	0.00%	0.00%
	CRR 2	12.21%	-21.70%	9.41%	-0.08%	0.17%	-0.01%	0.00%	0.00%	0.00%
	CRR 3	-0.32%	6.28%	-24.16%	15.02%	2.45%	0.47%	0.08%	0.08%	0.10%
	CRR 4	0.05%	-0.60%	42.04%	-61.09%	16.39%	2.50%	0.30%	0.26%	0.15%
	CRR 5	0.02%	-0.21%	12.18%	45.63%	-72.82%	11.98%	2.17%	0.68%	0.38%
	CRR 6	0.00%	0.07%	2.19%	18.28%	41.71%	-74.02%	7.03%	3.23%	1.50%
	CRR 7	0.01%	-0.09%	4.47%	11.02%	20.36%	35.72%	-85.46%	10.87%	3.12%
	CRR 8	0.01%	-0.10%	5.11%	7.22%	21.84%	24.43%	27.89%	-92.49%	6.08%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

3m TM

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	98.01%	1.95%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	CRR 2	2.94%	94.77%	2.22%	0.02%	0.04%	0.00%	0.00%	0.00%	0.00%
	CRR 3	-0.05%	1.48%	94.35%	3.41%	0.62%	0.12%	0.02%	0.02%	0.03%
	CRR 4	0.01%	-0.06%	9.52%	86.23%	3.53%	0.59%	0.08%	0.06%	0.04%
	CRR 5	0.00%	-0.03%	3.24%	9.79%	83.71%	2.55%	0.48%	0.16%	0.10%
	CRR 6	0.00%	0.01%	0.87%	4.41%	8.83%	83.34%	1.49%	0.68%	0.36%
	CRR 7	0.00%	-0.01%	1.23%	2.77%	4.68%	7.47%	80.92%	2.21%	0.74%
	CRR 8	0.00%	-0.01%	1.33%	1.99%	4.90%	5.30%	5.64%	79.46%	1.40%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Q = G/4

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	-2.04%	2.02%	0.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	CRR 2	3.05%	-5.42%	2.35%	-0.02%	0.04%	0.00%	0.00%	0.00%	0.00%
	CRR 3	-0.08%	1.57%	-6.04%	3.75%	0.61%	0.12%	0.02%	0.02%	0.03%
	CRR 4	0.01%	-0.15%	10.51%	-15.27%	4.10%	0.63%	0.07%	0.06%	0.04%
	CRR 5	0.00%	-0.05%	3.04%	11.41%	-18.21%	3.00%	0.54%	0.17%	0.09%
	CRR 6	0.00%	0.02%	0.55%	4.57%	10.43%	-18.50%	1.76%	0.81%	0.38%
	CRR 7	0.00%	-0.02%	1.12%	2.75%	5.09%	8.93%	-21.37%	2.72%	0.78%
	CRR 8	0.00%	-0.02%	1.28%	1.81%	5.46%	6.11%	6.97%	-23.12%	1.52%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

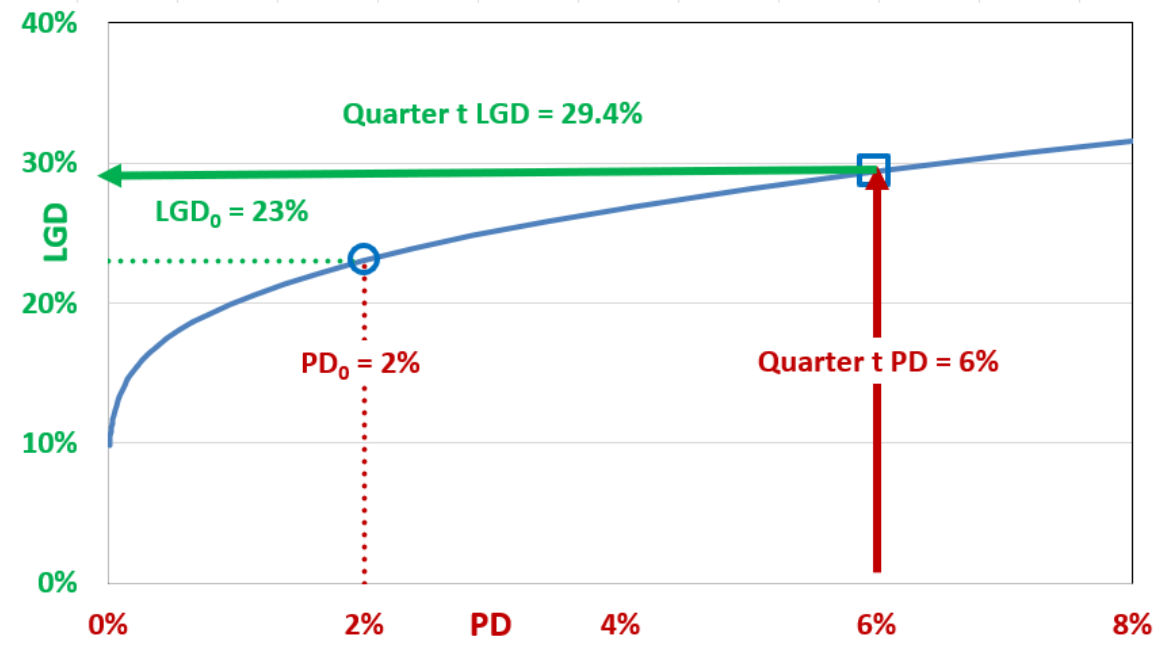
0 Then Smooth the matrix to take care of the small negative entries

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	98.01%	1.95%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	CRR 2	2.94%	94.77%	2.22%	0.02%	0.04%	0.00%	0.00%	0.00%	0.00%
	CRR 3	0.00%	1.48%	94.35%	3.41%	0.62%	0.12%	0.02%	0.02%	0.03%
	CRR 4	0.01%	0.00%	9.52%	86.23%	3.53%	0.59%	0.08%	0.06%	0.04%
	CRR 5	0.00%	0.00%	3.24%	9.79%	83.71%	2.55%	0.48%	0.16%	0.10%
	CRR 6	0.00%	0.01%	0.87%	4.41%	8.83%	83.34%	1.49%	0.68%	0.36%
	CRR 7	0.00%	0.00%	1.23%	2.77%	4.68%	7.47%	80.92%	2.21%	0.74%
	CRR 8	0.00%	0.00%	1.33%	1.99%	4.90%	5.30%	5.64%	79.46%	1.40%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

PIT TM		to grade								
		CRR 1	CRR 2	CRR 3	CRR 4	CRR 5	CRR 6	CRR 7	CRR 8	CRR 9
from grade	CRR 1	98.00%	1.95%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	CRR 2	2.94%	94.77%	2.22%	0.02%	0.04%	0.00%	0.00%	0.00%	0.00%
	CRR 3	0.00%	1.48%	94.30%	3.41%	0.62%	0.12%	0.02%	0.02%	0.03%
	CRR 4	0.01%	0.00%	9.52%	86.18%	3.53%	0.59%	0.08%	0.06%	0.04%
	CRR 5	0.00%	0.00%	3.24%	9.78%	83.69%	2.55%	0.48%	0.16%	0.10%
	CRR 6	0.00%	0.01%	0.87%	4.41%	8.83%	83.34%	1.49%	0.68%	0.36%
	CRR 7	0.00%	0.00%	1.23%	2.77%	4.68%	7.46%	80.91%	2.21%	0.74%
	CRR 8	0.00%	0.00%	1.33%	1.99%	4.90%	5.29%	5.64%	79.45%	1.40%
	CRR 9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

15.1 PIT LGD- Jacob Frye

0.02	PD_0	
0.25	$ELGD_0$	
	0.05	
	PD	LGD
k	0	
0.55032	0.0001	0.09803
	0.00012	0.10044
	0.00014	0.10293
	0.00017	0.10552
	0.00021	0.10819
	0.00025	0.11097
	0.0003	0.11385
	0.00036	0.11684
	0.00043	0.11995
	0.00052	0.12318
	0.00062	0.12653
	0.00074	0.13002
	0.00089	0.13366
	0.00107	0.13745
	0.00128	0.1414
	0.00154	0.14552
	0.00185	0.14983
	0.00222	0.15433
	0.00266	0.15903
	0.00319	0.16396
	0.00383	0.16913
	0.0046	0.17455
	0.00552	0.18024
	0.00662	0.18622
	0.00795	0.19253
	0.00954	0.19917
	0.01145	0.20619
	0.01374	0.2136
	0.01648	0.22146



This spreadsheet illustrates the LGD function of Frye and Jacobs.
Neither the Federal Reserve nor any other agency endorses this model.
Only a bank's management and supervisors can determine whether a particular model is appropriate for a particular use.
Any views expressed are the author's and do not necessarily represent the views of the management of the Federal Reserve Bank of Chicago or the Federal Reserve System.

15.2 PIT LGD- Jacob Frye

PEAKS2TAILS

PIT LGD Model (JF)

PD	3.10%
ELGD	62.46%

K
0.214053

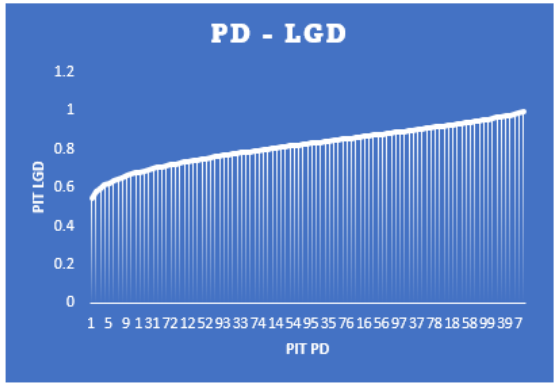
Model Output

PIT PD %	PIT LGD
30.00	76.71%

Rating	Corporates							
	Inputs			Forward Looking PIT PDs				
	TTC PD	E-LGD	K					
1	0.008%	62.13%	12.43%	0.01%	0.01%	0.01%	0.01%	0.01%
2	0.008%	55.96%	15.12%	0.01%	0.01%	0.01%	0.01%	0.01%
3	0.083%	61.85%	14.70%	0.03%	0.03%	0.03%	0.03%	0.03%
4	0.327%	58.29%	18.54%	0.04%	0.04%	0.04%	0.04%	0.04%
5	0.705%	63.76%	16.81%	0.62%	0.61%	0.61%	0.61%	0.62%
6	1.186%	62.98%	18.36%	1.42%	1.41%	1.40%	1.41%	1.42%
7	2.183%	60.66%	21.50%	2.23%	2.21%	2.20%	2.21%	2.22%
8	4.174%	64.37%	21.12%	3.56%	3.52%	3.51%	3.53%	3.55%
9	6.420%	61.46%	25.22%	6.71%	6.61%	6.58%	6.63%	6.68%
10	7.142%	63.26%	24.33%	7.80%	7.70%	7.67%	7.73%	7.77%

Final Result

Corporate Model					
Rating	Forward Looking LGDs				
	Mar-23	Jun-23	Sep-23	Dec-23	Mar-24
A1	61.21%	61.21%	61.21%	61.21%	61.21%
A2	54.94%	54.94%	54.94%	54.94%	54.94%
A3	57.57%	57.57%	57.57%	57.57%	57.57%
A4	50.43%	50.37%	50.36%	50.39%	50.42%
B1	61.41%	61.36%	61.34%	61.37%	61.39%
B2	61.75%	61.71%	61.70%	61.72%	61.74%
B3	58.71%	58.67%	58.66%	58.68%	58.70%
C1	61.61%	61.55%	61.54%	61.57%	61.59%
C2	59.70%	59.60%	59.58%	59.63%	59.68%
C3	61.88%	61.79%	61.77%	61.81%	61.85%

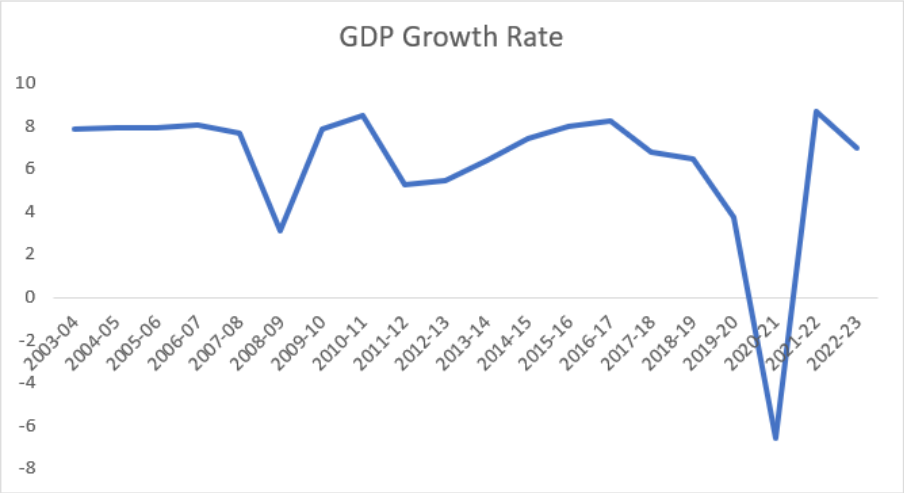


15.3 PIT CCF Regression

Year	GDP (Y-o-Y %) at Market Prices	CCF	Economy Indicator
2003-04	7.86	44.57%	Moderate
2004-05	7.92	69.67%	Moderate
2005-06	7.92	50.92%	Moderate
2006-07	8.06	46.58%	Good
2007-08	7.66	46.84%	Moderate
2008-09	3.09	69.26%	Poor
2009-10	7.86	57.02%	Moderate
2010-11	8.50	44.57%	Good
2011-12	5.24	54.88%	Poor
2012-13	5.46	50.91%	Poor
2013-14	6.39	56.92%	Moderate
2014-15	7.41	46.65%	Moderate
2015-16	8		
2016-17	8.26		
2017-18	6.8		
2018-19	6.45		
2019-20	3.74		
2020-21	-6.6		
2021-22	8.68		
2022-23	6.95		

Min	3.09	44.57%
1st Quartile	6.16	
2nd Quartile	7.76	
3rd Quartile	7.92	
Max	8.5	69.67%
	-0.533	

GDP_{Good}	CCF_{Good}	$GDP_{Moderate}$	$CCF_{Moderate}$	GDP_{Poor}	CCF_{Poor}
8.28	45.58%	7.57	53.23%	4.60	58.35%



Year	GDP (Y-o-Y %) at Constant Prices
2005-06	16.35
2006-07	8.06
2007-08	7.66
2008-09	3.09
2009-10	7.86
2010-11	8.5
2011-12	5.24
2012-13	5.46
2013-14	6.39
2014-15	7.41
2015-16	8
2016-17	8.26
2017-18	6.8
2018-19	6.45
2019-20	3.74
2020-21	-6.6
2021-22	8.68
2022-23	6.72
2023-24	5.7
2024-25	6.13
2025-26	6.04
2026-27	6.22

15.4 EAD Term Structure

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EAD TERM STRUCTURE MODELLING

Limit	1,00,000.00
Utilized	50,000.00

CCF ^{D}	75%

	High	Moderate	Low
CCF ND	46%	53%	58%

Macro Economic Forecast

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Moderate	Low	High	Low	High	Moderate	Low	High	Low	High	Moderate	Moderate	Low	High	Low	High	Moderate	Low	High	Low

Model Output

Year	3	EAD	INR 97,564.89

	EAD ^{T}	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
1	EAD^{1}	87,500.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	EAD^{2}	76,613.57	94,153.39	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	EAD^{3}	76,613.57	90,259.55	97,564.89	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	EAD^{4}	76,613.57	90,259.55	94,698.76	98,674.69	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	EAD^{5}	76,613.57	90,259.55	94,698.76	97,792.03	99,448.01	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	EAD^{6}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,699.58	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	EAD^{7}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,859.48	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	EAD^{8}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,941.48	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	EAD^{9}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,968.15	-	-	-	-	-	-	-	-	-	-	-	-	-
10	EAD^{10}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,986.73	-	-	-	-	-	-	-	-	-	-	-	-
11	EAD^{11}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,992.78	-	-	-	-	-	-	-	-	-	-	-
12	EAD^{12}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,996.62	-	-	-	-	-	-	-	-	-	-
13	EAD^{13}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,998.42	-	-	-	-	-	-	-	-	-
14	EAD^{14}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,999.34	-	-	-	-	-	-	-	-
15	EAD^{15}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.64	-	-	-	-	-	-	-
16	EAD^{16}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.85	-	-	-	-	-	-
17	EAD^{17}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.92	-	-	-	-	-
18	EAD^{18}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.85	99,999.96	-	-	-	-
19	EAD^{19}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.85	99,999.94	99,999.98	-	-	-
20	EAD^{20}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.85	99,999.94	99,999.97	99,999.99	-	-
21	EAD^{21}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.85	99,999.94	99,999.97	99,999.99	1,00,000.00	-
22	EAD^{22}	76,613.57	90,259.55	94,698.76	97,792.03	98,798.31	99,437.94	99,765.90	99,872.59	99,946.93	99,971.12	99,986.49	99,993.68	99,997.37	99,998.57	99,999.40	99,999.68	99,999.85	99,999.94	99,999.97	99,999.99	99,999.99	1,00,000.00

15.5 EAD For Credit Cards

PEAKS2TAILS

EAD Model for Credit Card

uti_def_orig	actual utilization % at the time of default
uti_def	Same as uti_def_orig , but we replace values which are >=1 with 0.999 and negative values with 0.2
uti_ini	initial utilization % (12m before default date)

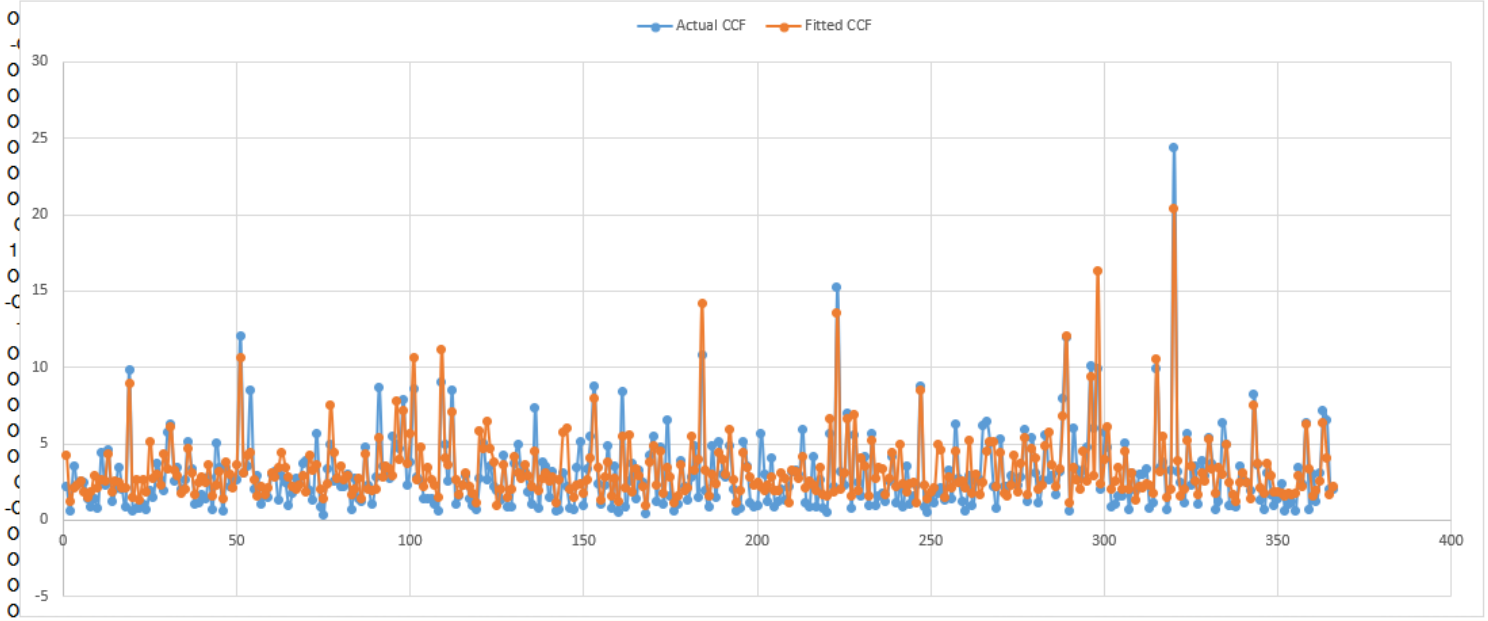
We will do beta regresion with uti_def as y and uti_ini and gdp as X

$$f(y;\mu,\phi)=\frac{\Gamma(\phi)}{\Gamma(\mu\phi)\Gamma((1-\mu)\phi)}y^{\mu\phi-1}(1-y)^{(1-\mu)\phi-1},\quad 0<y<1,$$

Maximize sum (ln(f)) 153.0454

	Intercept	0.155374228		
2.233406	uti_ini	0.673596171		
	gdp	-0.18282171		
	ϕ	2.62056145		

SL	id	report_date	year	balance	default_flag	x2 gdp	uer	cpi	hpi	ir	uti_def_orig	y uti_def	x1 uti_ini	ccf_ratio	βX	μ = N(βX)	f	ln(f)	fitted uti_def
1	335679	9/30/08	2008	89182.01	1	-1.449	5.9	4.8	-11.483	4.75	0.371927306	0.371927306	0.166529187	2.233406126	0.532456	0.702795	0.624296	-0.47113	0.702795001
2	342540	6/30/08	2008	291269.44	1	1.005	5.3	3.4	-5.192	4.87	0.288526753	0.288526753	0.483251564	0.597052912	0.297155	0.616826	0.754364	-0.28188	0.616825841
3	343423	3/31/11	2011	177314.43	1	2.262	7.8	4.1	-1.533	3.89	0.736082494	0.736082494	0.210035337	3.504565023	-0.11669	0.453553	0.944353	-0.05725	0.453553065
4	346117	12/31/04	2004	109932.69	1	1.844	4.7	1.4	14.396	4.62	0.490712517	0.490712517	0.21016179	2.334927382	-0.04018	0.483973	1.175854	0.161995	0.483972888
5	358929	3/31/07	2007	66443	1	1.751	5.5	2.9	10.433	4.75	0.478016321	0.478016321	0.187625839	2.547710496	-0.03836	0.484699	1.177796	0.163645	0.484699316
6	362661	12/31/09	2009	186012.76	1	-1.386	7.8	2.1	2.246	3.85	1.185579496	0.9999	0.4008267	2.494594299	0.67876	0.751355	26.34468	3.271266	0.75135517
7	364252	12/31/07	2007	29589.94	1	3.302	5.2	2.1	6.161	4.74	0								
8	367025	9/30/07	2007	46356.11	1	2.897	5.3	1.8	10.197	5.09	-								
9	376285	9/30/10	2010	112054.93	1	2.553	7.8	3.1	3.576	3.39	0								
10	407177	9/30/12	2012	22779.79	1	1.754	7.9	2.4	-1.425	1.72	0								
11	412545	12/31/13	2013	119627.07	1	2.425	7.2	2.1	7.32	2.88	0								
12	414995	12/31/10	2010	115716.33	1	2.255	7.9	3.4	-0.388	3.44	0								
13	415746	12/31/08	2008	87500	1	-4.397	6.4	3.9	-15.513	4.2	0								
14	433040	12/31/10	2010	136223.3	1	2.255	7.9	3.4	-0.388	3.44	0								
15	433853	9/30/09	2009	175055	1	-4.008	7.8	1.5	-4.912	3.83	(								
16	440289	9/30/08	2008	180441.2	1	-1.449	5.9	4.8	-11.483	4.75	1								
17	456152	12/31/10	2010	60173.21	1	2.255	7.9	3.4	-0.388	3.44	0								
18	473639	3/31/07	2007	103050.87	1	1.751	5.5	2.9	10.433	4.75	-(								
19	474606	3/31/09	2009	94418.64	1	-6.091	7.1	3	-17.12	3.62	-								
20	484958	12/31/12	2012	23916.75	1	1.266	7.8	2.7	-0.778	1.84	0								
21	492015	9/30/08	2008	70071.48	1	-1.449	5.9	4.8	-11.483	4.75	0								
22	509650	3/31/06	2006	136601.83	1	3.901	5.2	1.9	5.503	4.13	0								
23	512591	6/30/11	2011	73290.04	1	1.305	7.9	4.4	-2.281	3.66	0								
24	517307	3/31/05	2005	51863.97	1	1.859	4.7	1.7	9.882	4.6	0								
25	517742	12/31/12	2012	22954.59	1	1.266	7.8	2.7	-0.778	1.84	(								
26	527970	9/30/05	2005	37163.07	1	3.386	4.7	2.4	2.853	4.24	-(								
27	533838	6/30/12	2012	119196.5	1	1.038	8	2.8	-0.908	2	0								
28	552027	6/30/04	2004	42659.93	1	2.89	4.8	1.4	19.598	5.04	0								
29	566508	9/30/06	2006	55550.86	1	2.07	5.5	2.4	7.53	4.54	0								
30	574440	9/30/06	2006	51478.17	1	2.07	5.5	2.4	7.53	4.54	0								



16.1 Logistic Regression for Prepayment Modelling

PEAKS2TAILS

Logistic regression for Full Prepayment Modeling

We use a panel data which contains BVs and MVs and a prepayment flag (1, 0)
Each account is observed quarterly until prepayment occurs.
So our model will help predict accounts will be prepaying fully in one quarter or not.

y = prepayment flag

Maximize Sum (likelihood)
-7415.749934

(The logistic model here is developed as is without splitting the data set or doing WOE transformation for simplicity)

X	id	report_date	year	quarter	month	balance	origination_date	tob	remaining_term	default_flag	prep_flag	part_prep_flag	gdp	uer	cpi	hpi	ir	multi_flag	pp_new	fppp_perc	ltv_utd
1	1000170	3/31/04	2004	1	3	44577	9/17/99	54	186	0	0	0	3.3	5	1	17	5	0	0	0	58
2	1000170	6/30/04	2004	2	6	44145	9/17/99	57	183	0	0	0	2.9	5	1	20	5	0	0	0	57
3	1000170	9/30/04	2004	3	9	43715	9/17/99	60	180	0	0	0	2.1	5	1	19	5	0	0	0	56
4	1000170	12/31/04	2004	4	12	43277	9/17/99	63	177	0	0	0	1.8	5	1	14	5	0	0	0	55
5	1000170	3/31/05	2005	1	3	42817	9/17/99	66	174	0	0	0	1.9	5	2	10	5	0	0	0	55
6	1000170	6/30/05	2005	2	6	42357	9/17/99	69	171	0	0	0	2.5	5	2	5	4	0	0	0	56
7	1000170	9/30/05	2005	3	9	41897	9/17/99	72	168	0	0	0	3.4	5	2	3	4	0	0	0	56
8	1000170	12/31/05	2005	4	12	41430	9/17/99	75	165	0	0	0	4.2	5	2	4	4	0	0	0	55
9	1000170	3/31/06	2006	1	3	40940	9/17/99	78	162	0	0	0	3.9	5	2	6	4	0	0	0	53
10	1000170	6/30/06	2006	2	6	40450	9/17/99	81	159	0	0	0	3	6	2	7	5	0	0	0	52
11	1000170	9/30/06	2006	3	9	39959	9/17/99	84	156	0	0	0	2.1	6	2	8	5	0	0	0	51
12	1000170	12/31/06	2006	4	12	39460	9/17/99	87	153	0	0	0	1.1	6	3	10	5	0	0	0	51
13	1000170	3/31/07	2007	1	3	38939	9/17/99	90	150	0	0	0	1.8	6	3	10	5	0	0	0	52
14	1000170	6/30/07	2007	2	6	38416	9/17/99	93	147	0	0	0	2.3	5	3	11	5	0	0	0	52
15	1000170	9/30/07	2007	3	9	37892	9/17/99	96	144	0	0	0	2.9	5	2	10	5	0	0	0	53
16	1000170	12/31/07	2007	4	12	37359	9/17/99	99	141	0	0	0	3.3	5	2	6	5	0	0	0	53
17	1000170	3/31/08	2008	1	3	36811	9/17/99	102	138	0	0	0	2.4	5	2	2	5	0	0	0	53
18	1000170	6/30/08	2008	2	6	36254	9/17/99	105	135	0	0	0	1	5	3	-5	5	0	0	0	53
19	1000170	9/30/08	2008	3	9	35694	9/17/99	108	132	0	0	0	-1	6	5	-11	5	0	0	0	53
20	1000170	12/31/08	2008	4	12	35125	9/17/99	111	129	0	0	0	-4	6	4	-16	4	0	0	0	52
21	1000170	3/31/09	2009	1	3	34535	9/17/99	114	126	0	0	0	-6	7	3	-17	4	0	0	0	51
22	1000170	6/30/09	2009	2	6	33941	9/17/99	117	123	0	0	0	-6	8	2	-13	4	0	0	0	49
23	1000170	9/30/09	2009	3	9	33312	9/17/99	120	120	0	0	0	-4	8	2	-5	4	0	0	0	47
24	1000170	12/31/09	2009	4	12	32658	9/17/99	123	117	0	0	0	-1	8	2	2	4	0	0	0	46
25	1000170	3/31/10	2010	1	3	31995	9/17/99	126	114	0	0	0	0.8	8	3	7	4	0	0	0	46
26	1000170	6/30/10	2010	2	6	31340	9/17/99	129	111	0	0	0	2.1	8	4	8	4	0	0	0	47
27	1000170	9/30/10	2010	3	9	30681	9/17/99	132	108	0	1	0	2.6	8	3	4	3	2	0	1	47
28	1000527	3/31/04	2004	1	3	43130	7/04/03	8	292	0	0	0	3.3	5	1	17	5	0	0	0	58
29	1000527	6/30/04	2004	2	6	42909	7/04/03	11	289	0	0	0	2.9	5	1	20	5	0	0	0	57
30	1000527	9/30/04	2004	3	9	42714	7/04/03	14	286	0	0	0	2.1	5	1	19	5	0	0	0	56
31	1000527	12/31/04	2004	4	12	42508	7/04/03	17	283	0	0	0	1.8	5	1	14	5	0	0	0	55
32	1000527	3/31/05	2005	1	3	42285	7/04/03	20	280	0	0	0	1.9	5	2	10	5	0	0	0	55
33	1000527	6/30/05	2005	2	6	42065	7/04/03	23	277	0	0	0	2.5	5	2	5	4	0	0	0	56
34	1000527	9/30/05	2005	3	9	41840	7/04/03	26	274	0	0	0	3.4	5	2	3	4	0	0	0	56
35	1000527	12/31/05	2005	4	12	41615	7/04/03	29	271	0	0	0	4.2	5	2	4	4	0	0	0	55
36	1000527	3/31/06	2006	1	3	41374	7/04/03	32	268	0	0	0	3.9	5	2	6	4	0	0	0	53
37	1000527	6/30/06	2006	2	6	41136	7/04/03	35	265	0	0	0	3	6	2	7	5	0	0	0	52
38	1000527	9/30/06	2006	3	9	40910	7/04/03	38	262	0	1	0	2.1	6	2	8	5	2	0	1	49

Intercept	93.47239
tob	0.002554
ltv_utd	-1.305934
uer	-3.353891
gdp	0.21933
cpi	-0.628574
hpi	0.092015
ir	-2.011749

P(full_prepayment)	ylnp+(1-y)ln)1-p)
0.001941188	-0.001943074
0.00406608	-0.004074369
0.021524829	-0.021759867
0.083053319	-0.086705953
0.049533342	-0.050802196
0.009145439	-0.009187515
0.013754431	-0.013849899
0.020797941	-0.021017265
0.245272103	-0.281397996
0.120850742	-0.128800592
0.297935607	-0.353730151
0.274233896	-0.320527487
0.063723399	-0.065844332
0.061201896	-0.063154834
0.044356849	-0.045370709
0.07627497	-0.079340838
0.056006617	-0.057636122
0.004315336	-0.004324674
0.000100865	-0.00010087
0.000134987	-0.000134996
0.000161441	-0.000161454
0.000522189	-0.000522326
0.023218888	-0.023492693
0.166678549	-0.182335816
0.056038813	-0.05767023
0.046307649	-0.047414143
0.166707067	-1.791517094
0.001726386	-0.001727878
0.003617001	-0.003623558
0.019184658	-0.019371072
0.074533343	-0.077457175
0.044285918	-0.045296488
0.008139954	-0.008173264
0.01224848	-0.01232411
0.018535301	-0.018709232
0.22417951	-0.253834113
0.108913897	-0.11531422
0.837158429	-0.177741944

16.2 Competing Risk Modelling

PEAKS2TAILS

Competing risk Modeling

In recent years, jointly modeling risks has gathered attention.
In this framework, we try to jointly model default or prepayments in framework.

ln(P(y=default)/P(y=active)) = a1 + b1 x tob + c1 x gdp
ln(P(y=Fullyprepaid)/P(y=active)) = a2 + b2 x tob + c2 x gdp

In this example, we will try to jointly predict Full prepayment or Default using Multinomial Logistic Regression.

Within a time interval t to t + τ

(i) a loan can remain active
(ii)a loan can become default
(iii)a loan can fully prepay

The following data set contains joint observation of default flag and full repayment flag.
Let's use only tob and gdp for illustration.

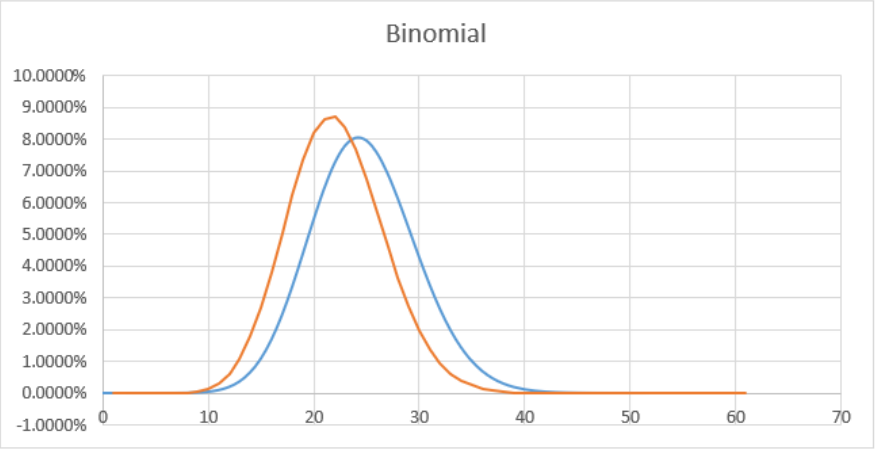
a1	b1	c1	a2	b2	c2
-4.474	0.0022	-0.14195	-2.7542	0.0019043	0.12043

Maximize sum (ln(p))
-12455.20268

id	report_date	year	quarter	month	balance	default_flag	prep_flag	part_prep_flag	gdp	uer	cpi	hpi	ir	multi_flag	ltv_utd	pp_new	fppp_perc	P(y=active)	P(y=default)	P(y=FullyPrepaid)	P	ln(p)
1000170	3/31/04	2004	1	3	44576.69	0	0	0	3.332	4.8	1.3	17.301	4.73	0	58	0	0	0.89815451	0.007188316	0.094657174	0.8982	-0.1074
1000170	6/30/04	2004	2	6	44145.22	0	0	0	2.89	4.8	1.4	19.598	5.04	0	57	0	0	0.901649784	0.007734513	0.090615703	0.9016	-0.1035
1000170	9/30/04	2004	3	9	43714.65	0	0	0	2.069	4.7	1.3	19.287	4.95	0	56	0	0	0.908048791	0.008810259	0.08314095	0.908	-0.0965
1000170	12/31/04	2004	4	12	43276.97	0	0	0	1.844	4.7	1.4	14.396	4.62	0	55	0	0	0.909333497	0.009169457	0.081497046	0.9093	-0.095
1000170	3/31/05	2005	1	3	42816.81	0	0	0	1.859	4.7	1.7	9.882	4.6	0	55	0	0	0.908737117	0.009204571	0.082058312	0.9087	-0.0957
1000170	6/30/05	2005	2	6	42356.88	0	0	0	2.471	4.8	2	5.042	4.4	0	56	0	0	0.903251965	0.008443332	0.088304703	0.9033	-0.1018
1000170	9/30/05	2005	3	9	41897.08	0	0	0	3.386	4.7	2.4	2.853	4.24	0	56	0	0	0.894421565	0.007391063	0.098187372	0.8944	-0.1116
1000170	12/31/05	2005	4	12	41429.7	0	0	0	4.157	5.1	2.1	4.06	4.25	0	55	0	0	0.88604957	0.006606326	0.107344104	0.886	-0.121
1000170	3/31/06	2006	1	3	40940.01	0	0	0	3.901	5.2	1.9	5.503	4.13	0	53	0	0	0.88815695	0.006912637	0.104930412	0.8882	-0.1186
1000170	6/30/06	2006	2	6	40449.74	0	0	0	3.031	5.5	2.3	6.8	4.55	0	52	0	0	0.896165036	0.007944148	0.095890816	0.8962	-0.1096
1000170	9/30/06	2006	3	9	39958.77	0	0	0	2.07	5.5	2.4	7.53	4.54	0	51	0	0	0.90409303	0.009246724	0.086660246	0.9041	-0.1008
1000170	12/31/06	2006	4	12	39459.71	0	0	0	1.066	5.5	2.7	9.753	4.5	0	51	0	0	0.911331448	0.010819757	0.077848795	0.9113	-0.0928
1000170	3/31/07	2007	1	3	38938.52	0	0	0	1.751	5.5	2.9	10.433	4.75	0	52	0	0	0.905678693	0.009820974	0.084500333	0.9057	-0.0991
1000170	6/30/07	2007	2	6	38415.9	0	0	0	2.266	5.4	2.6	10.612	5.1	0	52	0	0	0.900912742	0.009140772	0.089946486	0.9009	-0.1043
1000170	9/30/07	2007	3	9	37891.69	0	0	0	2.897	5.3	1.8	10.197	5.09	0	53	0	0	0.894712419	0.008355106	0.096932475	0.8947	-0.1113
1000170	12/31/07	2007	4	12	37358.83	0	0	0	3.302	5.2	2.1	6.161	4.74	0	53	0	0	0.89024898	0.007900986	0.101850034	0.8902	-0.1163
1000170	3/31/08	2008	1	3	36810.65	0	0	0	2.418	5.2	2.4	1.613	4.51	0	53	0	0	0.898012829	0.009095377	0.092891794	0.898	-0.1076
1000170	6/30/08	2008	2	6	36253.62	0	0	0	1.005	5.3	3.4	-5.192	4.87	0	53	0	0	0.908913274	0.011325097	0.079761628	0.9089	-0.0955
1000170	9/30/08	2008	3	9	35694.06	0	0	0	-1.449	5.9	4.8	-11.483	4.75	0	53	0	0	0.922982003	0.016401059	0.060616938	0.923	-0.0801
1000170	12/31/08	2008	4	12	35125.28	0	0	0	-4.397	6.4	3.9	-15.513	4.2	0	52	0	0	0.93153707	0.025321718	0.043141213	0.9315	-0.0709
1000170	3/31/09	2009	1	3	34534.79	0	0	0	-6.091	7.1	3	-17.12	3.62	0	51	0	0	0.932154984	0.032440398	0.035404618	0.9322	-0.0703
1000170	6/30/09	2009	2	6	33940.96	0	0	0	-5.694	7.8	2.1	-13.095	3.68	0	49	0	0	0.931808097	0.030854536	0.037337367	0.9318	-0.0706
1000170	9/30/09	2009	3	9	33311.94	0	0	0	-4.008	7.8	1.5	-4.912	3.83	0	47	0	0	0.929705503	0.024393007	0.04590149	0.9297	-0.0729
1000170	12/31/09	2009	4	12	32658.17	0	0	0	-1.386	7.8	2.1	2.246	3.85	0	46	0	0	0.920559569	0.016756938	0.062683493	0.9206	-0.0828
1000170	3/31/10	2010	1	3	31994.74	0	0	0	0.798	8	3.3	7.114	4.21	0	46	0	0	0.907008375	0.012189325	0.0808023	0.907	-0.0976
1000170	6/30/10	2010	2	6	31339.68	0	0	0	2.054	7.9	3.5	8.082	3.95	0	47	0	0	0.896420918	0.010146525	0.093432557	0.8964	-0.1093
1000170	9/30/10	2010	3	9	30680.54	0	1	0	2.553	7.8	3.1	3.576	3.39	2	47	0	1	0.891318741	0.009461164	0.099220095	0.0992	-2.3104
1000527	3/31/04	2004	1	3	43129.89	0	0	0	3.332	4.8	1.3	17.301	4.73	0	58	0	0	0.905974514	0.006552264	0.087473223	0.906	-0.0987
1000527	6/30/04	2004	2	6	42908.69	0	0	0	2.89	4.8	1.4	19.598	5.04	0	57	0	0	0.909237607	0.007048095	0.083714298	0.9092	-0.0951
1000527	9/30/04	2004	3	9	42714.37	0	0	0	2.069	4.7	1.3	19.287	4.95	0	56	0	0	0.91520754	0.008024138	0.076768322	0.9152	-0.0886

17.1 LDP Binomial Approach

	n	3018			n	3018			
	p	0.795%			p	0.820%	0.700%		
	0	0.0000%	0.0000%		0	0.0000%	0.0000%		
	1	0.0000%	0.0000%		1	0.0000%	0.0000%		
	2	0.0000%	0.0000%		2	0.0000%	0.0000%		
	3	0.0000%	0.0000%		3	0.0000%	0.0001%		
	4	0.0000%	0.0001%		4	0.0000%	0.0005%		
	5	0.0002%	0.0003%		5	0.0001%	0.0022%		
	6	0.0010%	0.0012%		6	0.0005%	0.0080%		
	7	0.0033%	0.0045%		7	0.0019%	0.0241%		
	8	0.0099%	0.0144%		8	0.0060%	0.0640%		
	9	0.0265%	0.0409%		9	0.0165%	0.1509%		
	10	0.0639%	0.1049%		10	0.0410%	0.3201%		
	11	0.1402%	0.2450%		11	0.0927%	0.6170%		
	12	0.2816%	0.5266%		12	0.1920%	1.0899%		
	13	0.5219%	1.0485%		13	0.3671%	1.7766%		
	14	0.8979%	1.9464%		14	0.6516%	2.6882%		
	15	1.4415%	3.3879%		15	1.0788%	3.7951%		
	16	2.1687%	5.5567%		16	1.6741%	5.0211%		
	17	3.0699%	8.6266%		17	2.4441%	6.2505%		
	18	4.1028%	12.7294%		18	3.3690%	7.3461%		
	19	5.1929%	17.9223%		19	4.3981%	8.1766%		
	20	6.2419%	24.1642%		20	5.4526%	8.6431%		
	21	7.1431%	31.3074%		21	6.4358%	8.6982%		
	22	7.8003%	39.1077%		22	7.2486%	8.3530%		
	23	8.1449%	47.2526%		23	7.8066%	7.6702%		
	24	8.1476%	55.4002%		24	8.0544%	6.7474%	49.3316%	77.4429%
	25	7.8217%	63.2219%		25	7.9751%	5.6964%		
	26	7.2176%	70.4395%		26	7.5903%	4.6225%		
	27	6.4114%	76.8509%		27	6.9542%	3.6110%		
	28	5.4900%	82.3408%		28	6.1418%	2.7192%		
	29	4.5373%	86.8782%		29	5.2355%	1.9763%		
	30	3.6238%	90.5020%		30	4.3127%	1.3881%		
	31	2.7999%	93.3019%		31	3.4369%	0.9431%		



17.2 LDP Pluto Tasche Approach

Series 2 Point 54
(54, 0.0000%)

Step 1 : Pluto-Tashe method

1985 - 2019									
								0.25	
Ratings	#Obligors	#Defaults	DR(obs)%	Cum #Obligors	Cum #Defaults	p	Shock x	Beta PD	X shocked
Aaa	485	0	0%	3018	24	0.80%	27.00	0.933%	0.895%
Aa1	128	0	0%	2533	24	0.95%	27.00	1.111%	1.066%
Aa2	156	0	0%	2405	24	1.00%	27.00	1.170%	1.123%
Aa3	147	0	0%	2249	24	1.07%	27.00	1.251%	1.201%
A1	138	0	0%	2102	24	1.14%	27.00	1.339%	1.284%
A2	152	0	0%	1964	24	1.22%	27.00	1.433%	1.375%
A3	139	0	0%	1812	24	1.32%	27.00	1.553%	1.490%
Baa1	141	0	0%	1673	24	1.43%	27.00	1.682%	1.614%
Baa2	153	0	0%	1532	24	1.57%	27.00	1.836%	1.762%
Baa3	215	0	0%	1379	24	1.74%	27.00	2.039%	1.958%
Ba1	186	0	0%	1164	24	2.06%	27.00	2.415%	2.320%
Ba2	125	0	0%	978	24	2.45%	27.00	2.874%	2.761%
Ba3	142	0	0%	853	24	2.81%	27.00	3.294%	3.165%
B1	230	0	0%	711	24	3.38%	27.00	3.950%	3.797%
B2	189	0	0%	481	24	4.99%	27.00	5.830%	5.613%
B3	158	8	5%	292	24	8.22%	27.00	9.576%	9.247%
Caa1	78	5	6%	134	16	11.94%	18.00	14.407%	13.433%
Caa2	26	3	12%	56	11	19.64%	13.00	24.496%	23.214%
Caa3	21	5	24%	30	8	26.67%	10.00	34.283%	33.333%
Ca	6	2	33%	9	3	33.33%	4.00	50.199%	44.444%
C	3	1	33%	3	1	33.33%	2.00	67.365%	66.667%
Total	3018	24	0.795%					3.419%	

Confidence Interval (γ)	75%
1 - γ	25%

If the average PD of a portfolio is p, the Probability that number of defaults observed in a portfolio of n accounts will be
1-γ = BINOM.DIST(k,n,p,CUMULATIVE)

Cum Binomial is related to Cum Beta distribution
p = BETA.INV(γ, k+1, n-k)

Scaling (Optional)

We can see that the estimate of PD for Aaa is even higher than the average portfolio DR
This is not surprising as all default events are taken into the estimation of PD for the highest rating
We can scale the PDs such that the average estimate PD matches the average observed PD
For our exercise we will not do scaling at this stage to preserve the conservatism

PD(est)	PD(scaled)
0.933%	0.217%
1.111%	0.258%
1.170%	0.272%
1.251%	0.291%
1.339%	0.311%
1.433%	0.333%
1.553%	0.361%
1.682%	0.391%
1.836%	0.427%
2.039%	0.474%
2.415%	0.562%
2.874%	0.668%
3.294%	0.766%
3.950%	0.919%
5.830%	1.356%
9.576%	2.227%

Average scaled PD 0.795%

17.3 LDP Bayesian Approach

PEAKS2TAILS

Step 2 : Bayesian Approach - Beta Prior and posterior

We will keep the estimated PDs from Pluto-Tasche to retain the conservative upper bounds
Assuming these PDs come from a beta (α , β) prior , we can estimate the parameters (α , β) using Method of moments
So the idea is to use portfolio level time series data to frame our prior belief about PD and then use it on recent data (2010-2019) to frame our posterior

0.044389044
0.030648247
0.044601118
0.039458048
0.03851014
0.037483843
0.042854384
0.043756434
0.023727261
0.037276949
0.039370433
0.034611492
0.034047128
0.044640436
0.049988263
0.039384294
0.028750607
0.03703634
0.040686074
0.034746584
0.039840121
0.035180302
0.039949982
0.046913309
0.036826342
0.038626602
0.033753354
0.040684339
0.028501575
0.044983199

Mean and variance of beta (α , β) distribution

$$\bar{x} \approx \frac{\alpha}{\alpha+\beta} \qquad s^2 \approx \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\beta = \frac{\alpha(1-\bar{x})}{\bar{x}}$$

$$s^2 = \frac{\alpha\left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right]}{\left(\alpha+\left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right]\right)^2\left(\alpha+\left[\frac{\alpha(1-\bar{x})}{\bar{x}}\right]+1\right)} = \frac{\alpha^2\bar{x}^2(1-\bar{x})}{(\alpha\bar{x}+\alpha(1-\bar{x}))^2(\alpha\bar{x}+\alpha(1-\bar{x})+\bar{x})}$$

$$= \frac{\alpha^2\bar{x}^2(1-\bar{x})}{\alpha^2(\alpha+\bar{x})} = \frac{\bar{x}^2(1-\bar{x})}{\alpha+\bar{x}}$$

$$\alpha = \frac{\bar{x}^2(1-\bar{x})-s^2\bar{x}}{s^2} = \bar{x}\left[\frac{\bar{x}(1-\bar{x})}{s^2}-1\right]$$

x_bar	4.000%
s^2	0.003%

α	β
61.400	1473.600

parameters of prior beta (α , β)

recent data

2010 - 2019			
Ratings	#Obligors	#Defaults	DR(obs)%
Aaa	140	0	0%
Aa1	28	0	0%
Aa2	64	0	0%
Aa3	70	0	0%
A1	60	0	0%
A2	39	0	0%
A3	54	0	0%
Baa1	52	0	0%
Baa2	78	0	0%
Baa3	96	0	0%
Ba1	69	0	0%
Ba2	36	0	0%
Ba3	83	0	0%
B1	126	0	0%
B2	90	0	0%
B3	110	2	2%
Caa1	33	1	3%
Caa2	20	3	15%
Caa3	18	3	17%
Ca	2	2	100%

$$Prior : f(PD) = Beta\ (\alpha,\beta)$$

$$Posterior :\ f(PD|X) = Beta(\alpha + \textstyle\sum_1^n x_i, \beta + n - \sum_1^n x_i)$$

n =	1271
∑X =	12

α	β
73.400	2732.600

parameters of posterior beta (α , β)

Posterior Mean = 2.616%

Portfolio reference PD

*This is the bayesian estimate for overall PD of the LDP portfolio based on recent data where the prior belief was framed looking at the entire history by incoporating conservatism via Pluto Tasche

17.4 LDP CAP Approach

Step 3 : Cumulative Accuracy Profile (CAP)

Minimize SSE 0.01680973 Concavity(k) = 39.27142902

2010 - 2019							Observed	Function(k)		
Ratings	#Obligors	#Defaults	DR(obs)%	Cum #Obligors	Cum #Defaults	Cum %Obligors	Cum %Defaults	Cum %Defaults	xR	CAP
Aaa	140	0	0%	1271	12	1	100.00%	100.00%	94.49%	0.000%
Aa1	28	0	0%	1131	12	0.889850511	100.00%	100.00%	87.88%	0.000%
Aa2	64	0	0%	1103	12	0.867820614	100.00%	100.00%	84.26%	0.000%
Aa3	70	0	0%	1039	12	0.817466562	100.00%	100.00%	78.99%	0.000%
A1	60	0	0%	969	12	0.762391817	100.00%	100.00%	73.88%	0.000%
A2	39	0	0%	909	12	0.715184894	100.00%	100.00%	69.98%	0.000%
A3	54	0	0%	870	12	0.684500393	100.00%	100.00%	66.33%	0.000%
Baa1	52	0	0%	816	12	0.642014162	100.00%	100.00%	62.16%	0.000%
Baa2	78	0	0%	764	12	0.601101495	100.00%	100.00%	57.04%	0.000%
Baa3	96	0	0%	686	12	0.539732494	100.00%	100.00%	50.20%	0.000%
Ba1	69	0	0%	590	12	0.464201416	100.00%	100.00%	43.71%	0.000%
Ba2	36	0	0%	521	12	0.409913454	100.00%	100.00%	39.58%	0.000%
Ba3	83	0	0%	485	12	0.3815893	100.00%	100.00%	34.89%	0.000%
B1	126	0	0%	402	12	0.316286389	100.00%	100.00%	26.67%	0.001%
B2	90	0	0%	276	12	0.217151849	100.00%	99.98%	18.17%	0.029%
B3	110	2	2%	186	12	0.146341463	100.00%	99.68%	10.31%	0.648%
Caa1	33	1	3%	76	10	0.059795437	83.33%	90.45%	4.68%	5.898%
Caa2	20	3	15%	43	9	0.033831629	75.00%	73.52%	2.60%	13.375%
Caa3	18	3	17%	23	6	0.018095987	50.00%	50.87%	1.10%	24.057%
Ca	2	2	100%	5	3	0.00393391	25.00%	14.31%	0.31%	32.767%
C	3	1	33%	3	1	0.002360346	8.33%	8.85%	0.12%	35.398%

<D> 0.944%

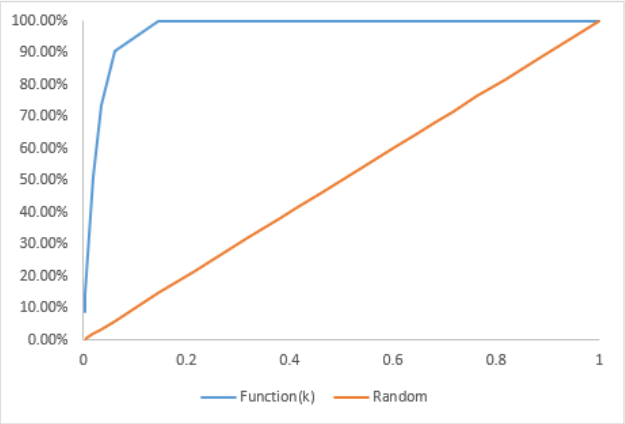
CAP curve $y(x) = \frac{1 - e^{-kx}}{1 - e^{-k}}$ PD (Rating) $PD(R) = \frac{k\langle D\rangle}{1 - e^{-k}} exp\{-kx_R\}$

with x_R denoting the cumulative percentage of obligors in rating class R, computed as

$$x_R = \frac{Z_N + Z_{N-1} + \cdots + Z_{R-1} + \left(\frac{Z_R}{2}\right)}{Z}$$

The parameter k represents the concavity of the curve and when related with the Figure 3.2 - CAP Curve can be interpreted as follows:

- When $k \to \infty$, $y \approx 1$ and the CAP curve corresponds to the perfect model with perfect discriminatory power and the area under the curve is closed to 1;
- When $k \to 0$, $y \approx x$, and the CAP curve corresponds to the random model with no discriminatory power and the area under the curve is equal to 0.5.

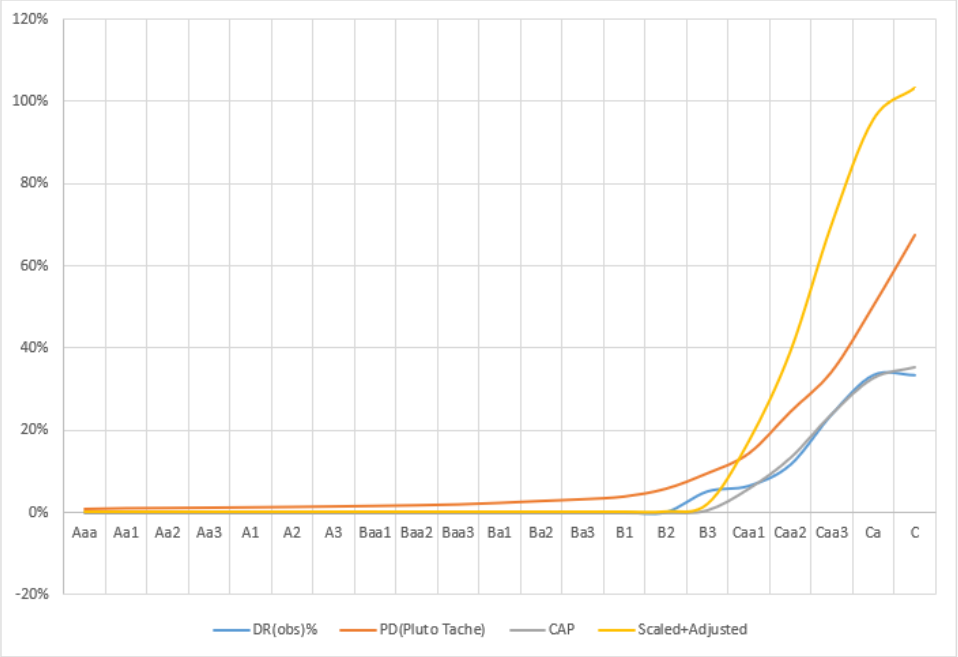


17.5 LDP Comparisons

Step 6 : Comparison of various models

Comparison

1985 - 2019						
Ratings	#Obligors	#Defaults	DR(obs)%	PD(Pluto Tache)	CAP	Scaled+Adjusted
Aaa	485	0	0%	0.933%	0.000%	0.03%
Aa1	128	0	0%	1.111%	0.000%	0.03%
Aa2	156	0	0%	1.170%	0.000%	0.03%
Aa3	147	0	0%	1.251%	0.000%	0.03%
A1	138	0	0%	1.339%	0.000%	0.03%
A2	152	0	0%	1.433%	0.000%	0.03%
A3	139	0	0%	1.553%	0.000%	0.03%
Baa1	141	0	0%	1.682%	0.000%	0.03%
Baa2	153	0	0%	1.836%	0.000%	0.03%
Baa3	215	0	0%	2.039%	0.000%	0.03%
Ba1	186	0	0%	2.415%	0.000%	0.03%
Ba2	125	0	0%	2.874%	0.000%	0.03%
Ba3	142	0	0%	3.294%	0.000%	0.03%
B1	230	0	0%	3.950%	0.001%	0.03%
B2	189	0	0%	5.830%	0.029%	0.09%
B3	158	8	5%	9.576%	0.648%	1.89%
Caa1	78	5	6%	14.407%	5.898%	17.19%
Caa2	26	3	12%	24.496%	13.375%	38.98%
Caa3	21	5	24%	34.283%	24.057%	70.11%
Ca	6	2	33%	50.199%	32.767%	95.49%
C	3	1	33%	67.365%	35.398%	103.15%



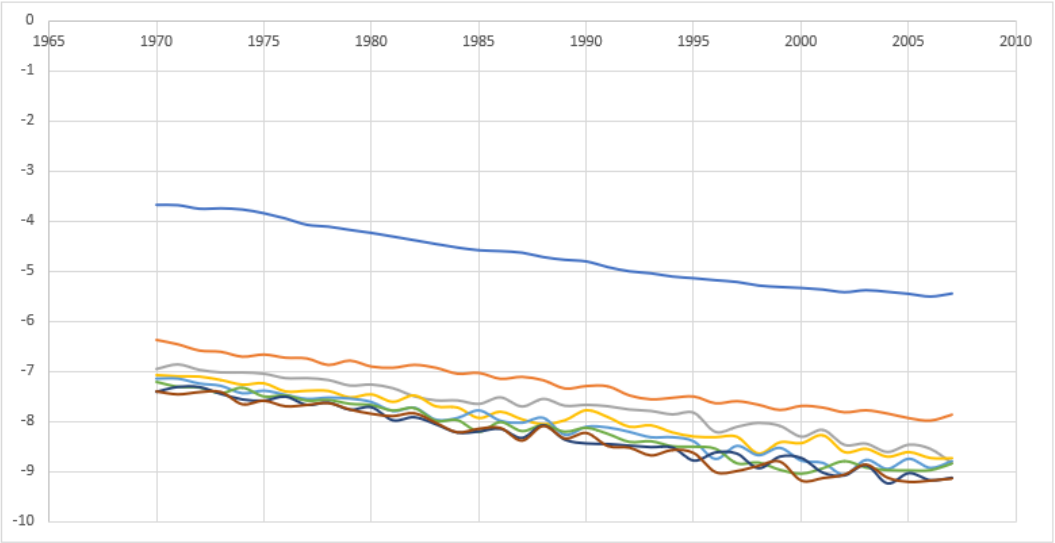
Observation

Pluto Tasche is the most conservative as expected

18.1 Actuarial Models- Lee Carter

PEAKS2TAILS

The Model



$$\ln m(x,t) = a(x) + b(x)k(t) + \varepsilon(x,t)$$

$$\sum_{x=1}^{x=\omega} b(x) = 1$$

$$\sum_{t=1}^{t=n} k(t) = 0$$

Lee Carter constraints

t = 1 to 38 n = 38
x = 0 to 95 ω = 96

t →

x ↓

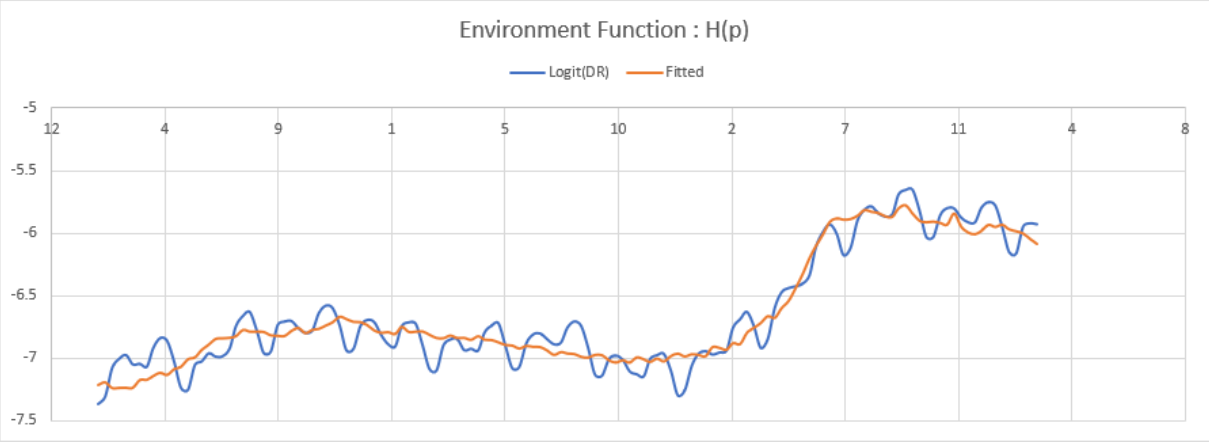
Age/Year	1970	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
0	-3.67796	-3.68306	-3.75388	-3.74546	-3.76931	-3.84339	-3.94663	-4.07268	-4.10943	-4.17449	-4.23406	-4.30612	-4.37973	-4.45451	-4.52448	-4.57825	-4.59577	-4.62606	-4.71446	-4.76864	-4.80079	-4.91376	-4.9963	-5.03592	-5.10081	-5.13369	-5.17327
1	-6.3762	-6.46447	-6.58857	-6.6137	-6.71113	-6.66763	-6.73134	-6.74617	-6.87607	-6.78914	-6.90797	-6.93134	-6.87385	-6.93206	-7.05032	-7.03455	-7.15187	-7.11308	-7.18154	-7.34529	-7.29585	-7.30195	-7.48365	-7.56062	-7.53074	-7.50536	-7.63935
2	-6.9654	-6.87354	-6.98231	-7.03257	-7.03413	-7.0589	-7.14621	-7.14671	-7.18426	-7.29407	-7.27418	-7.34766	-7.50878	-7.5899	-7.59039	-7.66076	-7.53012	-7.71113	-7.56001	-7.69702	-7.67841	-7.70678	-7.77136	-7.80069	-7.86893	-7.83714	-8.21745
3	-7.06204	-7.0857	-7.09195	-7.16015	-7.25325	-7.22589	-7.38878	-7.37609	-7.37953	-7.50821	-7.44819	-7.59653	-7.46963	-7.68771	-7.71094	-7.92032	-7.7952	-7.9469	-8.04536	-7.96606	-7.7645	-7.90355	-8.09657	-8.06717	-8.2094	-8.29246	-8.30435
4	-7.15094	-7.15139	-7.24878	-7.29464	-7.44263	-7.39095	-7.4733	-7.55322	-7.52817	-7.54747	-7.61618	-7.7952	-7.7295	-7.96754	-7.93241	-7.78345	-7.98828	-8.02764	-7.93153	-8.26616	-8.10681	-8.12177	-8.20936	-8.31573	-8.31475	-8.39926	-8.74656
5	-7.20993	-7.30834	-7.32319	-7.43565	-7.32724	-7.5028	-7.48162	-7.58533	-7.57212	-7.64906	-7.67557	-7.78451	-7.73533	-7.98995	-7.97652	-8.20819	-8.01512	-8.19548	-8.07279	-8.20822	-8.12935	-8.25259	-8.41243	-8.39921	-8.49968	-8.50855	-8.54946
6	-7.4044	-7.30569	-7.30874	-7.44463	-7.55728	-7.58183	-7.5009	-7.66701	-7.62093	-7.76299	-7.71402	-7.97611	-7.91368	-8.0565	-8.22313	-8.19974	-8.14739	-8.33088	-8.07566	-8.36568	-8.44092	-8.4529	-8.48555	-8.5151	-8.52274	-8.78632	-8.62271
7	-7.4086	-7.46706	-7.42306	-7.42073	-7.66267	-7.5926	-7.70259	-7.6754	-7.63905	-7.76628	-7.85057	-7.89564	-7.83965	-8.02918	-8.216	-8.14534	-8.12986	-8.38073	-8.09043	-8.33747	-8.22966	-8.48822	-8.51984	-8.67223	-8.56463	-8.62722	-9.00115
8	-7.53604	-7.49923	-7.50393	-7.68945	-7.76544	-7.72405	-7.77843	-7.86322	-7.78846	-7.79987	-7.92132	-7.92979	-7.9714	-8.21518	-8.15613	-8.14938	-8.25487	-8.4414	-8.36277	-8.1968	-8.26911	-8.34354	-8.58132	-8.60241	-8.9089	-8.80911	-9.03917
9	-7.58872	-7.6899	-7.60501	-7.71451	-7.82374	-7.89104	-7.95765	-7.90047	-7.91413	-7.97495	-7.97618	-8.06792	-8.29231	-7.95373	-8.40634	-8.45189	-8.4893	-8.30813	-8.32981	-8.46995	-8.60923	-8.67418	-8.68772	-8.59311	-8.63461	-8.96615	-8.83009
10	-7.70902	-7.67866	-7.67963	-7.85297	-7.94127	-7.92565	-8.0801	-7.994	-7.95026	-8.12081	-8.02526	-8.20616	-8.32238	-8.40732	-8.33239	-8.56467	-8.2672	-8.33264	-8.66762	-8.49397	-8.47645	-8.68845	-8.69909	-8.81054	-8.75682	-8.86574	-8.93966
11	-7.73591	-7.88558	-7.79239	-7.8481	-7.91948	-7.87487	-7.95448	-7.98412	-7.99582	-8.09518	-8.18657	-8.00098	-8.21002	-8.3375	-8.38162	-8.54965	-8.38197	-8.42726	-8.51083	-8.4504	-8.3565	-8.53087	-8.68691	-8.5193	-8.58301	-8.77905	-8.74175
12	-7.59681	-7.6362	-7.65841	-7.75936	-7.94477	-7.85303	-7.81088	-7.98943	-7.96396	-8.05571	-8.11753	-8.21074	-8.33479	-8.12127	-8.44225	-8.32892	-8.37353	-8.51698	-8.38398	-8.50987	-8.60103	-8.38921	-8.58948	-8.51991	-8.62918	-8.85528	-8.69095

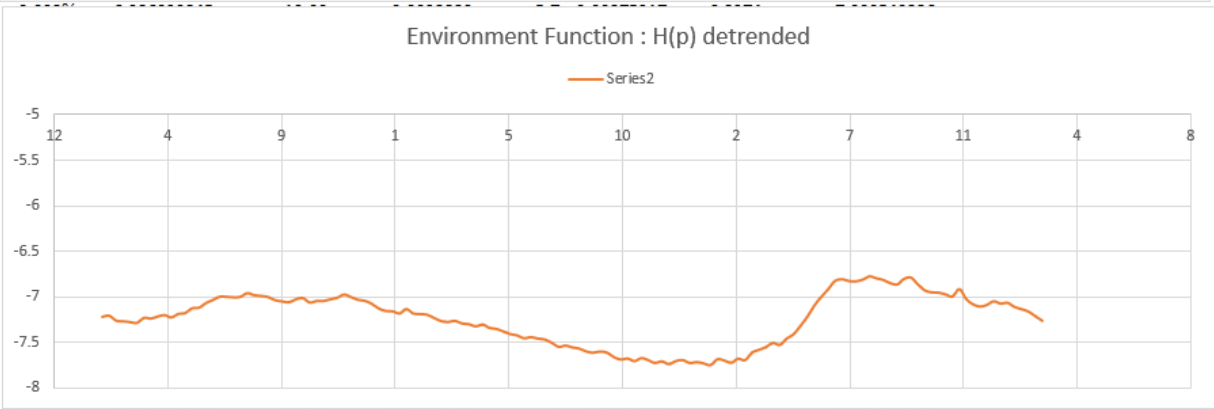
18.2 Actuarial Models- APC Analysis

α	β1	β2	β3	β4
-8.071991301	0.00157312	2.59369174	0.209754	0.22076184

intercept	-7.24133087
slope	0.00858723

Period(p)	DR	logit(DR) : H(p)	t	HPI_rate	urate	GDP_rate	fitted :H(p)	detrended : H(p)
7/01/00	0.063%	-7.36932476	1.00	0.00826856	4	0.00033247	-7.2099	-7.218470812
8/01/00								
9/01/00								
10/01/00								
11/01/00								
12/01/00								
1/01/01								
2/01/01								
3/01/01								
4/01/01								
5/01/01								
6/01/01								
7/01/01								
8/01/01								
9/01/01								
10/01/01								
11/01/01								
12/01/01								
1/01/02								
2/01/02								
3/01/02								
4/01/02								
5/01/02								
6/01/02								
7/01/02								
8/01/02								
9/01/02								
10/01/02								
11/01/02								
12/01/02								
1/01/03								
2/01/03								
3/01/03								
4/01/03								
5/01/03	0.136%	-6.600900381	35.00	0.01035344	6.1	0.0029691	-6.7099	-7.010477972
6/01/03	0.118%	-6.739448567	36.00	0.01051282	6.3	0.0029691	-6.6660	-6.975127943
7/01/03	0.097%	-6.939841163	37.00	0.01067367	6.2	0.00552573	-6.6844	-7.002135828
8/01/03	0.098%	-6.927595843	38.00	0.00978856	6.1	0.00552573	-6.7061	-7.032421028





In practice, we use splines to approximate the function
Also feature selection involves lags of macroeconomic variables and various transformations
Fit is not great but R^2 is high and will serve fine as an illustration

Over a full economic cycle, we assume no trend for environment function
We detrend the environment function so as to eliminate 'identification problem'
This is same as applying the constraint that linear trend for period effect = 0

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.956306597
R Square	0.914522308
Adjusted R Squar	0.911932075
Standard Error	0.136708047
Observations	137

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	26.39390224	6.598476	353.0656	1.95056E-69
Residual	132	2.46695989	0.018689		
Total	136	28.86086213			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-8.071991301	0.043083774	-187.356	5E-162	-8.157215265	-7.98677
X Variable 1	0.00157312	0.000523259	3.006389	0.003166	0.000538062	0.002608
X Variable 2	2.593691735	1.96363252	1.320864	0.188832	-1.29056748	6.477951
X Variable 3	0.209753775	0.009159601	22.89988	8.19E-48	0.191635178	0.227872
X Variable 4	0.220761843	6.275653114	0.035178	0.971991	-12.19310031	12.63462

18.3 Actuarial Models- Kaplan Meir

PEAKS2TAILS

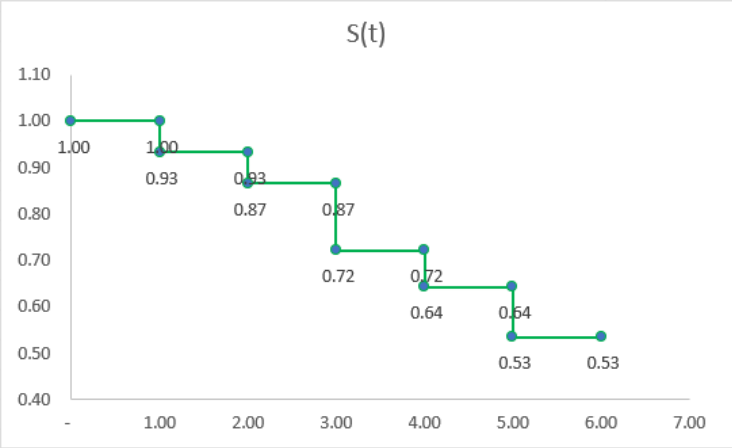
Kaplan-Meier Estimate of Survival Function

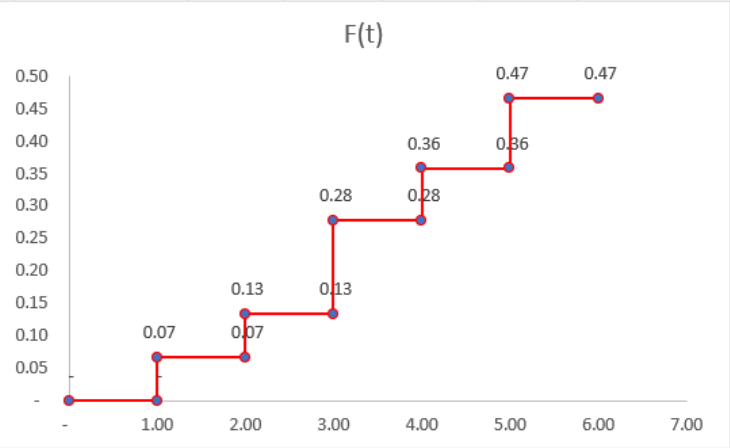
Total Obligors	15
Deaths	6
Censored	9

Month on Books	Event	Event Type
1	loan #4 defaults	D
2	loan #13 defaults	D
2	loan #7 matures	C
3	loan #6 and #9 default	2D
3	loan #1 has no reported data beyond this due to data entry error	C
4	loan #10 defaults	D
4	loan #8 prepays the entire amount	C
4	loan #12 matures	C
5	loan #5 defaults	D
6	Close of Investigation. Remaining loans are current.	5C

j	t	Survivors beg	defaulted	censored	Survivors end	λ	1 - λ	S
1	1	15	1	0	14	6.67%	0.9333	0.9333
2	2	14	1	1	12	7.14%	0.9286	0.8667
3	3	12	2	1	9	16.67%	0.8333	0.7222
4	4	9	1	2	6	11.11%	0.8889	0.6420
5	5	6	1		5	16.67%	0.8333	0.5350

time	S(t)	F(t)
[0, 1)	1	0
[1, 2)	0.9333	0.0667
[2, 3)	0.8667	0.1333333
[3, 4)	0.7222	0.2777778
[4, 5)	0.6420	0.3580247
[5, 6)	0.5350	0.4650206





0	0
3	0.0667
4	0.0714
11	0.1667
21	0.1111
26	0.1667

-	1.00
1.00	1.00
1.00	0.93
2.00	0.93
2.00	0.87
3.00	0.87
3.00	0.72
4.00	0.72
4.00	0.64
5.00	0.64
5.00	0.53

2.00	0.13
3.00	0.13
3.00	0.28
4.00	0.28
4.00	0.36
5.00	0.36
5.00	0.47

18.4 Actuarial Models- Cox Regression

PEAKS2TAILS

							betas																
							0.95354	-0.4173	0.79091		Breslow's formulation												Maximize
ID	time	last status_time	LTV_orig	FICO_orig	LTV	FICO1	FICO2	exp(βx)	Month	Survivors_before	Defaults	censored	Survivors_after	ΣLTV	ΣFICO1	ΣFICO2	Factor_Numerator	Factor_Denominator	Factor	log (Factor)	ln (Partial Likelihood)
1	24	1	69.4	715	0	0	1	2.20539223	0	46207	0	0	46207	0	0	0	1	114255.4544	8.75232E-06	-11.646192	9380.41094
2	2	2	80	558	1	0	0	2.594892	1	46207	0	0	46207	0	0	0	1	114255.4544	8.75232E-06	-11.646192	
3	5	2	83.6	680	1	1	0	1.7096004	2	46207	896	3137	42174	724	452	110	4.9233E+255	114255.4544	4.309E+250	577.1069777	
4	35	0	81.8	587	1	0	0	2.594892	3	42174	997	2725	38452	804	546	115	3.1923E+273	104961.1512	3.0414E+268	618.2051307	
5	3	2	80	527	1	0	0	2.594892	4	38452	964	2568	34920	775	503	148	4.1912E+280	96440.07565	4.3459E+275	634.6801395	
6	31	1	80	670	1	1	0	1.7096004	5	34920	999	1979	31942	805	577	173	1.6689E+288	88059.76273	1.8952E+283	652.2709195	
7	2	2	95	601	1	1	0	1.7096004	6	31942	954	1905	29083	752	499	208	2.7018E+292	80928.45227	3.3384E+287	662.0474266	
9	13	1	80	501	1	0	0	2.594892	7	29083	946	2066	26071	756	577	183	2.3198E+271	73972.15941	3.1361E+266	613.6306002	
10	5	1	80	665	1	1	0	1.7096004	8	26071	981	1555	23535	770	535	250	6.1392E+307	66809.70437	9.189E+302	697.5987091	
11	3	2	95	564	1	0	0	2.594892	9	23535	895	2241	20399	706	514	223	6.5475E+275	60539.33927	1.0815E+271	624.0789355	
12	3	2	80	583	1	0	0	2.594892	10	20399	765	680	18954	563	409	204	1.2571E+229	53245.25416	2.361E+224	516.6381476	
13	3	2	80	635	1	1	0	1.7096004	11	18954	703	662	17589	533	410	176	7.533E+206	49692.95441	1.5159E+202	465.5382106	
14	3	2	80	506	1	0	0	2.594892	12	17589	599	586	16404	439	321	168	2.1422E+181	46267.44853	4.6301E+176	406.7875578	
15	4	2	95	561	1	0	0	2.594892	13	16404	555	427	15422	396	294	160	4.6657E+165	43233.45387	1.0792E+161	370.7924097	
16	10	1	70	572	0	0	0	1	14	15422	488	387	14547	349	266	136	1.0863E+143	40710.32836	2.6683E+138	318.7381899	
17	3	2	80	608	1	1	0	1.7096004	15	14547	430	269	13848	303	231	153	1.4743E+136	38474.20505	3.8318E+131	302.9819931	
18	10	1	95	571	1	0	0	2.594892	16	13848	389	296	13163	263	180	129	4.005E+120	36565.94689	1.0953E+116	267.1908859	
19	4	1	80	630	1	1	0	1.7096004	17	13163	336	266	12561	225	187	96	1.83006E+92	34654.67347	5.28085E+87	201.9889906	
20	3	2	85	647	1	1	0	1.7096004	18	12561	316	281	11964	219	138	129	9.8354E+109	33097.19441	2.9717E+105	242.8605592	
21	4	2	79.8	614	0	1	0	0.65883297	19	11964	293	257	11414	183	153	89	4.23344E+78	31456.20745	1.34582E+74	170.6883016	
22	5	1	75	582	0	0	0	1	20	11414	273	254	10887	184	149	80	4.72432E+76	30029.956	1.5732E+72	166.23924	
24	8	1	85	615	1	1	0	1.7096004	21	10887	256	256	10375	165	124	88	1.215E+76	28709.57534	4.23204E+71	164.9262256	
25	4	1	80	557	1	0	0	2.594892	22	10375	209	184	9982	144	117	55	2.09706E+57	27367.29714	7.66266E+52	121.770784	
26	33	2	75	630	0	1	0	0.65883297	23	9982	194	183	9605	132	97	54	4.29722E+55	26304.82543	1.63362E+51	117.922641	
27	3	2	80	641	1	1	0	1.7096004	24	9605	172	159	9274	106	80	63	1.09214E+51	25299.51275	4.31684E+46	107.3814383	
28	35	0	85	575	1	0	0	2.594892	25	9274	150	195	8929	108	77	45	1.68828E+46	24439.36549	6.90806E+41	96.33867704	
29	9	1	80	696	1	1	0	1.7096004	26	8929	120	205	8604	77	73	29	4.15789E+28	23503.03336	1.76909E+24	55.8325059	
30	4	2	80	633	1	1	0	1.7096004	27	8604	109	216	8279	70	60	26	1.11046E+27	22654.15792	4.90178E+22	52.24646965	
31	35	0	95	639	1	1	0	1.7096004	28	8279	84	176	8019	56	37	23	2.42964E+24	21852.82502	1.11182E+20	46.1576986	
33	6	2	75	713	0	0	1	2.20539223	29	8019	82	259	7678	56	44	25	6.36708E+23	21110.92873	3.01601E+19	44.85305204	
34	2	2	80	543	1	0	0	2.594892	30	7678	84	426	7168	54	48	23	3.66295E+21	20201.83332	1.81318E+17	39.73902806	
35	6	2	80	584	1	0	0	2.594892	31	7168	73	834	6261	56	40	22	3.15051E+23	18870.00492	1.66959E+19	44.2616941	
36	3	2	72.2	509	0	0	0	1	32	6261	51	634	5576	34	28	13	2.95856E+13	16557.74778	1786814614	21.30370033	
37	3	2	80	656	1	1	0	1.7096004	33	5576	37	854	4685	28	16	16	1.55424E+14	14816.71277	10489790028	23.07366824	
38	3	2	80	586	1	0	0	2.594892	34	4685	22	650	4013	19	12	2	2401828.684	12580.78284	190.9124983	5.251815199	
39	3	2	90	563	1	0	0	2.594892	35	4013	20	491	3502	12	14	4	6400.727629	10835.1267	0.590738605	-0.52638165	
40	3	2	90	588	1	0	0	2.594892	36	3502	23	744	2735	18	9	7	168859679.9	9465.330829	17839.8075	9.789187616	
42	4	2	76.2	584	0	0	0	1	37	2735	13	581	2141	9	8	4	4479.39682	7369.44814	0.607833414	-0.49785443	
43	3	2	69	702	0	0	1	2.20539223	38	2141	13	405	1723	8	2	6	102664.4252	5696.318678	18.02294271	2.891645541	
44	3	2	80	505	1	0	0	2.594892	39	1723	15	410	1298	6	6	5	1302.557387	4460.171066	0.292042024	-1.23085757	
45	5	1	80	618	1	1	0	1.7096004	40	1298	4	286	1008	2	2	0	2.92273354	3310.520873	0.000882862	-7.0323415	
46	5	1	80	581	1	0	0	2.594892	41	1008	4	327	677	2	2	0	2.92273354	2550.232041	0.001146066	-6.77142031	
47	3	2	80	600	1	1	0	1.7096004	42	677	3	198	476	2	1	1	9.783623641	1627.230645	0.006012438	-5.11392493	
48	2	2	80	636	1	1	0	1.7096004	43	476	1	208	267	0	1	0	0.658832971	1077.795302	0.000611278	-7.39995808	

18.5 Actuarial Models- AFT

$$\log L(\alpha,\mu,\sigma) = \sum_{i=1}^{n}\{-\delta_i \log(\sigma t_i) + \delta_i \log f_{\varepsilon_i}(z_i) + (1-\delta_i)\log S_{\varepsilon_i}(z_i))\}$$

Assumption ε is Normal

α	σ	β1	β2
17.23403373	5	-0.06599947	-0.0091752

Sum of LL -133.524

ID	time	LTV_orig	FICO_orig	last status_time	e	PDe	Se	LL
1	48	69.4	715	1	-2.222184066	0.001506	0.671637	-6.49834
4	60	81.8	587	0	-2.35507581	0.00119	0.681185	-0.38392
6	56	80	670	1	-1.781324103	0.001337	0.639179	-6.61719
9	37	80	501	0	-3.746370803	0.001629	0.773154	-0.25728
10	29	80	665	1	-2.485256085	0.002432	0.690424	-6.0192
16	34	70	572	0	-3.839481959	0.001747	0.778726	-0.2501
18	34	95	571	0	-2.198670435	0.00213	0.669935	-0.40057
19	28	80	630	0	-2.841480259	0.002425	0.715083	-0.33536
22	29	75	582	1	-3.57679706	0.00213	0.762807	-6.15154
24	32	85	615	1	-2.515579883	0.002197	0.692559	-6.12068
25	28	80	557	0	-3.51127164	0.002227	0.75874	-0.2761
28	60	85	575	0	-2.253980199	0.001201	0.673932	-0.39463
29	33	80	696	1	-2.071612398	0.002219	0.660681	-6.11072
31	60	95	639	0	-1.006771139	0.001303	0.579789	-0.54509
45	29	80	618	1	-2.916491632	0.002321	0.720154	-6.06579
46	29	80	581	1	-3.255974934	0.002226	0.74254	-6.1077
49	29	95	573	1	-2.339384681	0.002466	0.680064	-6.00513
56	60	52.5	664	0	-3.582368	0.001029	0.763151	-0.2703
57	60	54.6	669	0	-3.397892992	0.001056	0.751614	-0.28553
58	60	56.9	618	0	-3.714030655	0.001009	0.7712	-0.25981
61	60	64.3	652	0	-2.913676948	0.001122	0.719964	-0.32855
65	60	80	606	0	-2.299545592	0.001196	0.677209	-0.38977
66	60	70	514	0	-3.803660936	0.000996	0.776591	-0.25284
69	60	75	540	0	-3.235107752	0.001079	0.741191	-0.2995
70	40	80	594	1	-2.815113393	0.001702	0.71329	-6.37575
71	38	80	553	0	-3.242590888	0.001701	0.741675	-0.29884
72	37	69.6	536	1	-4.111632437	0.001538	0.794554	-6.4774
74	43	80	626	0	-2.449185551	0.001646	0.687875	-0.37415
76	36	80	634	1	-2.553464933	0.001945	0.695217	-6.2423
77	35	80	654	1	-2.398131322	0.002032	0.684253	-6.19875
80	60	80	544	0	-2.868409505	0.001128	0.716908	-0.33281
82	52	60.6	552	1	-4.21849827	0.001075	0.800581	-6.83553
83	40	80	598	1	-2.778412496	0.001709	0.710786	-6.37165
84	43	80	660	0	-2.137227922	0.001694	0.665473	-0.40726

19.1 CCAR Variable Transformation

PEAKS2TAILS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
1	observation_date	INF	UR	HPI_g	INCOME_g	GDP	INF_lag0	UR_lag0	HPI_lag0	INC_lag0	GDP_lag0	INF_lag1	UR_lag1	HPI_lag1	INC_lag1	GDP_lag1	INF_lag4	UR_lag4	HPI_lag4	INC_lag4	GDP_lag4	QQ4MA_INF_lead	QQ4MA_UR_lead	QQ4MA_HPI_lead	QQ4MA_INC_lead	QQ4MA_GDP_lead	QQ4MA_INF_lag	QQ4MA_UR_lag	QQ4MA_HPI_lag	QQ4MA_INC_lag
2	1/01/99	0.015	4.300	0.064	0.043	0.050																								
3	4/01/99	0.030	4.267	0.071	0.005	0.048																								
4	7/01/99	0.030	4.233	0.077	0.030	0.067																								
5	10/01/99	0.029	4.067	0.079	0.053	0.087																								
6	1/01/00	0.039	4.033	0.086	0.069	0.041	0.039	4.033	0.086	0.069	0.041	0.029	4.067	0.079	0.053	0.087	0.015	4.300	0.064	0.043	0.050	0.034	3.967	0.088	0.048	0.053	0.032	4.150	0.078	0.039
7	4/01/00	0.031	3.933	0.091	0.047	0.097	0.031	3.933	0.091	0.047	0.097	0.039	4.033	0.086	0.069	0.041	0.030	4.267	0.071	0.005	0.048	0.034	4.017	0.086	0.039	0.046	0.032	4.067	0.084	0.050
8	7/01/00	0.036	4.000	0.081	0.051	0.027	0.036	4.000	0.081	0.051	0.027	0.031	3.933	0.091	0.047	0.097	0.030	4.233	0.077	0.030	0.067	0.033	4.133	0.079	0.025	0.034	0.034	4.008	0.084	0.055
9	10/01/00	0.028	3.900	0.093	0.025	0.045	0.028	3.900	0.093	0.025	0.045	0.036	4.000	0.081	0.051	0.027	0.029	4.067	0.079	0.053	0.087	0.026	4.342	0.076	0.033	0.027	0.034	3.967	0.088	0.048
10	1/01/01	0.038	4.233	0.081	0.032	0.013	0.038	4.233	0.081	0.032	0.013	0.028	3.900	0.093	0.025	0.045	0.039	4.033	0.086	0.069	0.041	0.019	4.742	0.068	0.010	0.021	0.034	4.017	0.086	0.039
11	4/01/01	0.028	4.400	0.062	-0.010	0.049	0.028	4.400	0.062	-0.010	0.049	0.038	4.233	0.081	0.032	0.013	0.031	3.933	0.091	0.047	0.097	0.012	5.108	0.065	0.025	0.029	0.033	4.133	0.079	0.025
12	7/01/01	0.011	4.833	0.069	0.086	0.000	0.011	4.833	0.069	0.086	0.000	0.028	4.400	0.062	-0.010	0.049	0.036	4.000	0.081	0.051	0.027	0.013	5.467	0.073	0.036	0.027	0.026	4.342	0.076	0.033
13	10/01/01	-0.003	5.500	0.060	-0.067	0.023	-0.003	5.500	0.060	-0.067	0.023	0.011	4.833	0.069	0.086	0.000	0.028	3.900	0.093	0.025	0.045	0.016	5.692	0.081	0.016	0.036	0.019	4.742	0.068	0.010
14	1/01/02	0.013	5.700	0.069	0.093	0.046	0.013	5.700	0.069	0.093	0.046	-0.003	5.500	0.060	-0.067	0.023	0.038	4.233	0.081	0.032	0.013	0.022	5.783	0.089	0.039	0.037	0.012	5.108	0.065	0.025
15	4/01/02	0.031	5.833	0.095	0.032	0.038	0.031	5.833	0.095	0.032	0.038	0.013	5.700	0.069	0.093	0.046	0.028	4.400	0.062	-0.010	0.049	0.029	5.825	0.093	0.015	0.036	0.013	5.467	0.073	0.036
16	7/01/02	0.021	5.733	0.102	0.006	0.035	0.021	5.733	0.102	0.006	0.035	0.031	5.833	0.095	0.032	0.038	0.011	4.833	0.069	0.086	0.000	0.020	5.900	0.088	0.020	0.038	0.016	5.692	0.081	0.016
17	10/01/02	0.024	5.867	0.091	0.025	0.028	0.024	5.867	0.091	0.025	0.028	0.021	5.733	0.102	0.006	0.035	-0.003	5.500	0.060	-0.067	0.023	0.022	6.000	0.086	0.036	0.052	0.022	5.783	0.089	0.039
18	1/01/03	0.041	5.867	0.083	-0.002	0.041	0.041	5.867	0.083	-0.002	0.041	0.024	5.867	0.091	0.025	0.028	0.013	5.700	0.069	0.093	0.046	0.020	5.992	0.092	0.032	0.062	0.029	5.825	0.093	0.015
19	4/01/03	-0.007	6.133	0.075	0.050	0.049	-0.007	6.133	0.075	0.050	0.049	0.041	5.867	0.083	-0.002	0.041	0.031	5.833	0.095	0.032	0.038	0.018	5.950	0.103	0.037	0.065	0.020	5.900	0.088	0.020
20	7/01/03	0.030	6.133	0.098	0.069	0.089	0.030	6.133	0.098	0.069	0.089	-0.007	6.133	0.075	0.050	0.049	0.021	5.733	0.102	0.006	0.035	0.027	5.817	0.118	0.035	0.068	0.022	6.000	0.086	0.036
21	10/01/03	0.015	5.833	0.112	0.011	0.070	0.015	5.833	0.112	0.011	0.070	0.030	6.133	0.098	0.069	0.089	0.024	5.867	0.091	0.025	0.028	0.026	5.642	0.123	0.025	0.062	0.020	5.992	0.092	0.032
22	1/01/04	0.034	5.700	0.127	0.018	0.051	0.034	5.700	0.127	0.018	0.051	0.015	5.833	0.112	0.011	0.070	0.041	5.867	0.083	-0.002	0.041	0.033	5.542	0.126	0.034	0.062	0.018	5.950	0.103	0.037
23	4/01/04	0.031	5.600	0.135	0.041	0.063	0.031	5.600	0.135	0.041	0.063	0.034	5.700	0.127	0.018	0.051	-0.007	6.133	0.075	0.050	0.049	0.030	5.442	0.132	0.018	0.068	0.027	5.817	0.118	0.035
24	7/01/04	0.025	5.433	0.120	0.029	0.063	0.025	5.433	0.120	0.029	0.063	0.031	5.600	0.135	0.041	0.063	0.030	6.133	0.098	0.069	0.089	0.029	5.317	0.133	0.017	0.065	0.026	5.642	0.123	0.025
25	10/01/04	0.043	5.433	0.124	0.050	0.071	0.043	5.433	0.124	0.050	0.071	0.025	5.433	0.120	0.029	0.063	0.015	5.833	0.112	0.011	0.070	0.037	5.200	0.134	0.014	0.066	0.033	5.542	0.126	0.034
26	1/01/05	0.020	5.300	0.148	-0.049	0.076	0.020	5.300	0.148	-0.049	0.076	0.043	5.433	0.124	0.050	0.071	0.034	5.700	0.127	0.018	0.051	0.036	5.083	0.131	0.010	0.062	0.030	5.442	0.132	0.018
27	4/01/05	0.027	5.100	0.141	0.039	0.048	0.027	5.100	0.141	0.039	0.048	0.020	5.300	0.148	-0.049	0.076	0.031	5.600	0.135	0.041	0.063	0.036	4.942	0.115	0.042	0.063	0.029	5.317	0.133	0.017
28	7/01/05	0.060	4.967	0.122	0.017	0.068	0.060	4.967	0.122	0.017	0.068	0.027	5.100	0.141	0.039	0.048	0.025	5.433	0.120	0.029	0.063	0.038	4.825	0.083	0.036	0.062	0.037	5.200	0.134	0.014
29	10/01/05	0.037	4.967	0.112	0.033	0.055	0.037	4.967	0.112	0.033	0.055	0.060	4.967	0.122	0.017	0.068	0.043	5.433	0.124	0.050	0.071	0.033	4.742	0.046	0.034	0.054	0.036	5.083	0.131	0.010
30	1/01/06	0.021	4.733	0.083	0.080	0.082	0.021	4.733	0.083	0.080	0.082	0.037	4.967	0.112	0.033	0.055	0.020	5.300	0.148	-0.049	0.076	0.019	4.608	0.023	0.038	0.052	0.036	4.942	0.115	0.042
31	4/01/06	0.036	4.633	0.015	0.015	0.045	0.036	4.633	0.015	0.015	0.045	0.021	4.733	0.083	0.080	0.082	0.027	5.100	0.141	0.039	0.048	0.024	4.550	0.006	0.026	0.044	0.038	4.825	0.083	0.036
32	7/01/06	0.038	4.633	-0.026	0.008	0.034	0.038	4.633	-0.026	0.008	0.034	0.036	4.633	0.015	0.015	0.045	0.060	4.967	0.122	0.017	0.068	0.026	4.517	-0.013	0.027	0.046	0.033	4.742	0.046	0.034
33	10/01/06	-0.016	4.433	0.018	0.051	0.049	-0.016	4.433	0.018	0.051	0.049	0.038	4.633	-0.026	0.008	0.034	0.037	4.967	0.112	0.033	0.055	0.023	4.525	-0.025	0.026	0.049	0.019	4.608	0.023	0.038
34	1/01/07	0.039	4.500	0.017	0.030	0.050	0.039	4.500	0.017	0.030	0.050	-0.016	4.433	0.018	0.051	0.049	0.021	4.733	0.083	0.080	0.082	0.040	4.617	-0.046	0.015	0.047	0.024	4.550	0.006	0.026
35	4/01/07	0.045	4.500	-0.061	0.018	0.052	0.045	4.500	-0.061	0.018	0.052	0.039	4.500	0.017	0.030	0.050	0.036	4.633	0.015	0.015	0.045	0.041	4.742	-0.072	0.010	0.034	0.026	4.517	-0.013	0.027
36	7/01/07	0.025	4.667	-0.075	0.007	0.045	0.025	4.667	-0.075	0.007	0.045	0.045	4.500	-0.061	0.018	0.052	0.038	4.633	-0.026	0.008	0.034	0.042	4.950	-0.086	0.024	0.032	0.023	4.525	-0.025	0.026
37	10/01/07	0.049	4.800	-0.065	0.006	0.041	0.049	4.800	-0.065	0.006	0.041	0.025	4.667	-0.075	0.007	0.045	-0.016	4.433	0.018	0.051	0.049	0.051	5.283	-0.096	0.002	0.023	0.040	4.617	-0.046	0.015
38	1/01/08	0.043	5.000	-0.087	0.007	-0.002	0.043	5.000	-0.087	0.007	-0.002	0.049	4.800	-0.065	0.006	0.041	0.039	4.500	0.017	0.030	0.050	0.016	5.800	-0.117	0.011	-0.007	0.041	4.742	-0.072	0.010
39	4/01/08	0.052	5.333	-0.116	0.077	0.043	0.052	5.333	-0.116	0.077	0.043	0.043	5.000	-0.087	0.007	-0.002	0.045	4.500	-0.061	0.018	0.052	-0.002	6.617	-0.134	0.008	-0.019	0.042	4.950	-0.086	0.024
40	7/01/08	0.061	6.000	-0.115	-0.081	0.009	0.061	6.000	-0.115	-0.081	0.009	0.052	5.333	-0.116	0.077	0.043	0.025	4.667	-0.075	0.007	0.045	-0.009	7.608	-0.119	-0.006	-0.033	0.051	5.283	-0.096	0.002
41	10/01/08	-0.093	6.867	-0.149	0.043	-0.079	-0.093	6.867	-0.149	0.043	-0.079	0.061	6.000	-0.115	-0.081	0.009	0.049	4.800	-0.065	0.006	0.041	-0.016	8.517	-0.089	0.002	-0.031	0.016	5.800	-0.117	0.011
42	1/01/09	-0.028	8.267	-0.156	-0.009	-0.049	-0.028	8.267	-0.156	-0.009	-0.049	-0.093	6.867	-0.149	0.043	-0.079	0.043	5.000	-0.087	0.007	-0.002	0.015	9.283	-0.053	-0.007	0.003	-0.002	6.617	-0.134	0.008

19.2 CCAR Variable Selection

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1	CORR TEST	INF_lag0	UR_lag0	HPI_lag0	INC_lag0	GDP_lag0	INF_lag1	UR_lag1	HPI_lag1	INC_lag1	GDP_lag1	INF_lag4	UR_lag4	HPI_lag4	INC_lag4	GDP_lag4	QQ4MA_INF lead	QQ4MA_UR lead	QQ4MA_HPI lead	QQ4MA_INC lead	QQ4MA_GD P lead	QQ4MA_INF lag	QQ4MA_UR lag
2		-0.22721497	0.872478518	-0.41017942	-0.1212038	-0.27152237	-0.20083878	0.886043429	-0.48216771	-0.15414491	-0.31881251	-0.19455073	0.836888891	-0.70999982	-0.16571884	-0.44675217	-0.36160073	0.828827776	-0.31567752	-0.20993942	-0.25716746	-0.37341959	0.893747271
3																							
4																							
5	FINAL IV	UR_lag0	HPI_lag0	GDP_lag0	UR_lag1	HPI_lag1	GDP_lag1	UR_lag4	HPI_lag4	GDP_lag4	QQ4MA_INF lead	QQ4MA_UR lead	QQ4MA_HPI lead	QQ4MA_GD P lead	QQ4MA_INF lag	QQ4MA_UR lag	QQ4MA_HPI lag	QQ4MA_INC lag	QQ4MA_GD P lag				
6		0.872478518	-0.41017942	-0.27152237	0.886043429	-0.48216771	-0.31881251	0.836888891	-0.70999982	-0.44675217	-0.36160073	0.828827776	-0.31567752	-0.25716746	-0.37341959	0.893747271	-0.55647044	-0.36042295	-0.46492305				
7																							
8																							
9	SIGN TEST	UR_lag0	HPI_lag0	GDP_lag0	UR_lag1	HPI_lag1	GDP_lag1	UR_lag4	HPI_lag4	GDP_lag4	QQ4MA_INF lead	QQ4MA_UR lead	QQ4MA_HPI lead	QQ4MA_GD P lead	QQ4MA_INF lag	QQ4MA_UR lag	QQ4MA_HPI lag	QQ4MA_INC lag	QQ4MA_GD P lag				
10		Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass	Pass				
11																							
12																							
13	FINAL IV	UR_lag0	HPI_lag0	GDP_lag0	UR_lag1	HPI_lag1	GDP_lag1	UR_lag4	HPI_lag4	GDP_lag4	QQ4MA_INF lead	QQ4MA_UR lead	QQ4MA_HPI lead	QQ4MA_GD P lead	QQ4MA_INF lag	QQ4MA_UR lag	QQ4MA_HPI lag	QQ4MA_INC lag	QQ4MA_GD P lag				
14		0.872478518	-0.41017942	-0.27152237	0.886043429	-0.48216771	-0.31881251	0.836888891	-0.70999982	-0.44675217	-0.36160073	0.828827776	-0.31567752	-0.25716746	-0.37341959	0.893747271	-0.55647044	-0.36042295	-0.46492305				
15																							
16																							
17																							
18																							
19																							
20																							
21																							

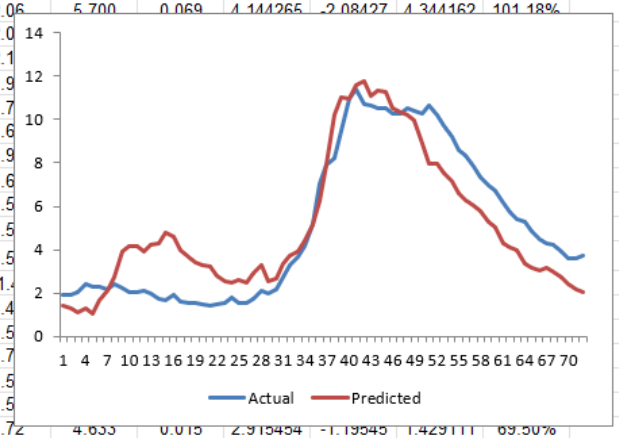
19.3 CCAR Exhaustive Model Search

Exhaustive Model Research

	Champion Model	Challenger Model	Model 3	Model 4	Full Model
Sign test	Pass	Pass	Pass	Pass	Pass
Adjusted r^2 >30%	Pass	Pass	Pass	Pass	Pass
P-Value significance	Pass	Pass	Pass	Fail	Pass
Duplicate Variables	Pass	Pass	Pass	Pass	Pass
Multicollinearity	Pass	Pass	Pass	Pass	Pass
Heteroskedasticity	Fail	Fail	Fail	Fail	Fail
Auo corr test	Fail	Fail	Fail	Fail	Fail
RMSE	1.474109978	1.350748527	1.453620654	1.635432937	1.404534513
MAPE	43.38%	38.11%	39.91%	47.24%	49.60%

19.4 CCAR Champion Model

observation_date	DR	UR_lag1	HPI_lag1	y hat	error	error sq	error %
1/01/00	1.95	4.067	0.079	1.440706	0.509294	0.259381	26.12%
4/01/00	1.89	4.033	0.086	1.326484	0.563516	0.31755	29.82%
7/01/00	2.07	3.933	0.091	1.124666	0.945334	0.893656	45.67%
10/01/00	2.42	4.000	0.081	1.323201	1.096799	1.202968	45.32%
1/01/01	2.28	3.900	0.093	1.054726	1.225274	1.501297	53.74%
4/01/01	2.3	4.233	0.081	1.695338	0.604662	0.365616	26.29%
7/01/01	2.19	4.400	0.062	2.128836	0.061164	0.003741	2.79%
10/01/01	2.41	4.833	0.069	2.758592	-0.34859	0.121516	14.46%
1/01/02	2.21	5.500	0.060	3.906969	-1.69697	2.879705	76.79%
4/01/02	2.06	5.700	0.069	4.144265	-2.08427	4.344162	101.18%
7/01/02	2.0						
10/01/02	2.1						
1/01/03	1.9						
4/01/03	1.7						
7/01/03	1.6						
10/01/03	1.9						
1/01/04	1.6						
4/01/04	1.5						
7/01/04	1.5						
10/01/04	1.5						
1/01/05	1.						
4/01/05	1.4						
7/01/05	1.5						
10/01/05	1.7						
1/01/06	1.5						
4/01/06	1.5						
7/01/06	1.72	4.633	0.015	2.915454	-1.19545	1.429111	69.50%
10/01/06	2.11	4.633	-0.026	3.270267	-1.16027	1.34622	54.99%
1/01/07	2.01	4.433	0.018	2.565292	-0.55529	0.308349	27.63%
4/01/07	2.17	4.500	0.017	2.683962	-0.51396	0.264157	23.68%
7/01/07	2.73	4.500	-0.061	3.368348	-0.63835	0.407488	23.38%
10/01/07	3.3	4.667	-0.075	3.753541	-0.45354	0.205699	13.74%
1/01/08	3.67	4.800	-0.065	3.885685	-0.21569	0.04652	5.88%
4/01/08	4.15	5.000	-0.087	4.399068	-0.24907	0.062035	6.00%
7/01/08	5.16	5.333	-0.116	5.183032	-0.02303	0.00053	0.45%
10/01/08	7.05	6.000	-0.115	6.239005	0.810995	0.657712	11.50%
1/01/09	7.88	6.867	-0.149	7.926673	-0.04667	0.002178	0.59%
4/01/09	8.21	8.267	-0.156	10.2275	-2.0175	4.070316	24.57%
7/01/09	9.49	9.300	-0.058	11.01652	-1.52652	2.330272	16.09%
10/01/09	10.97	9.633	0.009	10.96639	0.003606	1.3E-05	0.03%
1/01/10	11.36	9.933	-0.007	11.58657	-0.22657	0.051335	1.99%
4/01/10	10.72	9.833	-0.046	11.76347	-1.04347	1.088827	9.73%
7/01/10	10.67	9.633	-0.003	11.07413	-0.40413	0.163321	3.79%



RMSE	
SSE	156.456
MSE	2.173
RMSE	1.474109978

MAPE	
	3123.35%
MAPE	43.38%

Multicollinearity			
	VIF	SDx	Result
UR_lag1	1.15302719	1.763989961	Pass
HPI_lag1	1.15302719	0.072116594	Pass

Heteroskedasticity				
	R Square	BP statistic	p-value	Result
$et^2 \sim X$	0.254062776	18.29251986	1.8945E-05	Fail

Auto Correlation		
DW	r	Result
0.11833952	0.94083024	Fail

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.902419769
R Square	0.81436144
Adjusted R Square	0.808980612
Standard Error	1.505814895
Observations	72

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	686.3431114	343.1715557	151.3450098	5.8756E-26
Residual	69	156.4560164	2.267478499		
Total	71	842.7991278			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-4.367987603	0.730016413	-5.983410137	8.72441E-08	-5.824330498	-2.911644708
UR_lag1	1.59982197	0.108784264	14.706373	5.92528E-23	1.3828033	1.816840639
HPI_lag1	-8.779420639	2.660890358	-3.299429686	0.001534926	-14.08775166	-3.471089619

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.504046402
R Square	0.254062776
Adjusted R Square	0.232441407
Standard Error	1.815266925
Observations	72

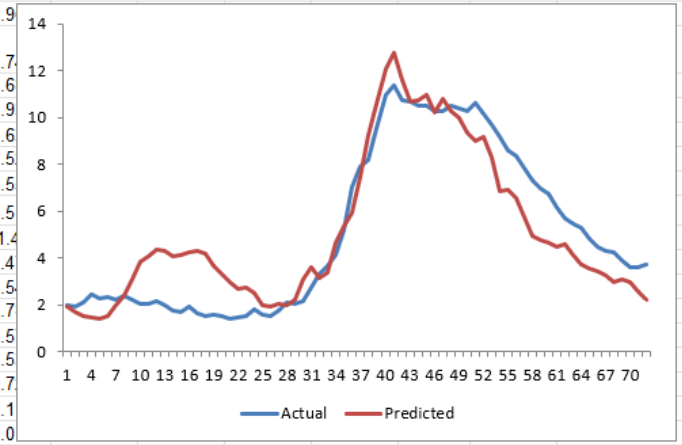
ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	77.44062313	38.72031157	11.75054077	4.05722E-05
Residual	69	227.3683867	3.295194011		
Total	71	304.8090099			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.214326257	0.880038213	-0.24354199	0.808308186	-1.969954455	1.54130194
UR_lag1	0.294442152	0.131139942	2.245251515		0.032825127	0.556059178

19.5 CCAR Challenger Model

observation_date	DR	QQ4MA_UR_lag	HPI_lag4	y hat	error	error sq	error %
1/01/00	1.95	4.150	0.064	1.897	0.053	0.0029	2.74%
4/01/00	1.89	4.067	0.071	1.691	0.199	0.0396	10.54%
7/01/00	2.07	4.008	0.077	1.524	0.546	0.2985	26.39%
10/01/00	2.42	3.967	0.079	1.434	0.986	0.9726	40.75%
1/01/01	2.28	4.017	0.086	1.418	0.862	0.7426	37.80%
4/01/01	2.3	4.133	0.091	1.527	0.773	0.5968	33.59%
7/01/01	2.19	4.342	0.081	1.964	0.226	0.051	10.31%
10/01/01	2.41	4.742	0.093	2.389	0.021	0.0004	0.86%
1/01/02	2.21	5.108	0.081	3.079	-0.869	0.7549	39.31%
4/01/02	2.06	5.467	0.062	3.843	-1.783	3.178	86.54%
7/01/02	2.05	5.692	0.069	4.078	-2.028	4.1138	98.94%
10/01/02	2.13	5.783	0.060	4.330	-2.200	4.8419	103.31%
1/01/03	1.9						
4/01/03	1.7						
7/01/03	1.6						
10/01/03	1.9						
1/01/04	1.6						
4/01/04	1.5						
7/01/04	1.5						
10/01/04	1.5						
1/01/05	1.4						
4/01/05	1.4						
7/01/05	1.5						
10/01/05	1.7						
1/01/06	1.5						
4/01/06	1.5						
7/01/06	1.7						
10/01/06	2.1						
1/01/07	2.0						
4/01/07	2.17	4.517	0.015	3.059	-0.889	0.7901	40.96%
7/01/07	2.73	4.525	-0.026	3.586	-0.856	0.7323	31.35%
10/01/07	3.3	4.617	0.018	3.161	0.139	0.0194	4.22%
1/01/08	3.67	4.742	0.017	3.360	0.310	0.096	8.44%
4/01/08	4.15	4.950	-0.061	4.656	-0.506	0.2564	12.20%
7/01/08	5.16	5.283	-0.075	5.314	-0.154	0.0236	2.98%
10/01/08	7.05	5.800	-0.065	5.948	1.102	1.2137	15.63%
1/01/09	7.88	6.617	-0.087	7.418	0.462	0.2133	5.86%
4/01/09	8.21	7.608	-0.116	9.226	-1.016	1.0319	12.37%
7/01/09	9.49	8.517	-0.115	10.533	-1.043	1.0883	10.99%
10/01/09	10.97	9.283	-0.149	12.086	-1.116	1.2466	10.18%
1/01/10	11.36	9.675	-0.156	12.745	-1.385	1.9194	12.20%



RMSE	
SSE	131.366
MSE	1.825
RMSE	1.350748527

MAPE	
	2743.93%
MAPE	38.11%

Multicollinearity	VIF	SDx	Result
QQ4MA_UR_lag	1.57622605	1.732132107	Pass
HPI_lag4	1.57622605	0.072327627	Pass

Heteroskedasticity	R Square	BP statistic	p-value	Result
$et^2 \sim X$	0.209026612	15.04991607	0.000104705	Fail

Auto_Correlation		
DW	r	Result
0.1271821	0.936408939	Fail

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.918766474
R Square	0.844131834
Adjusted R Square	0.839613917
Standard Error	1.379800206
Observations	72

ANOVA

	df	SS	MS	F	ignificance F
Regression	2	711.4335737	355.7167869	186.8409	1.41E-28
Residual	69	131.365554	1.903848609		
Total	71	842.7991278			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	ower 95.0%	pper
Intercept	-3.331053922	0.813039187	-4.09703982	0.000112	-4.95302	-1.70909	-4.95302	-1.7
QQ4MA_UR_lag	1.456200088	0.118690312	12.26890443	5.24E-19	1.219419	1.692981	1.219419	1.69
HPI_lag4	-12.73547925	2.84244496	-4.480466441	2.88E-05	-18.406	-7.06496	-18.406	-7.0

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.457194283
R Square	0.209026612
Adjusted R Square	0.186099847
Standard Error	1.770833515
Observations	72

ANOVA

	df	SS	MS	F	ignificance F
Regression	2	57.18001511	28.59000756	9.117144	0.000307
Residual	69	216.3737422	3.135851336		
Total	71	273.5537573			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	ower 95.0%	pper

19.6 CCAR K-Fold CV

PEAKS2TAILS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB
1									Fold1			y hat	error				Fold2			y hat	error				Fold3			y hat
2		observation_date	DR	UR_lag1	HPI_lag1		1	1/01/00	1.95	4.066667	0.079419	1.440706	0.509294		1	1/01/00	1.95	4.066667	0.079419	1.509041	0.440959		1	1/01/00	1.95	4.066667	0.079419	1.575664
3	1	1/01/00	1.95	4.067	0.079		2	4/01/00	1.89	4.033333	0.086355	1.326484	0.563516		2	4/01/00	1.89	4.033333	0.086355	1.391285	0.498715		2	4/01/00	1.89	4.033333	0.086355	1.447843
4	2	4/01/00	1.89	4.033	0.086		3	7/01/00	2.07	3.933333	0.09112	1.124666	0.945334		3	7/01/00	2.07	3.933333	0.09112	1.191272	0.878728		3	7/01/00	2.07	3.933333	0.09112	1.250141
5	3	7/01/00	2.07	3.933	0.091		4	10/01/00	2.42	4	0.080654	1.323201	1.096799		4	10/01/00	2.42	4	0.080654	1.394242	1.025758		4	10/01/00	2.42	4	0.080654	1.466338
6	4	10/01/00	2.42	4.000	0.081		5	1/01/01	2.28	3.9	0.093012	1.054726	1.225274		5	1/01/01	2.28	3.9	0.093012	1.121698	1.158302		5	1/01/01	2.28	3.9	0.093012	1.180722
7	5	1/01/01	2.28	3.900	0.093		6	4/01/01	2.3	4.233333	0.080786	1.695338	0.604662		6	4/01/01	2.3	4.233333	0.080786	1.753465	0.546535		6	4/01/01	2.3	4.233333	0.080786	1.797374
8	6	4/01/01	2.30	4.233	0.081		7	7/01/01	2.19	4.4	0.06178	2.128836	0.061164		7	7/01/01	2.19	4.4	0.06178	2.192513	-0.00251		7	7/01/01	2.19	4.4	0.06178	2.254988
9	7	7/01/01	2.19	4.400	0.062		8	10/01/01	2.41	4.833333	0.069013	2.758592	-0.34859		8	10/01/01	2.41	4.833333	0.069013	2.792879	-0.38288		8	10/01/01	2.41	4.833333	0.069013	2.788848
10	8	10/01/01	2.41	4.833	0.069		9	1/01/02	2.21	5.5	0.059692	3.906969	-1.69697		9	1/01/02	2.21	5.5	0.059692	3.911859	-1.70186		9	1/01/02	2.21	5.5	0.059692	3.846943
11	9	1/01/02	2.21	5.500	0.060		10	4/01/02	2.06	5.7	0.069108	4.144265	-2.08427		10	4/01/02	2.06	5.7	0.069108	4.130889	-2.07089		10	4/01/02	2.06	5.7	0.069108	4.022964
12	10	4/01/02	2.06	5.700	0.069		11	7/01/02	2.05	5.833333	0.094698	4.132912	-2.08291		11	7/01/02	2.05	5.833333	0.094698	4.092419	-2.04242		11	7/01/02	2.05	5.833333	0.094698	3.916694
13	11	7/01/02	2.05	5.833	0.095		12	10/01/02	2.13	5.733333	0.101973	3.909063	-1.77906		12	10/01/02	2.13	5.733333	0.101973	3.868434	-1.73843		12	10/01/02	2.13	5.733333	0.101973	3.689935
14	12	10/01/02	2.13	5.733	0.102		13	1/01/03	1.96	5.866667	0.090774	4.220692	-2.26069		13	1/01/03	1.96	5.866667	0.090774	4.181404	-2.2214		13	1/01/03	1.96	5.866667	0.090774	4.009641
15	13	1/01/03	1.96	5.867	0.091		14	4/01/03	1.74	5.866667	0.082702	4.29156	-2.55156		14	4/01/03	1.74	5.866667	0.082702	4.258517	-2.51852		14	4/01/03	1.74	5.866667	0.082702	4.103109
16	14	4/01/03	1.74	5.867	0.083		15	7/01/03	1.68	6.133333	0.074596	4.789348	-3.10935		15	7/01/03	1.68	6.133333	0.074596	4.747933	-3.06793		15	7/01/03	1.68	6.133333	0.074596	4.57704
17	15	7/01/03	1.68	6.133	0.075		16	10/01/03	1.93	6.133333	0.097667	4.58679	-2.65679		16	10/01/03	1.93	6.133333	0.097667	4.527527	-2.59753		16	10/01/03	1.93	6.133333	0.097667	4.309889
18	16	10/01/03	1.93	6.133	0.098		17	1/01/04	1.63	5.833333	0.11168	3.983822	-2.35382		17	1/01/04	1.63	5.833333	0.11168	3.930193	-2.30019		17	1/01/04	1.63	5.833333	0.11168	3.720062
19	17	1/01/04	1.63	5.833	0.112		18	4/01/04	1.52	5.7	0.126579	3.639706	-2.11971		18	4/01/04	1.52	5.7	0.126579	3.581873	-2.06187		18	4/01/04	1.52	5.7	0.126579	3.35751
20	18	4/01/04	1.52	5.700	0.127		19	7/01/04	1.55	5.6	0.13476	3.407898	-1.8579		19	7/01/04	1.55	5.6	0.13476	3.349227	-1.79923		19	7/01/04	1.55	5.6	0.13476	3.120253
21	19	7/01/04	1.55	5.600	0.135		20	10/01/04	1.51	5.433333	0.119844	3.272221	-1.76222		20	10/01/04	1.51	5.433333	0.119844	3.234241	-1.72424		20	10/01/04	1.51	5.433333	0.119844	3.055431
22	20	10/01/04	1.51	5.433	0.120		21	1/01/05	1.4	5.433333	0.124296	3.233136	-1.83314		21	1/01/05	1.4	5.433333	0.124296	3.191712	-1.79171		21	1/01/05	1.4	5.433333	0.124296	3.003883
23	21	1/01/05	1.40	5.433	0.124		22	4/01/05	1.47	5.3	0.148162	2.81029	-1.34029		22	4/01/05	1.47	5.3	0.148162	2.757725	-1.28772		22	4/01/05	1.47	5.3	0.148162	2.537494
24	22	4/01/05	1.47	5.300	0.148		23	7/01/05	1.54	5.1	0.14102	2.553031	-1.01303		23	7/01/05	1.54	5.1	0.14102	2.516973	-0.97697		23	7/01/05	1.54	5.1	0.14102	2.335145
25	23	7/01/05	1.54	5.100	0.141		24	10/01/05	1.78	4.966667	0.121702	2.509323	-0.72932		24	10/01/05	1.78	4.966667	0.121702	2.49553	-0.71553		24	10/01/05	1.78	4.966667	0.121702	2.368795
26	24	10/01/05	1.78	4.967	0.122		25	1/01/06	1.57	4.966667	0.112068	2.593906	-1.02391		25	1/01/06	1.57	4.966667	0.112068	2.587567	-1.01757		25	1/01/06	1.57	4.966667	0.112068	2.480351
27	25	1/01/06	1.57	4.967	0.112		26	4/01/06	1.53	4.733333	0.083421	2.472114	-0.94211		26	4/01/06	1.53	4.733333	0.083421	2.500747	-0.97075		26	4/01/06	1.53	4.733333	0.083421	2.47949
28	26	4/01/06	1.53	4.733	0.083		27	7/01/06	1.72	4.633333	0.014701	2.915454	-1.19545		27	7/01/06	1.72	4.633333	0.014701	3.002737	-1.28274		27	7/01/06	1.72	4.633333	0.014701	3.132677
29	27	7/01/06	1.72	4.633	0.015		28	10/01/06	2.11	4.633333	-0.02571	3.270267	-1.16027		28	10/01/06	2.11	4.633333	-0.02571	3.388813	-1.27881		28	10/01/06	2.11	4.633333	-0.02571	3.600634
30	28	10/01/06	2.11	4.633	-0.026		29	1/01/07	2.01	4.433333	0.018141	2.565292	-0.55529		29	1/01/07	2.01	4.433333	0.018141	2.660896	-0.6509		29	1/01/07	2.01	4.433333	0.018141	2.807798
31	29	1/01/07	2.01	4.433	0.018		30	4/01/07	2.17	4.5	0.016772	2.683962	-0.51396		30	4/01/07	2.17	4.5	0.016772	2.776964	-0.60696		30	4/01/07	2.17	4.5	0.016772	2.918662
32	30	4/01/07	2.17	4.500	0.017		31	7/01/07	2.73	4.5	-0.06118	3.368348	-0.63835		31	7/01/07	2.73	4.5	-0.06118	3.521652	-0.79165		31	7/01/07	2.73	4.5	-0.06118	3.821287
33	31	7/01/07	2.73	4.500	-0.061		32	10/01/07	3.3	4.666667	-0.07469	3.753541	-0.45354		32	10/01/07	3.3	4.666667	-0.07469	3.908139	-0.60814		32	10/01/07	3.3	4.666667	-0.07469	4.215191
34	32	10/01/07	3.30	4.667	-0.075		33	1/01/08	3.67	4.8	-0.06544	3.885685	-0.21569		33	1/01/08	3.67	4.8	-0.06544	4.02581	-0.35581		33	1/01/08	3.67	4.8	-0.06544	4.298178
35	33	1/01/08	3.67	4.800	-0.065		34	4/01/08	4.15	5	-0.08747	4.399068	-0.24907		34	4/01/08	4.15	5	-0.08747	4.545253	-0.39525		34	4/01/08	4.15	5	-0.08747	4.838326
36	34	4/01/08	4.15	5.000	-0.087		35	7/01/08	5.16	5.333333	-0.11603	5.183032	-0.02303		35	7/01/08	5.16	5.333333	-0.11603	5.333002	-0.173		35	7/01/08	5.16	5.333333	-0.11603	5.644042
37	35	7/01/08	5.16	5.333	-0.116		36	10/01/08	7.05	6	-0.11482	6.239005	0.810995		36	10/01/08	7.05	6	-0.11482	6.351437	0.698563		36	10/01/08	7.05	6	-0.11482	6.580266
38	36	10/01/08	7.05	6.000	-0.115		37	1/01/09	7.88	6.866667	-0.14912	7.926673	-0.04667		37	1/01/09	7.88	6.866667	-0.14912	8.01805	-0.13805		37	1/01/09	7.88	6.866667	-0.14912	8.212677
39	37	1/01/09	7.88	6.867	-0.149		38	4/01/09	8.21	8.266667	-0.15608	10.2275	-2.0175		38	4/01/09	8.21	8.266667	-0.15608	10.24739	-2.03739		38	4/01/09	8.21	8.266667	-0.15608	10.28859
40	38	4/01/09	8.21	8.267	-0.156		39	7/01/09	9.49	9.3	-0.05765	11.01652	-1.52652		39	7/01/09	9.49	9.3	-0.05765	10.90353	-1.41353		39	7/01/09	9.49	9.3	-0.05765	10.62167
41	39	7/01/09	9.49	9.300	-0.058		40	10/01/09	10.97	9.633333	0.008797	10.96639	0.003606		40	10/01/09	10.97	9.633333	0.008797	10.78369	0.18631		40	10/01/09	10.97	9.633333	0.008797	10.32732
42	40	10/01/09	10.97	9.633	0.009		41	1/01/10	11.36	9.933333	-0.00718	11.58657	-0.22657		41	1/01/10	11.36	9.933333	-0.00718	11.39975	-0.03975		41	1/01/10	11.36	9.933333	-0.00718	10.93984
43	41	1/01/10	11.36	9.933	-0.007		42	4/01/10	10.72	9.833333	-0.04555	11.76347	-1.04347		42	4/01/10	10.72	9.833333	-0.04555	11.61182	-0.89182		42	4/01/10	10.72	9.833333	-0.04555	11.24162
44	42	4/01/10	10.72	9.833	-0.046		43	7/01/10	10.67	9.633333	-0.00347	11.07413	-0.40413		43	7/01/10	10.67	9.633333	-0.00347	10.90092	-0.23092		43	7/01/10	10.67	9.633333	-0.00347	10.46941

19.7 CCAR Coefficient Stability Test

Stability Test

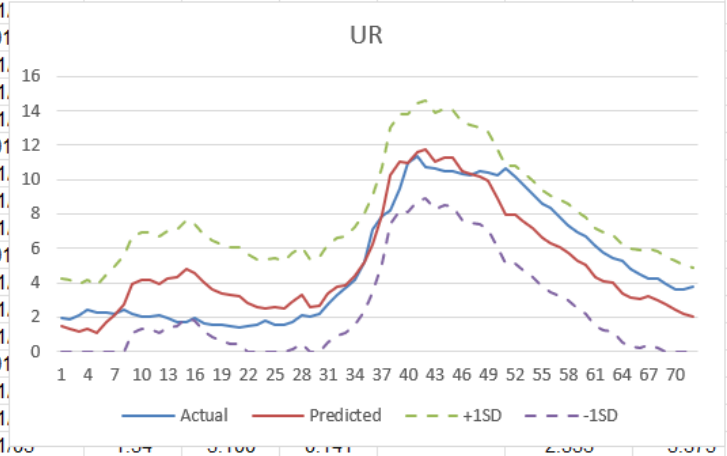
	Training data coefficients	*Standard Error*	LB	UB	Full model coefficients	
Intercept	-4.367987603	0.730016413	-5.824330498	-2.911644708	-3.567381312	TRUE
UR_lag1	1.59982197	0.108784264	1.3828033	1.816840639	1.494378419	TRUE
HPI_lag1	-8.779420639	2.660890358	-14.08775166	-3.471089619	-9.506599679	TRUE

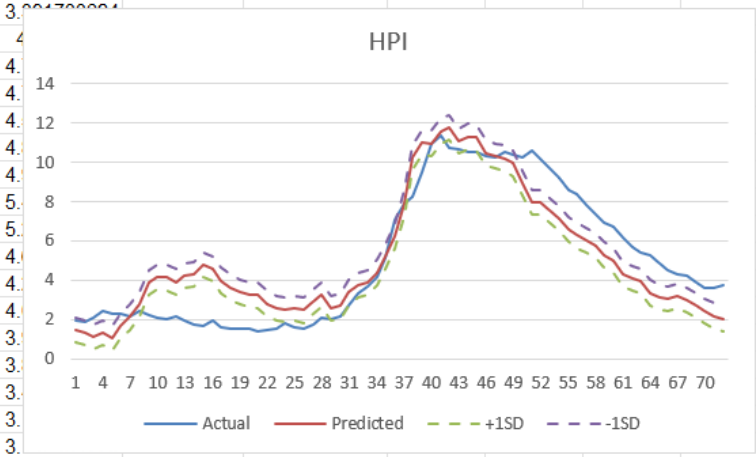
	β1	Lb	Ub	Within interval
Fold 1	1.59982197	1.382253441	1.817390499	Yes
Fold 2	1.544911221	1.382253441	1.817390499	Yes
Fold 3	1.425256169	1.382253441	1.817390499	Yes
Fold 4	1.312264236	1.382253441	1.817390499	No
Fold 5	1.620037416	1.382253441	1.817390499	Yes
Fold 6	1.404278513	1.382253441	1.817390499	Yes
Fold 7	1.495943397	1.382253441	1.817390499	Yes
Fold 8	1.55647073	1.382253441	1.817390499	Yes
Fold 9	1.510673869	1.382253441	1.817390499	Yes

	β2	Lb	Ub	Within interval
Fold 1	-8.779420639	-14.10120136	-3.457639923	Yes
Fold 2	-9.552980959	-14.10120136	-3.457639923	Yes
Fold 3	-11.57903675	-14.10120136	-3.457639923	Yes
Fold 4	-11.54294311	-14.10120136	-3.457639923	Yes
Fold 5	-12.56728564	-14.10120136	-3.457639923	Yes
Fold 6	-12.8685152	-14.10120136	-3.457639923	Yes
Fold 7	-5.887270693	-14.10120136	-3.457639923	Yes
Fold 8	-4.849590088	-14.10120136	-3.457639923	Yes
Fold 9	-9.626294418	-14.10120136	-3.457639923	Yes

19.8 CCAR Variable Sensitivity Analysis

observation_date	DR	UR_lag1	HPI_lag1						
1/01/00	1.95	4.067	0.079	1.441	4.263	0.00	0.80756358	2.073847417	
4/01/00	1.89	4.033	0.086	1.326	4.149	0.00	0.69334222	1.959626054	Intercept -4.36799
7/01/00	2.07	3.933	0.091	1.125	3.947	0.00	0.49152436	1.757808192	UR_lag1 1.599822
10/01/00	2.42	4.000	0.081	1.323	4.145	0.00	0.69005911	1.956342945	HPI_lag1 -8.77942
1/01/01	2.28	3.900	0.093	1.055	3.877	0.00	0.42158388	1.687867715	
4/01/01	2.3	4.233	0.081	1.695	4.517	0.00	1.06219628	2.328480111	**1.76399**
7/01						0.00	1.49569438	2.761978217	0.072117
10/01						0.00	2.12544979	3.	
1/01						1.08	3.27382746	4	
4/01						1.32	3.51112353	4.	
7/01						1.31	3.49976977	4.	
10/01						1.09	3.27592085	4.	
1/01						1.40	3.58754961	4.	
4/01						1.47	3.65841853	4.	
7/01						1.97	4.15620622	5.	
10/01						1.76	3.95364786	5.	
1/01						1.16	3.3506804	4.	
4/01						0.82	3.00656444	4.	
7/01						0.59	2.77475586	4.	
10/01						0.45	2.63907868	3.	
1/01						0.41	2.59999377	3.	
4/01						0.00	2.17714762	3.	
7/01						0.00	1.91988911	3.	
10/01/05	1.78	4.967	0.122	2.509	5.331	0.00	1.87618095	3.	
1/01/06	1.57	4.967	0.112	2.594	5.416	0.00	1.96076443	3.227048263	
4/01/06	1.53	4.733	0.083	2.472	5.294	0.00	1.83897221	3.105256047	
7/01/06	1.72	4.633	0.015	2.915	5.738	0.09	2.28231243	3.548596265	
10/01/06	2.11	4.633	-0.026	3.270	6.092	0.45	2.63712538	3.903409208	
1/01/07	2.01	4.433	0.018	2.565	5.387	0.00	1.93214983	3.19843366	
4/01/07	2.17	4.500	0.017	2.684	5.506	0.00	2.05082011	3.317103942	
7/01/07	2.73	4.500	-0.061	3.368	6.190	0.55	2.73520606	4.001489897	
10/01/07	3.3	4.667	-0.075	3.754	6.576	0.93	3.12039867	4.386682499	
1/01/08	3.67	4.800	-0.065	3.886	6.708	1.06	3.25254331	4.518827144	
4/01/08	4.15	5.000	-0.087	4.399	7.221	1.58	3.76592619	5.032210023	
7/01/08	5.16	5.333	-0.116	5.183	8.005	2.36	4.54988962	5.816173456	
10/01/08	7.05	6.000	-0.115	6.239	9.061	3.42	5.60586353	6.872147365	
1/01/09	7.88	6.867	-0.149	7.927	10.749	5.10	7.29353089	8.559814718	
4/01/09	8.21	8.267	-0.156	10.228	13.050	7.41	9.59436044	10.86064427	



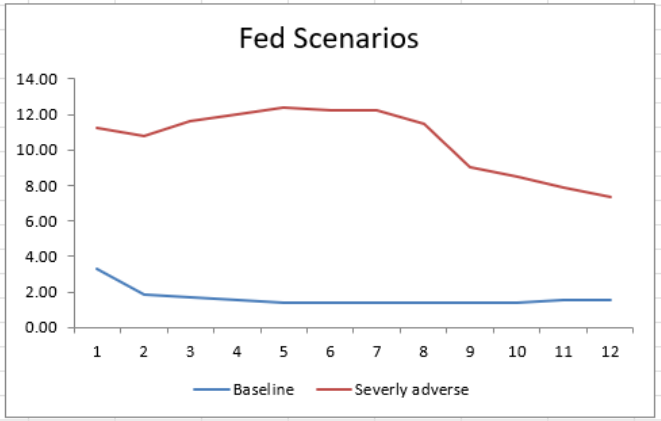


19.9 CCAR Scenario Analysis

Scenario Name	Date	Unemployment rate	House Price Index (Level)	HPIg	UR_lag1	HPIg_lag1	DR
Baseline	2022 Q1	4	269.8	-0.098	4.23	0.195	0.90
Baseline	2022 Q2	3.8	271.8	0.030	4.00	-0.098	3.34
Baseline	2022 Q3	3.7	273.8	0.029	3.80	0.030	1.83
Baseline	2022 Q4	3.6	275.9	0.031	3.70	0.029	1.68
Baseline	2023 Q1	3.5	277.9	0.029	3.60	0.031	1.52
Baseline	2023 Q2	3.5	280	0.030	3.50	0.029	1.39
Baseline	2023 Q3	3.5	282.1	0.030	3.50	0.030	1.38
Baseline	2023 Q4	3.5	284.2	0.030	3.50	0.030	1.38
Baseline	2024 Q1	3.5	286.4	0.031	3.50	0.030	1.38
Baseline	2024 Q2	3.5	288.5	0.029	3.50	0.031	1.37
Baseline	2024 Q3	3.6	290.7	0.030	3.50	0.029	1.39
Baseline	2024 Q4	3.6	292.9	0.030	3.60	0.030	1.52
Baseline	2025 Q1	3.6	295.1	0.030	3.60	0.030	1.53

Scenario Name	Date	Unemployment rate	House Price Index (Level)	HPIg	UR_lag1	HPIg_lag1	DR
Severly Adverse	2022 Q1	7	246.4	-0.460	4.23	0.195	0.90
Severly Adverse	2022 Q2	8.1	232.3	-0.236	7	-0.460	11.27
Severly Adverse	2022 Q3	8.9	221.1	-0.198	8.1	-0.236	10.78
Severly Adverse	2022 Q4	9.4	212.5	-0.159	8.9	-0.198	11.61
Severly Adverse	2023 Q1	9.8	205.2	-0.140	9.4	-0.159	11.99
Severly Adverse	2023 Q2	9.9	200	-0.103	9.8	-0.140	12.41
Severly Adverse	2023 Q3	10	195.4	-0.093	9.9	-0.103	12.20
Severly Adverse	2023 Q4	9.5	191.2	-0.087	10	-0.093	12.26
Severly Adverse	2024 Q1	8.9	194.7	0.073	9.5	-0.087	11.46
Severly Adverse	2024 Q2	8.5	198.1	0.069	8.9	0.073	9.04
Severly Adverse	2024 Q3	8.1	201.4	0.066	8.5	0.069	8.48
Severly Adverse	2024 Q4	7.7	204.5	0.061	8.1	0.066	7.91
Severly Adverse	2025 Q1	7.4	207.6	0.060	7.7	0.061	7.36

	Coefficient	Standard Error
Intercept	-3.56738	0.629988
UR_lag1	1.494378	0.096793
HPI_lag1	-9.5066	2.576896



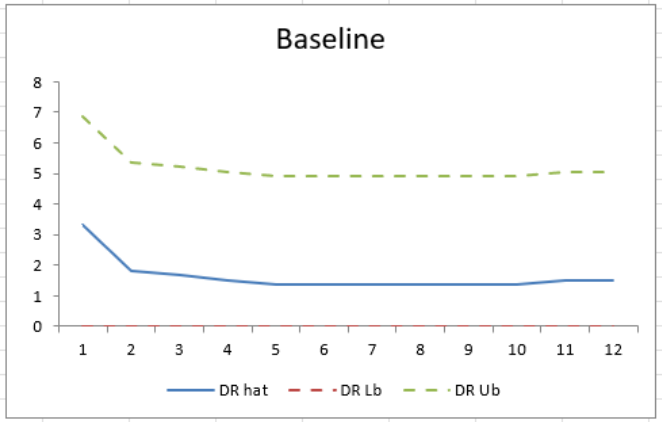
19.10 CCAR Forecast Error

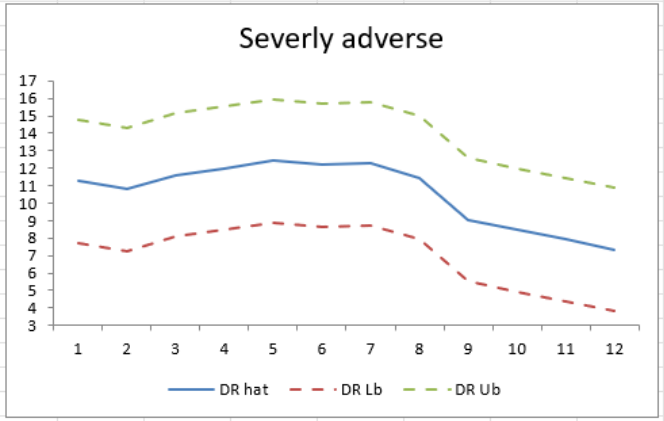
Scenario Name	Date	mployment	Price Index	HPIg	UR_lag1	HPIg_lag1	DR hat	DR Lb	DR Ub
Baseline	2022 Q1	4	269.8	-0.098	4.23	0.195	0.900611		
Baseline	2022 Q2	3.8	271.8	0.030	4.00	-0.098	3.337139	0	6.875792986
Baseline	2022 Q3	3.7	273.8	0.029	3.80	0.030	1.830411	0	5.369064841
Baseline	2022 Q4	3.6	275.9	0.031	3.70	0.029	1.683032	0	5.221686007
Baseline	2023 Q1	3.5	277.9	0.029	3.60	0.031	1.521838	0	5.060492029
Baseline	2023 Q2	3.5	280	0.030	3.50	0.029	1.388284	0	4.926938302
Baseline	2023 Q3	3.5	282.1	0.030	3.50	0.030	1.37667	0	4.915324626
Baseline	2023 Q4	3.5	284.2	0.030	3.50	0.030	1.378809	0	4.917463671
Baseline	2024 Q1	3.5	286.4	0.031	3.50	0.030	1.380917	0	4.919570987
Baseline	2024 Q2	3.5	288.5	0.029	3.50	0.031	1.369713	0	4.908367582
Baseline	2024 Q3	3.6	290.7	0.030	3.50	0.029	1.385136	0	4.923789894
Baseline	2024 Q4	3.6	292.9	0.030	3.60	0.030	1.523505	0	5.062159363
Baseline	2025 Q1	3.6	295.1	0.030	3.60	0.030	1.525683	0	5.064337338

	Coefficient	Standard Error
Intercept	-3.56738	0.629988
UR_lag1	1.494378	0.096793
HPI_lag1	-9.5066	2.576896

Prediction error
1.412265516
1.769327166

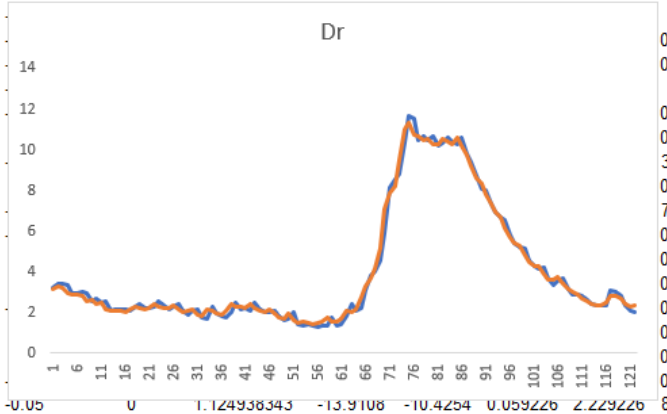
Scenario Name	Date	mployment	Price Index	HPIg	UR_lag1	HPIg_lag1	DR hat	DR Lb	DR Ub
Severely Adverse	2022 Q1	7	246.4	-0.460	4.23	0.195	0.900611		
Severely Adverse	2022 Q2	8.1	232.3	-0.236	7	-0.460	11.27021	7.731554	14.80886232
Severely Adverse	2022 Q3	8.9	221.1	-0.198	8.1	-0.236	10.77785	7.239196	14.31650429
Severely Adverse	2022 Q4	9.4	212.5	-0.159	8.9	-0.198	11.61164	8.07299	15.1502984
Severely Adverse	2023 Q1	9.8	205.2	-0.140	9.4	-0.159	11.9884	8.449747	15.52705526
Severely Adverse	2023 Q2	9.9	200	-0.103	9.8	-0.140	12.40681	8.868157	15.9454656
Severely Adverse	2023 Q3	10	195.4	-0.093	9.9	-0.103	12.20302	8.664363	15.74167134
Severely Adverse	2023 Q4	9.5	191.2	-0.087	10	-0.093	12.26122	8.722571	15.7998793
Severely Adverse	2024 Q1	8.9	194.7	0.073	9.5	-0.087	11.45548	7.916825	14.99413369
Severely Adverse	2024 Q2	8.5	198.1	0.069	8.9	0.073	9.042791	5.504137	12.58144543
Severely Adverse	2024 Q3	8.1	201.4	0.066	8.5	0.069	8.476521	4.937866	12.015175
Severely Adverse	2024 Q4	7.7	204.5	0.061	8.1	0.066	7.908849	4.370194	11.44750307
Severely Adverse	2025 Q1	7.4	207.6	0.060	7.7	0.061	7.358479	3.819825	10.89713379





19.11 Vector Error Correction Models

UR %	HPI %	DR_lag1	UR_lag1	HPI_lag1	ΔDR	ΔUR	ΔHPI	ΔDR_Lag1	ΔUR_Lag1	ΔHPI_Lag1	ce 1	ce 2	ΔDR	Dr hat	erro sq
6.4	-4.11497														
6.7	-1.83785	3.18	6.4	-4.11497	-0.11	0.3	2.277121								
6.8	2.71681	3.07	6.7	-1.83785	0.1	0.1	4.554656	-0.11	0.3	2.277120663	13.42497	17.01932	0.161455	3.231455	0.003777
7	-0.0368	3.17	6.8	2.71681	0.13	0.2	-2.75361	0.1	0.1	4.554655776	-12.1373	-8.45462	0.216902	3.386902	0.007552
7.3	0.052576	3.3	7	-0.0368	-0.07	0.3	0.08938	0.13	0.2	-2.753614514	3.507365	7.206651	0.131698	3.431698	0.040682
7.4	1.364562	3.23	7.3	0.052576	-0.27	0.1	1.311986	-0.07	0.3	0.089380326	2.93377	7.00479	0.111778	3.341778	0.145754
7.7	-0.42986	2.96	7.4	1.364562	-0.08	0.3	-1.79442								0.0111
7.3	0.592247	2.88	7.7	-0.42986	-0.02	-0.4	1.022107								0.013828
7.3	2.13111	2.86	7.3	0.592247	-0.04	0	1.538863								0.030783
7.1	0.791049	2.82	7.3	2.13111	-0.24	-0.2	-1.34006								0.12994
6.9	2.554992	2.58	7.1	0.791049	0.01	-0.2	1.763943								0.000565
6.8	2.636297	2.59	6.9	2.554992	-0.14	-0.1	0.081305								0.072379
6.6	3.322024	2.45	6.8	2.636297	0.02	-0.2	0.685726								3.03E-05
6.4	2.010437	2.47	6.6	3.322024	-0.29	-0.2	-1.31159								0.144677
6.1	3.067198	2.18	6.4	2.010437	-0.07	-0.3	1.056761								7.76E-08
5.8	2.355011	2.11	6.1	3.067198	0	-0.3	-0.71219								0.002816
5.6	1.804454	2.11	5.8	2.355011	0	-0.2	-0.55056								0.002701
5.8	1.162099	2.11	5.6	1.804454	-0.08	0.2	-0.64236								0.020282
5.7	1.366355	2.03	5.8	1.162099	0.12	-0.1	0.204256								0.004266
5.5	2.684425	2.15	5.7	1.366355	0.13	-0.2	1.31807								0.000293
5.6	1.729155	2.28	5.5	2.684425	-0.06	0.1	-0.95527								0.039392
5.6	2.854094	2.22	5.6	1.729155	-0.05	0	1.124938								0.006833
5.5	2.453001	2.17	5.6	2.854094	0.05	-0.1	-0.40109	-0.05	0	1.124938343	-13.9108	-10.4254	0.059226	2.229226	8.51E-05
5.2	2.159892	2.22	5.5	2.453001	0.19	-0.3	-0.29311	0.05	-0.1	-0.40109249	-11.6009	-8.27336	0.073375	2.293375	0.013601
5.3	2.768144	2.41	5.2	2.159892	-0.09	0.1	0.608252	0.19	-0.3	-0.29310885	-9.75948	-6.92758	0.126317	2.536317	0.046793
5.1	3.165298	2.32	5.3	2.768144	-0.13	-0.2	0.397154	-0.09	0.1	0.608251849	-13.2766	-10.2429	0.020821	2.340821	0.022747
4.9	3.337373	2.19	5.1	3.165298	0.03	-0.2	0.172075	-0.13	-0.2	0.397154025	-15.6442	-12.6728	-0.02054	2.169458	0.002554
4.7	3.802243	2.22	4.9	3.337373	0.12	-0.2	0.46487	0.03	-0.2	0.172075289	-16.5838	-13.839	0.038725	2.258725	0.006606
4.6	6.758831	2.34	4.7	3.802243	-0.18	-0.1	2.956587	0.12	-0.2	0.464870112	-19.083	-16.6492	0.068666	2.408666	0.061835
4.3	5.729471	2.16	4.6	6.758831	-0.13	-0.3	-1.02936	-0.18	-0.1	2.956587232	-35.9213	-33.3502	-0.06811	2.091888	0.00383
4.5	6.388921	2.03	4.3	5.729471	0.07	0.2	0.65945	-0.13	-0.3	-1.029359517	-30.2516	-27.8704	-0.1219	1.908105	0.036824
4.5	6.188182	2.1	4.5	6.388921	0.07	0	-0.20074	0.07	0.2	0.659449655	-33.8971	-31.3732	0.012557	2.112557	0.0033
4.3	6.445961	2.17	4.5	6.188182	-0.27	-0.2	0.257779	0.07	0	-0.20073845	-32.6961	-30.246	-0.00946	2.160539	0.067881
4.3	6.711046	1.9	4.3	6.445961	-0.1	0	0.265085	-0.27	-0.2	0.257778667	-34.4185	-31.8934	-0.1665	1.733504	0.004422
4.3	7.558042	1.8	4.3	6.711046	0.35	0	0.846997	-0.1	0	0.265084526	-36.012	-33.3818	-0.08425	1.71575	0.188573
4.1	7.917559	2.15	4.3	7.558042	-0.07	-0.2	0.359517	0.35	0	0.846996782	-40.4343	-38.1377	0.111532	2.261532	0.032954
4	8.149933	2.08	4.1	7.917559	-0.13	-0.1	0.232374	-0.07	-0.2	0.359517026	-42.5299	-40.3563	-0.11012	1.969879	0.000395
3.8	9.318565	1.95	4	8.149933	-0.06	-0.2	1.168632	-0.13	-0.1	0.232373922	-43.9692	-41.7611	-0.14179	1.808206	0.00669
4	8.197426	1.89	3.8	9.318565	0.18	0.2	-1.12114	-0.06	-0.2	1.168631948	-50.6136	-48.5228	-0.11848	1.771522	0.089089
3.9	8.771034	2.07	4	8.197426	0.35	-0.1	0.573608	0.18	0.2	-1.121139301	-44.1168	-42.0277	-0.02279	2.047209	0.138973
4.2	9.071084	2.42	3.9	8.771034	-0.14	0.3	0.30005	0.35	-0.1	0.573608253	-46.9986	-45.3485	0.060709	2.480709	0.040284
4.4	6.523061	2.28	4.2	9.071084	0.02	0.2	-2.54802	-0.14	0.3	0.300050147	-48.8292	-46.7332	-0.15405	2.12595	0.030294
4.6	6.538242	2.3	4.4	6.523061	-0.11	0.2	0.01518	0.02	0.2	-2.548022917	-34.4529	-32.2263	-0.0901	2.209897	0.000396



ce 1 0.28*DR_lag1 + 0.11* UR_lag1 +0.32 * HPI_lag1 = constant + epsilon — epsilon is stationary

ce 2 0.29*DR_lag1 + 0.26* UR_lag1 +0.14 * HPI_lag2 = constant + epsilon — epsilon is stationary

$$\begin{bmatrix} -0.02702169 & 0.030455 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & -5.6343 \\ 0 & 1 & -5.6149 \end{bmatrix}$$

$$\begin{bmatrix} 0.46376639 & 0.027591 & 0.02135836 \end{bmatrix}$$

α11	-0.02702169
α12	0.03045492
β11	1
β21	0
β31	-5.6343
β12	0
β22	1
β32	-5.6149
c11	0.46376639
c12	0.02759108
c13	0.02135836
SSE	8.14389187

20.1 Model Validation- Building Logistic Regression Model

PEAKS2TAILS

custom	age	ed	emplo	addre	incom	debti	credde	othde	defau	Y	Sigmo	Likelihoo	LL	Defaul	Score	Grad
10012	28	2	7	2	44	17.7	2.99	4.8	0	0.477332	0.617118	0.38288232	-0.9600276	1	473.3501	12
10017	64	5	34	17	116	14.7	5.05	12	0	-5.91126	0.002701	0.99729853	-0.0027051	0	657.6858	2
10030	40	1	20	12	61	4.8	1.04	1.89	0	-5.19604	0.005508	0.99449205	-0.0055232	0	637.049	4
10039	30	1	11	3	27	34.5	1.75	7.56	0	0.253566	0.563054	0.43694609	-0.8279455	1	479.8066	12
10069	25	1	2	2	30	22.4	0.76	5.96	1	0.865219	0.70375	0.70374982	-0.3513324	1	462.1581	12
10071	35	1	2	9	38	10.9	1.46	2.68	1	-0.16042	0.45998	0.45997985	-0.7765726	1	491.7519	12
10096	26	3	2	4	38	11.9	0.95	3.57	1	-0.07013	0.482476	0.48247553	-0.7288251	1	489.1464	12
10128	25	1	4	2	30	14.4	1.05	3.27	0	-0.10957	0.472635	0.52736454	-0.6398632	1	490.2845	12
10129	65	4	29	14	189	5	3.36	6.09	0	-6.70121	0.001228	0.99877209	-0.0012287	0	680.4791	1
10140	21	3	0	0	23	3.9	0.31	0.59	0	-0.39732	0.401957	0.59804339	-0.514092	1	498.5872	11
10142	62	1	33	13	96	17.5	6.69	10.11	0	-4.25269	0.014026	0.98597359	-0.0141257	0	609.8296	5
10169	30	4	4	3	39	10.6	2.39	1.74	1	0.079721	0.51992	0.51991961	-0.6540811	1	484.8227	12
10197	38	5	7	7	124	19.1	9.52	14.16	0	3.823748	0.978621	0.02137873	-3.8453589	1	376.7929	13
10200	18	2	0	0	35	3.9	0.17	1.19	0	-0.41803	0.396989	0.60301135	-0.5058193	0	499.1847	11
10218	53	1	9	13	41	13.3	2.33	3.12	0	-1.49529	0.183129	0.81687078	-0.2022744	0	530.2679	10
10234	18	2	0	0	15	7.4	0.83	0.28	1	0.249435	0.562038	0.56203751	-0.5761867	1	479.9258	12
10351	18	2	0	0	56	15.7	1.84	6.95	1	1.514771	0.819767	0.8197672	-0.1987349	1	443.4159	13
10423	19	1	2	0	31	0.3	0.02	0.07	1	-1.23438	0.225417	0.22541653	-1.4898054	0	522.7395	10
10428	19	2	0	0	26	5.8	0.59	0.92	1	-0.03572	0.49107	0.49106993	-0.7111687	1	488.1538	12
10449	56	2	27	21	125	8.5	2.9	7.72	0	-6.09442	0.00225	0.99774966	-0.0022529	0	662.9708	2
10455	21	2	1	1	40	7.6	1.52	1.52	1	0.303825	0.575377	0.5753772	-0.5527294	1	478.3565	12
10474	25	1	8	3	37	8.3	0.95	2.12	0	-1.62353	0.164718	0.83528187	-0.179986	0	533.9683	10
10525	47	3	8	10	43	6.1	0.36	2.26	0	-2.79861	0.057399	0.9426009	-0.0591123	0	567.8739	8
10564	40	4	17	11	103	13.3	10.41	3.29	1	1.23223	0.774209	0.7742087	-0.2559138	1	451.5683	13
10567	41	2	16	11	56	3.4	0.9	1.01	0	-4.49501	0.011041	0.9889587	-0.0111027	0	616.8216	5
10746	23	4	0	2	118	6.4	0.29	7.26	0	-0.43671	0.392525	0.60747549	-0.4984434	0	499.7239	11
10775	45	5	1	4	58	15.4	2.37	6.57	1	0.892811	0.70947	0.70946989	-0.3432372	1	461.3619	12
10776	37	3	15	10	65	5	1.3	1.95	0	-3.85208	0.020794	0.97920609	-0.0210132	0	598.2706	6
10797	47	2	23	14	46	11.7	0.84	4.54	0	-5.55493	0.003853	0.99614655	-0.0038609	0	647.4043	3
10798	23	4	0	2	42	6.7	1.58	1.23	0	0.354067	0.587604	0.41239648	-0.8857701	1	476.9068	12
10800	18	2	0	0	19	5.2	0.32	0.67	1	-0.20441	0.449076	0.44907555	-0.8005641	1	493.0209	12
10802	19	2	0	0	21	2	0.28	0.14	1	-0.50728	0.375832	0.37583225	-0.9786124	0	501.7599	11
10805	41	1	3	12	23	10.6	0.15	2.29	1	-1.31623	0.211447	0.21144669	-1.5537824	0	525.1013	10
10824	42	2	3	12	76	5.2	0.72	3.23	1	-1.57755	0.171143	0.17114286	-1.7652566	0	532.6414	10
10839	29	4	2	3	45	3	0.67	0.68	0	-1.04152	0.260858	0.73914243	-0.3022646	0	517.1748	11
10870	24	4	0	2	41	8.4	0.79	2.66	0	0.069641	0.517403	0.48259682	-0.7285737	1	485.1136	12
10884	45	5	9	11	48	10.2	3.53	1.37	0	-1.04415	0.260351	0.73964937	-0.301579	0	517.2507	11
10887	39	4	8	9	259	7.2	1.55	17.1	0	-2.11689	0.107466	0.89253419	-0.1136905	0	548.2036	9
10893	40	2	15	6	46	10.6	0.82	4.06	0	-3.44406	0.030947	0.96905342	-0.0314355	0	586.4975	7
10927	21	3	0	0	17	17.6	1.37	1.62	1	1.289235	0.784018	0.78401763	-0.2433238	1	449.9236	13
10935	55	3	34	16	150	9.9	0.73	14.12	0	-8.49228	0.000205	0.999795	-0.000205	0	732.1583	1
11045	29	2	6	4	35	3.3	0.2	0.96	1	-2.12864	0.106344	0.10634375	-2.2410785	0	548.5427	9
11057	22	3	0	0	24	5.2	0.28	0.97	1	-0.3222	0.42014	0.42014047	-0.8671662	1	496.4196	11

	0.416154
	868.4176
	900

b0	b1	b2	b3	b4	b5	b6	b7
-0.4149156	-0.01531098	-0.02847339	-0.225346	-0.03866459	-0.00276	0.076746213	0.5576
			-0.6402615				
0.3977341			0.3451874				
			-5.255%				

Median	LB	UB
12.534	0	16.5803324
21.934	16.5803324	29.0155817
38.384	29.0155817	50.77726798
67.172	50.77726798	88.86021896
117.551	88.86021896	155.5053832
205.714	155.5053832	272.1344206
360.000	272.1344206	476.235236
630.000	476.235236	833.411663
1102.500	833.411663	1458.47041
1929.375	1458.47041	2552.323218
3376.406	2552.323218	4466.565631
5908.711	4466.565631	7686.813994
10000.000	7686.813994	10000

1.75

LB	UB	Grade	Score
0.00%	0.17%	1	671.80
0.17%	0.29%	2	655.61
0.29%	0.51%	3	639.40
0.51%	0.89%	4	623.15
0.89%	1.56%	5	606.80
1.56%	2.72%	6	590.31
2.72%	4.76%	7	573.55
4.76%	8.33%	8	556.30
8.33%	14.58%	9	538.12
14.58%	25.52%	10	518.02
25.52%	44.67%	11	493.30
44.67%	76.87%	12	452.47
76.87%	99.99%	13	221.37

20.2 Model Validation- CAP & AR

PEAKS2TAILS

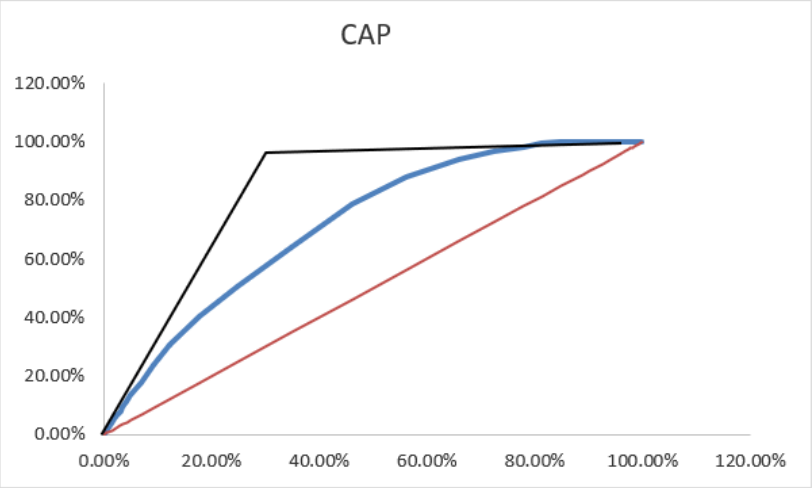
default	Score
0	473.3501
0	657.6858
0	637.049
0	479.8066
1	462.1581
1	491.7519
1	489.1464
0	490.2845
0	680.4791
0	498.5872
0	609.8296
1	484.8227
0	376.7929
0	499.1847
0	530.2679
1	479.9258
1	443.4159
1	522.7395
1	488.1538
0	662.9708
1	478.3565
0	533.9683
0	567.8739
1	451.5683
0	616.8216
0	499.7239
1	461.3619
0	598.2706
0	647.4043
0	476.9068
1	493.0209
1	501.7599
1	525.1013
1	532.6414
0	517.1748

Count	Good	Bad	Total
	571	329	900
67	63%	37%	

Validation

Bucket		Score	Cumulative Goods	Cumulative Bads	Cumulative Total	Cumulative f(x) Bad	Cumulative x Total	dx Incr total	f(x)dx Integral	For plot Bad	For plot Total
1	1%	60	0	1	1	0.30%	0.11%	0.11%		0.30%	0.11%
2	3%	72	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
3	4%	84	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
4	6%	96	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
5	7%	108	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
6	9%	120	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
7	10%	132	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
8	12%	144	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
9	13%	156	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
10	15%	168	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
11	16%	180	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
12	18%	192	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
13	19%	204	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
14	21%	216	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
15	22%	228	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
16	24%	240	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
17	25%	252	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
18	27%	264	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
19	28%	276	0	1	1	0.30%	0.11%	0.00%	-	0.30%	0.11%
20	30%	288	0	2	2	0.61%	0.22%	0.11%	0.00	0.61%	0.22%
21	31%	300	0	2	2	0.61%	0.22%	0.00%	-	0.61%	0.22%
22	33%	312	0	3	3	0.91%	0.33%	0.11%	0.00	0.91%	0.33%
23	34%	324	0	6	6	1.82%	0.67%	0.33%	0.00	1.82%	0.67%
24	36%	336	0	8	8	2.43%	0.89%	0.22%	0.00	2.43%	0.89%
25	37%	348	0	13	13	3.95%	1.44%	0.56%	0.00	3.95%	1.44%
26	39%	360	0	20	20	6.08%	2.22%	0.78%	0.00	6.08%	2.22%
27	40%	372	0	22	22	6.69%	2.44%	0.22%	0.00	6.69%	2.44%
28	42%	384	1	27	28	8.21%	3.11%	0.67%	0.00	8.21%	3.11%
29	43%	396	1	31	32	9.42%	3.56%	0.44%	0.00	9.42%	3.56%
30	45%	408	1	37	38	11.25%	4.22%	0.67%	0.00	11.25%	4.22%

CAPCAUC	0.72	AR
A_random	0.50	0.6827301
A_perfect	0.82	0.68273546



20.3 Model Validation- ROC & KS

PEAKS2TAILS

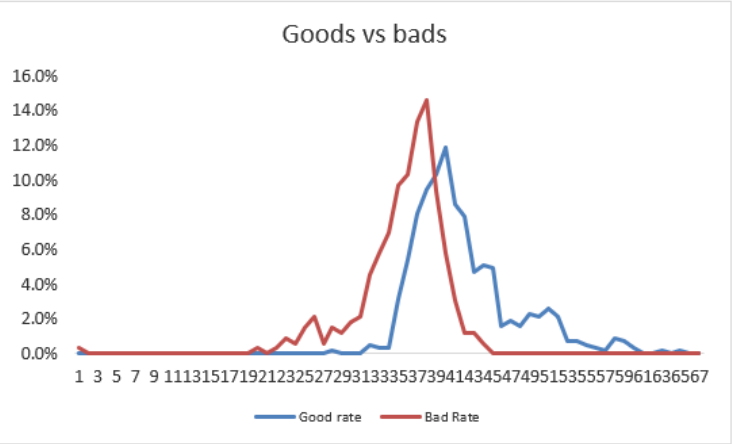
default	Score
0	473.3501
0	657.6858
0	637.049
0	479.8066
1	462.1581
1	491.7519
1	489.1464
0	490.2845
0	680.4791
0	498.5872
0	609.8296
1	484.8227
0	376.7929
0	499.1847
0	530.2679
1	479.9258
1	443.4159
1	522.7395
1	488.1538
0	662.9708
1	478.3565
0	533.9683
0	567.87
1	451.56
0	616.82
0	499.72
1	461.36
0	598.27
0	647.40
0	476.90
1	493.02
1	501.75
1	525.10
1	532.64
0	517.17
0	485.11
0	517.25
0	548.20

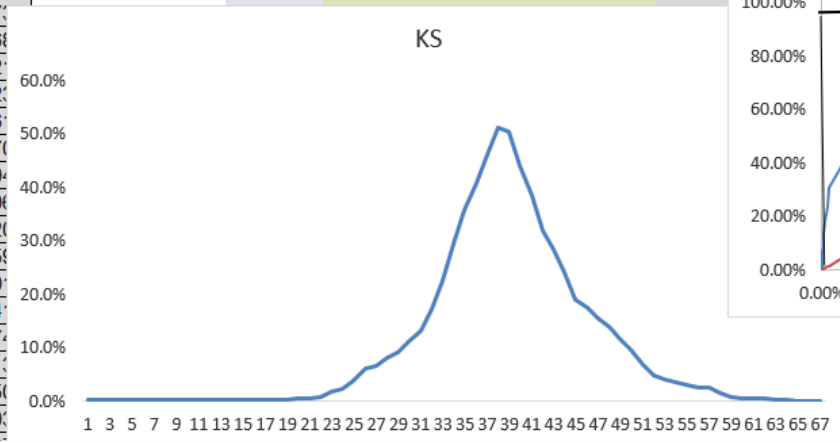
	Count	Good	Bad	Total
		571	329	900
	67	63%	37%	

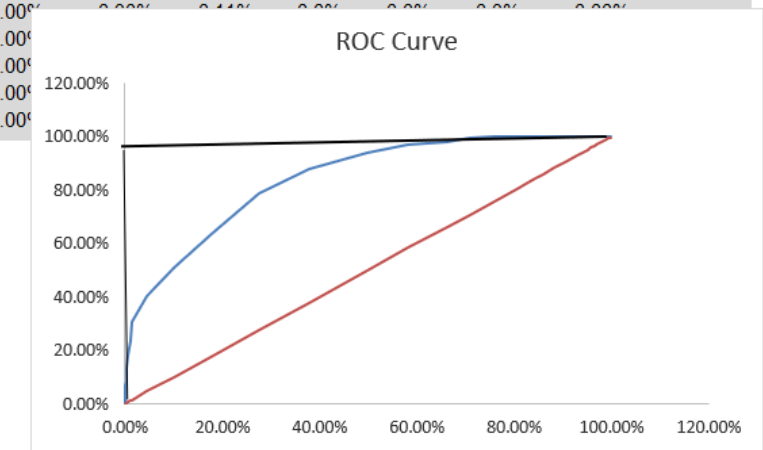
Validation

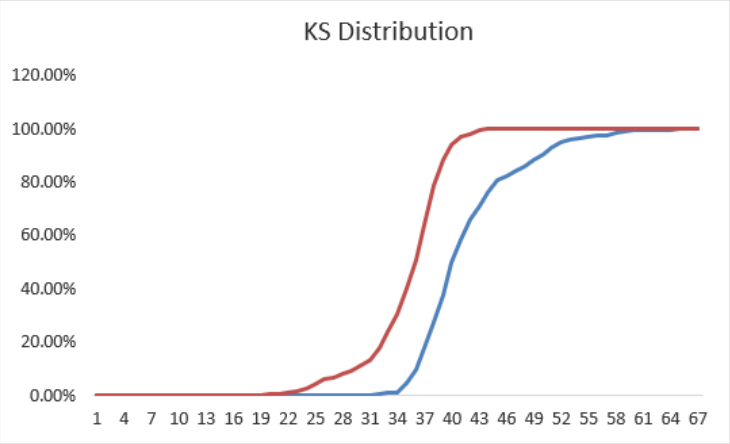
Bucket		Score	Cumulative Goods	Cumulative Bads	Cumulative Total	Cumulative prop % Good	f(x) Bad	x Total	Marginal % Good	Marginal % Bad	KS	dx Incr good	f(x)dx Integral
1	1%	60	0	1	1	0.00%	0.30%	0.11%	0.0%	0.3%	0.3%	0.00%	
2	3%	72	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
3	4%	84	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
4	6%	96	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
5	7%	108	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
6	9%	120	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
7	10%	132	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
8	12%	144	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
9	13%	156	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
10	15%	168	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
11	16%	180	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
12	18%	192	0	1	1	0.00%	0.30%	0.11%	0.0%	0.0%	0.3%	0.00%	-
13	19%	204	0	1	1	0.00							
14	21%	216	0	1	1	0.00							
15	22%	228	0	1	1	0.00							
16	24%	240	0	1	1	0.00							
17	25%	252	0	1	1	0.00							
							4.22%	0.0%	1.8%	11.1%	0.00%	-
							5.00%	0.0%	2.1%	13.2%	0.00%	-
							7.00%	0.5%	4.6%	17.2%	0.53%	0.00
							9.33%	0.4%	5.8%	22.7%	0.35%	0.00

ROCAUC	0.84	Gini
A_random	0.50	0.6827355
A_perfect	1.00	









20.4 Model Validation- Confusion Matrix

PEAKS2TAILS

A. default	Grade
0	12
0	2
0	4
0	12
1	12
1	12
1	12
0	12
0	1
0	11
0	5
1	12
0	13
0	11
0	10
1	12
1	13
1	10
1	12
0	2
1	12
0	10
0	8
1	13
0	5
0	11
1	12
0	6
0	3
0	12
1	12
1	11
1	10
1	10
0	11
0	12
0	11

Grade	TP	FP	FN	TN	FPR	TPR	FNR	BER	BER	KS
					0%	0%				
13	99	7	230	564	1%	30%	69.9%	0.26	0.26	29%
12	217	106	112	465	19%	66%	34.0%	0.24	0.24	47%
11	291	233	38	338	41%	88%	11.6%	0.30	0.30	48%
10	319	328	10	243	57%	97%	3.0%	0.38	0.38	40%
9	325	390	4	181	68%	99%	1.2%	0.44	0.44	30%
8	329	428	0	143	75%	100%	0.0%	0.48	0.48	25%
7	329	465	0	106	81%	100%	0.0%	0.52	0.52	19%
6	329	478	0	93	84%	100%	0.0%	0.53	0.53	16%
5	329	489	0	82	86%	100%	0.0%	0.54	0.54	14%
4	329	508	0	63	89%	100%	0.0%	0.56	0.56	11%
3	329	522	0	49	91%	100%	0.0%	0.58	0.58	9%
2	329	543	0	28	95%	100%	0.0%	0.60	0.60	5%
1	329	571	0	0	100%	100%	0.0%	0.63	0.63	0%
					100%	100%		0.242		48%

In our sample

Portion of Defaulte 0.365555556

		Predicted	
		1	0
Actual	1	TP	FN
	0	FP	TN

Confusion Matrix

11

		Predicted	
		1	0
Actual	1	291	38
	0	233	338

900

Accuracy	69.89%	$\frac{TP + TN}{N}$
Error rate	30.11%	$1 - accuracy$
Recall (TPR)	88.45%	$\frac{TP}{TP + FN}$

Disc. Power	F
Red	<63.8%
Amber	63.8% to 81.6%
Green	>81.6%

Precision	55.53%	$\frac{TP}{TP + FP}$	
Sensitivity (TPR)	88.45%	$\frac{TP}{TP + FN}$	0.59194
Specificity (TNR)	59.19%	$\frac{TN}{TN + FP}$	
F - Measure	68.23%	$\frac{(2 * Recall * Precision)}{Recall + Precision}$	

20.5 Model Validation- IV & KL Divergence

default	Grade
0	12
0	1
0	3
0	12
1	12
1	11
1	11
0	11
0	1
0	11
0	4
1	11
0	13
0	11
0	9
1	11
1	12
1	10
1	11
0	2
1	11
0	9
0	7
1	12
0	4
0	11
1	12
0	5
0	3
0	11
1	11
1	11
1	10
1	9
0	10
0	11
0	10

Grade	Bads	Goods	Total	ODR	Entropy	Marginal Bad %	Marginal Good %	IV	
1	0	41	41	0.0%	0.00	0.0%	7.2%	0.00	
2	0	14	14	0.0%	0.00	0.0%	2.5%	0.00	
3	0	18	18	0.0%	0.00	0.0%	3.2%	0.00	
4	0	14	14	0.0%	0.00	0.0%	2.5%	0.00	
5	0	14	14	0.0%	0.00	0.0%	2.5%	0.00	
6	0	27	27	0.0%	0.00	0.0%	4.7%	0.00	
7	2	39	41	4.9%	0.28	0.6%	6.8%	0.15	
8	7	46	53	13.2%	0.56	2.1%	8.1%	0.08	
9	14	88	102	13.7%	0.58	4.3%	15.4%	0.14	
10	48	108	156	30.8%	0.89	14.6%	18.9%	0.01	
11	104	116	220	47.3%	1.00	31.6%	20.3%	0.05	
12	106	44	150	70.7%	0.87	32.2%	7.7%	0.35	0.35
13	48	2	50	96.0%	0.24	14.6%	0.4%	0.53	
	329	571	900	36.6%	0.95				

Uncoditional Entropy	94.7%
Conditional Entropy	66.9%
Kullback-Leibler Divergence	27.9%
CIER	29.4%
Information Value	1.32

Disc. Power	IV
Red	<0.3
Amber	0.3 to 0.8
Green	>0.8

20.6 Model Validation- Somer's D & Kendall Tau

PEAKS2TAILS

custom	Default	PD	Filter
10012	0	0.61712	1
10017	0	0.0027	1
10030	0	0.00551	1
10039	0	0.56305	1
10069	1	0.70375	1
10071	1	0.45998	1
10096	1	0.48248	1
10128	0	0.47264	1
10129	0	0.00123	1
10140	0	0.40196	1
10142	0	0.01403	1
10169	1	0.51992	1
10197	0	0.97862	1
10200	0	0.39699	1
10218	0	0.18313	1
10234	1	0.56204	1
10351	1	0.81977	1
10423	1	0.22542	1
10428	1	0.49107	1
10449	0	0.00225	1
10455	1	0.57538	1
10474	0	0.16472	1
10525	0	0.0574	1
10564	1	0.77421	1
10567	0	0.01104	1
10746	0	0.39252	1
10775	1	0.70947	1
10776	0	0.02079	1
10797	0	0.00385	1
10798	0	0.5876	1
10800	1	0.44908	1
10802	1	0.37583	1
10805	1	0.21145	1
10824	1	0.17114	1
10839	0	0.26086	1
10870	0	0.5174	1
10884	0	0.26035	1
10887	0	0.10747	1
10893	0	0.03095	1
10927	1	0.78402	1
10935	0	0.00021	1
11045	1	0.10634	1
11057	1	0.42014	1
11080	0	0.00289	1
11199	0	0.01452	1
11267	0	0.16169	1
11288	0	0.22771	1
11307	1	0.82811	1
11329	1	0.91167	1
11337	0	0.00356	1
11344	1	0.47147	1
11363	1	0.83809	1

Total Pairs	24976	24976		
Concordant	9104			
Discordant	1840	ty	tx	
Neither	14032	14032	0	0

Gamma	0.66374	Gamma ignores all tied pairs of cases and tends to exaggerate the real strength of association
Kendall's tau- a	**0.29084**	Includes all tied cases, this is over conservative
Somer's D_Y	0.29084	Predicting Y from X (tied on Y)
Somer's D_X	0.66374	Predicting X from Y (tied on X)
Kendall's tau-b	**0.43937**	0.43937

Disc. Power	Somers'D
Red	<60%
Amber	60%-70%
Green	>70%

	Y	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
Y	X	0.61712	0.0027	0.00551	0.56305	0.70375	0.45998	0.482475534	0.47264	0.00123	0.40196	0.01403	0.51992	0.97862	0.39699	0.1831292	0.56204	0.81977	0.22542	0.49107	0.00225	0.57538	0.16472	0.0574	0.77421	0.01104
0	0.617118	0	0	0	0	1	-1	-1	0	0	0	0	-1	0	0	0	-1	1	-1	-1	0	-1	0	0	1	0
0	0.002701	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.005508	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.563054	0	0	0	0	1	-1	-1	0	0	0	0	-1	0	0	0	-1	1	-1	-1	0	1	0	0	1	0
1	0.70375	1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
1	0.45998	-1	1	1	-1	0	0	0	-1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
1	0.482476	-1	1	1	-1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.472635	0	0	0	0	1	-1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
0	0.001228	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.401957	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
0	0.014026	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.51992	-1	1	1	-1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.978621	0	0	0	0	-1	-1	-1	0	0	0	0	-1	0	0	0	-1	-1	-1	-1	0	-1	0	0	-1	0
0	0.396989	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
0	0.183129	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.562038	-1	1	1	-1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
1	0.819767	1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
1	0.225417	-1	1	1	-1	0	0	0	-1	1	-1	1	0	-1	-1	1	0	0	0	0	1	0	1	1	0	1
1	0.49107	-1	1	1	-1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.00225	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.575377	-1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.164718	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.057399	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.774209	1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.011041	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.392525	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
1	0.70947	1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.020794	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.003853	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.587604	0	0	0	0	1	-1	-1	0	0	0	0	-1	0	0	0	-1	1	-1	-1	0	-1	0	0	1	0
1	0.449076	-1	1	1	-1	0	0	0	-1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
1	0.375832	-1	1	1	-1	0	0	0	-1	1	-1	1	0	-1	-1	1	0	0	0	0	1	0	1	1	0	1
1	0.211447	-1	1	1	-1	0	0	0	-1	1	-1	1	0	-1	-1	1	0	0	0	0	1	0	1	1	0	1
1	0.171143	-1	1	1	-1	0	0	0	-1	1	-1	1	0	-1	-1	-1	0	0	0	0	1	0	1	1	0	1
0	0.260858	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
0	0.517403	0	0	0	0	1	-1	-1	0	0	0	0	1	0	0	0	1	1	-1	-1	0	1	0	0	1	0
0	0.260351	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	-1	1	0	1	0	0	1	0
0	0.107466	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
0	0.030947	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.784018	1	1	1	1	0	0	0	1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.000205	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0
1	0.106344	-1	1	1	-1	0	0	0	-1	1	-1	1	0	-1	-1	-1	0	0	0	0	1	0	-1	1	0	1
1	0.42014	-1	1	1	-1	0	0	0	-1	1	1	1	0	-1	1	1	0	0	0	0	1	0	1	1	0	1
0	0.002889	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	1	0

20.7 Model Validation- Stability Test

I	K	L	M	N	O	P	Q
othde	Y	Sigmo	Likelihc	LL	Default,	Scor	Grade
3.66	0.586544	0.642572	0.642572	-0.44228	1	470.1989	12
3.73	0.131718	0.532882	0.467118	-0.76117	1	483.3224	12
0.61	-0.66891	0.33874	0.33874	-1.08252	0	506.4238	11
2.55	1.482958	0.815019	0.815019	-0.20454	1	444.3339	13
14.96	0.501401	0.622789	0.622789	-0.47355	1	472.6556	12
0.88	-0.30881	0.423405	0.423405	-0.85943	1	496.0334	11
0.68	-0.23033	0.44267	0.55733	-0.5846	1	493.769	11
10.08	-2.38152	0.084593	0.915407	-0.08839	0	555.8391	9
1.93	0.084692	0.52116	0.52116	-0.6517	1	484.6793	12
1.8	-2.9044	0.051936	0.948064	-0.05333	0	570.9263	8
2.51	-3.52701	0.028553	0.971447	-0.02897	0	588.891	7
1.4	3.366597	0.966644	0.966644	-0.03392	1	389.9835	13
2.03	0.268153	0.56664	0.43336	-0.83619	1	479.3857	12
2.25	0.027428	0.506856	0.493144	-0.70695	1	486.3316	12
1.87	0.468191	0.614955	0.614955	-0.48621	1	473.6139	12
3.79	0.260182	0.564681	0.435319	-0.83168	1	479.6157	12
1.08	-0.26786	0.433432	0.433432	-0.83602	1	494.8518	11
2.48	-0.96571	0.275737	0.724263	-0.3226	0	514.9874	11
12.38	3.549213	0.972056	0.972056	-0.02834	1	384.7144	13
6.86	-3.80362	0.021804	0.978196	-0.02205	0	596.8722	6
1.15	-0.99001	0.270909	0.729091	-0.31596	0	515.6888	11
6.93	1.809174	0.859262	0.859262	-0.15168	1	434.9213	13
2.73	0.137739	0.53438	0.53438	-0.62665	1	483.1487	12
3.26	3.927112	0.98068	0.98068	-0.01951	1	373.8105	13
3.67	1.550583	0.824998	0.824998	-0.19237	1	442.3826	13
2.43	-4.36668	0.012534	0.987466	-0.01261	0	613.1187	5
14.88	0.100333	0.525062	0.525062	-0.64424	1	484.228	12
9.12	-5.616	0.003626	0.996374	-0.00363	0	649.1665	3
4.78	-0.51301	0.374487	0.625513	-0.46918	0	501.9255	11
7.44	1.910203	0.871042	0.871042	-0.13807	1	432.0062	13
1.6	-1.85887	0.134834	0.134834	-2.00371	0	540.7587	9
1.42	0.303352	0.575262	0.575262	-0.55293	1	478.3701	12
1.28	-2.9077	0.051774	0.948226	-0.05316	0	571.0214	8
7.64	0.045142	0.511284	0.511284	-0.67083	1	485.8205	12
0.68	-1.06416	0.256515	0.743485	-0.29641	0	517.8282	11
2.2	0.573971	0.639679	0.360321	-1.02076	1	470.5617	12
0.49	0.06949	0.517365	0.517365	-0.65901	1	485.1179	12
2.24	-2.12698	0.106502	0.893498	-0.11261	0	548.4946	9
1.09	-0.63947	0.345367	0.654633	-0.42368	0	505.5741	11
63.47	3.1743	0.959856	0.959856	-0.04097	1	395.5321	13
0.87	-0.52263	0.372237	0.372237	-0.98823	0	502.203	11
1.04	-0.28502	0.429223	0.570777	-0.56076	1	495.347	11
1.66	-0.30251	0.424943	0.575057	-0.55329	1	495.8517	11

T	U	V	W	X	Y	Z	AA	AB	AC
b0	b1	b2	b3	b4	b5	b6	b7	b8	Sum of LL
-0.41492	-0.015311	-0.02847	-0.22534598	-0.038664594	-0.00275587	0.076746213	0.557631	0.026705	-408.918

Grade	N_{DEV}	N_{VAL}	N_{DEV} %	N_{VAL} %	N_{DEV} - N_{VAL}	$\ln(N_{DEV}/N_{VAL})$	PSI
13	106	55	11.8%	9.17%	2.61%	25.06%	0.01
12	217	156	24.1%	26.00%	-1.89%	-7.54%	0.00
11	201	145	22.3%	24.17%	-1.83%	-7.89%	0.00
10	123	84	13.7%	14.00%	-0.33%	-2.41%	0.00
9	68	45	7.6%	7.50%	0.06%	0.74%	0.00
8	42	25	4.7%	4.17%	0.50%	11.33%	0.00
7	37	23	4.1%	3.83%	0.28%	7.00%	0.00
6	13	12	1.4%	2.00%	-0.56%	-32.54%	0.00
5	11	9	1.2%	1.50%	-0.28%	-20.48%	0.00
4	19	8	2.1%	1.33%	0.78%	45.95%	0.00
3	14	12	1.6%	2.00%	-0.44%	-25.13%	0.00
2	21	6	2.3%	1.00%	1.33%	84.73%	0.01
1	28	20	3.1%	3.33%	-0.22%	-6.90%	0.00
	900	600					0.03
		n	13				
		df	12				
		Xd	0%				
		Sd	1%				
		T.stat	0.00				
		p	1				
		Null	Accept	T Test on Means of difference			

Disc. Power	PSI
Red	>0.25
Amber	0.10-0.25
Green	<0.10

Age	N_{DEV}	N_{VAL}	N_{DEV} %	N_{VAL} %	N_{DEV} - N_{VAL}	$\ln(N_{DEV}/N_{VAL})$	CSI
25	246	172	27.3%	28.7%	-0.013	-0.048	0.001
30	149	106	16.6%	17.7%	-0.011	-0.065	0.001
35	128	92	14.2%	15.3%	-0.011	-0.075	0.001
40	101	65	11.2%	10.8%	0.004	0.035	0.000
45	80	58	8.9%	9.7%	-0.008	-0.084	0.001
50	67	29	7.4%	4.8%	0.026	0.432	0.011
55	43	26	4.8%	4.3%	0.004	0.098	0.000
60	31	17	3.4%	2.8%	0.006	0.195	0.001
65	25	16	2.8%	2.7%	0.001	0.041	0.000
70	17	8	1.9%	1.3%	0.006	0.348	0.002
75	9	7	1.0%	1.2%	-0.002	-0.154	0.000
80	4	4	0.4%	0.7%	-0.002	-0.405	0.001
	900	600					0.02

20.8 Model Validation- Calibration Tests

default	Grade
0	12
0	2
0	4
0	12
1	12
1	12
1	12
0	12
0	1
0	11
0	5
1	12
0	13
0	11
0	10
1	12
1	13
1	10
1	12
0	2
1	12
0	10
0	8
1	13
0	5
0	11
1	12
0	6
0	3
0	12
1	12
1	11
1	10
1	10
0	11
0	12
0	11
0	9
0	7
1	13
0	1
1	9
1	11
0	2
0	5
0	10
0	10
1	13
1	13
0	3
1	12
1	13

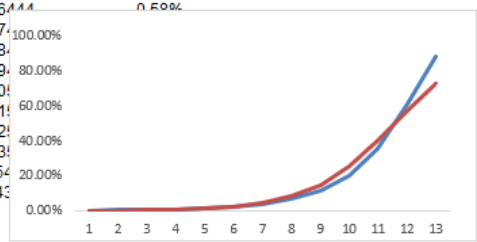
CI	95%		
CI	99%		

Estimated PD	37.04%	W.A. PD	36.56%
Observed DR	36.56%	Error	0.00

Bucket	Goods	Bads	Total	ODR
1	28	0	28	0.00%
2	21	0	21	0.00%
3	14	0	14	0.00%
4	19	0	19	0.00%
5	11	0	11	0.00%
6	13	0	13	0.00%
7	37	0	37	0.00%
8	38	4	42	9.52%
9	62	6	68	8.82%
10	95	28	123	22.76%
11	127	74	201	36.82%
12	99	118	217	54.38%
13	7	99	106	93.40%

Bucket	PD	Log Odds	Shifted	PD
1	0.08%	-7.0944	-7.180553391	0.08%
2	0.23%	-6.0814	-6.501604408	0.15%
3	0.40%	-5.5201	-5.822655426	0.30%
4	0.70%	-4.9574	-5.1437064	
5	1.22%	-4.3925	-4.464757	
6	2.14%	-3.8236	-3.785808	
7	3.74%	-3.2475	-3.106859	
8	6.55%	-2.6582	-2.427910	
9	11.46%	-2.0446	-1.748961	
10	20.05%	-1.3829	-1.070012	
11	35.09%	-0.6149	-0.391063	
12	60.77%	0.43752	0.2878854	
13	88.43%	2.0337	0.9668343	

Slope	0.67895
Alpha	-7.8595



Binomial — Non Calibrated

Bucket	Observed	Model PD	n	k* @ 95%	k* @ 99%	Calibration @ 95%	Calibration @ 99%	RAG
1	0.00	0.08%	28	0.00	1.00	correct	correct	Green
2	0.00	0.23%	21	0.00	1.00	correct	correct	Green
3	0.00	0.40%	14	1.00	1.00	correct	correct	Green
4	0.00	0.70%	19	1.00	1.00	correct	correct	Green
5	0.00	1.22%	11	1.00	1.00	correct	correct	Green
6	0.00	2.14%	13	1.00	2.00	correct	correct	Green
7	0.00	3.74%	37	3.00	5.00	correct	correct	Green
8	4.00	6.55%	42	6.00	7.00	correct	correct	Green
9	6.00	11.46%	68	12.00	14.00	correct	correct	Green
10	28.00	20.05%	123	32.00	35.00	correct	correct	Green
11	74.00	35.09%	201	82.00	86.00	correct	correct	Green
12	118.00	60.77%	217	144.00	148.00	correct	correct	Green
13	99.00	88.43%	106	99.00	101.00	correct	correct	Green

Normal

Bucket	Observed	Model PD	n	k* @ 95%	k* @ 99%	Calibration @ 95%	Calibration @ 99%	RAG
1	0.00	0.08%	28	0.27	0.38	correct	correct	Green
2	0.00	0.23%	21	0.41	0.56	correct	correct	Green
3	0.00	0.40%	14	0.44	0.60	correct	correct	Green
4	0.00	0.70%	19	0.73	0.98	correct	correct	Green
5	0.00	1.22%	11	0.73	0.98	correct	correct	Green
6	0.00	2.14%	13	1.14	1.49	correct	correct	Green
7	0.00	3.74%	37	3.28	4.07	correct	correct	Green
8	4.00	6.55%	42	5.39	6.48	correct	correct	Green
9	6.00	11.46%	68	12.11	13.90	correct	correct	Green
10	28.00	20.05%	123	31.97	35.00	correct	correct	Green
11	74.00	35.09%	201	81.67	86.28	correct	correct	Green
12	118.00	60.77%	217	143.70	148.60	correct	correct	Green
13	99.00	88.43%	106	99.15	101.40	correct	correct	Green

Ho : Model PD is correct
Ha : Model PD is underestimated
k>k* (Stat > crit)
We can use Confidence Interval Method
Create CI around Model PD and check whether default rates lies within it

Disc. Powe	Binomial	Normal	Vasicek	HL
Red	ODR > WCD @ 99%	ODR > WCD @ 99%	ODR > WCD @ 99%	p values < 0.01
Amber	WCD @ 95% < ODR < WCD @ 99%	D @ 95% < ODR < WCD @	@ 95% < ODR < WCD	0.01 < p values < 0.05
Green	ODR < WCD @ 95%	ODR < WCD @ 95%	ODR < WCD @ 95%	p values > 0.05

20.9 Model Validation- Brier Score

default	Grade	Estimated PD	$(p - j)^2$
0	12	0.607668981	0.369261591
0	2	0.002279796	5.19747E-06
0	4	0.006981874	4.87466E-05
0	12	0.607668981	0.369261591
1	12	0.607668981	0.153923628
1	12	0.607668981	0.153923628
1	12	0.607668981	0.153923628
0	12	0.607668981	0.369261591
0	1	0.000829017	6.87269E-07
0	11	0.350944442	0.123162002
0	5	0.01221828	0.000149286
1	12	0.607668981	0.153923628
0	13	0.8842907	0.781970042
0	11	0.350944442	0.123162002
0	10	0.200539681	0.040216164
1	12	0.607668981	0.153923628
1	13	0.8842907	0.013388642
1	10	0.200539681	0.639136801
1	12	0.607668981	0.153923628
0	2	0.002279796	5.19747E-06
1	12	0.607668981	0.153923628
0	10	0.200539681	0.040216164
0	8	0.065482345	0.004287938
1	13	0.8842907	0.013388642
0	5	0.01221828	0.000149286
0	11	0.350944442	0.123162002
1	12	0.607668981	0.153923628
0	6	0.02138199	0.00045719
0	3	0.003989642	1.59172E-05
0	12	0.607668981	0.369261591
1	12	0.607668981	0.153923628
1	11	0.350944442	0.421273117
1	10	0.200539681	0.639136801
1	10	0.200539681	0.639136801
0	11	0.350944442	0.123162002
0	12	0.607668981	0.369261591
0	11	0.350944442	0.123162002
0	9	0.114594104	0.013131809
0	7	0.037418483	0.001400143
1	13	0.8842907	0.013388642
0	1	0.000829017	6.87269E-07
1	9	0.114594104	0.783943601
1	11	0.350944442	0.421273117

Brier Score
0.154762266

20.10 Jeffrey's Prior Test

For a given rating category

True PD	π	
Estimated PD	0.03%	(Any LDP method(Pluto Tasche) or regulatory minimum)
Number of Obligors (n)	6	
Number of Defaults (d)	0	

Jeffrey's prior $\pi \sim Beta\left(\frac{1}{2}, \frac{1}{2}\right)$ (link for proof)

Likelihood of Default $P(D|\pi) \sim Bernouli(\pi)$

Posterior $P(\pi|D) \sim Beta\left(\frac{1}{2} + d, \quad \frac{1}{2} + n - d\right)$

	α	β
Jeffrey's prior	0.5	0.5
Posterior	0.5	6.5

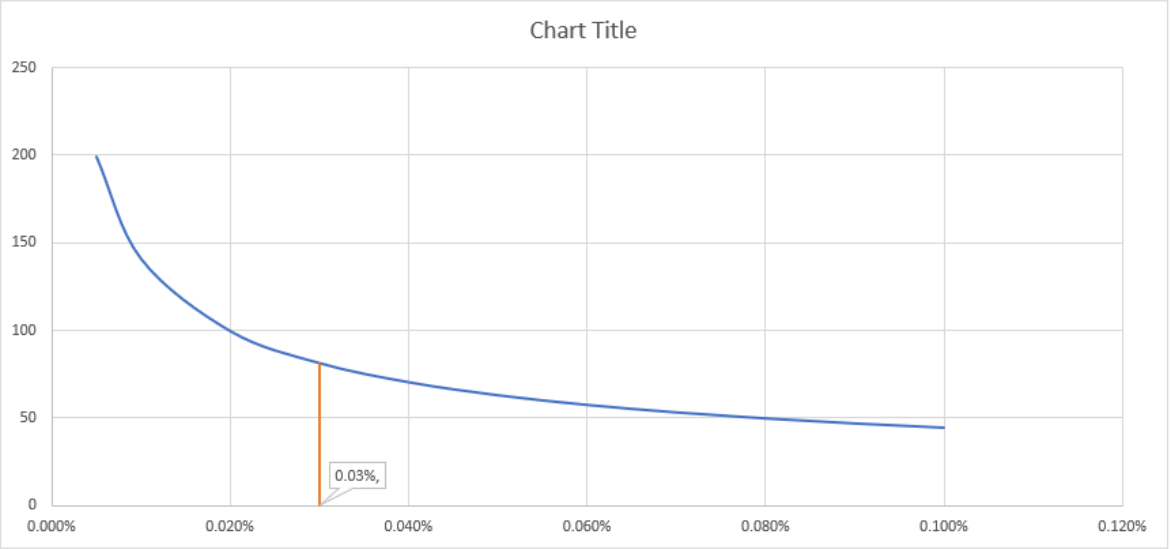
Our True PD comes from a distribution Beta(0.5, 6.5)

For LDP, we want to come up with a conservative estimate of PD

$H_0: \pi \leq \widehat{PD}$

$H_a: \pi > \widehat{PD}$

Reject the null if there is more than 95% probability of True PD exceeding the Estimated PD



π	P(π | D)
0.005%	199.4958
0.01%	141.026
0.02%	99.66561
0.03%	81.33187
0.04%	70.39673
0.05%	62.93011
0.06%	57.41546
0.07%	53.12714
0.08%	49.66855
0.09%	46.80218
0.10%	44.37601
0.11%	42.28758
0.12%	40.46499
0.13%	38.8561

Area to the left **0.048853** < 0.05 **Fail**

21.1 Margin of Conservatism type A

PEAKS2TAILS

PD - Adjusment Amount and Best Estimate (CAT A)

Baseline

				Cat A (Trigger 1)							Cat A (Trigger 2)							Cat A (			
Year ↓	**D**	**N**	**DR**	**d(1)**	**m(1)**	β^1	α^1	**AA(1)**	**DR(1)** direct	**DR(1)** Indirect	**d(2)**	**m(2)**	β^2	α^2	**AA(2)**	**DR(2)** direct	**DR(2)** Indirect	**d(3)**	**m(3)**	β^3	
1	6	1817	0.330%	1	-68	0.166667	-0.037424326	1.21202592	0.400%	0.400%	1	190	0.166667	0.104568	1.249441	0.413%	0.413%	1	181	0.166667	0.0
2	29	1673	1.733%	5	-188	0.172414	-0.112372983	1.32084059	2.290%	2.290%	2	54	0.068966	0.032277	1.349466	2.339%	2.339%	-5	166	-0.17241	0.0
3	8	1653	0.484%	3	169	0.375	0.102238355	1.247461581	0.604%	0.604%	1	-85	0.125	-0.05142	1.427461	0.691%	0.691%	5	-47	0.625	-0.
4	2	1761	0.114%	5	99	2.5	0.056218058	3.313709677	0.376%	0.376%	-3	74	-1.5	0.042022	1.821096	0.207%	0.207%	-2	-61	-1	-0.
5	**0**	**1881**	**0.000%**	**2**	**128**	**0**	**0**	**1**	**0.100%**	**0.000%**	**3**	**-169**	**1.5**	**-0.08412**	**2.72962**	**0.272%**	**0.272%**	0	**34**	0	0.0
6	16	1582	1.011%	-2	-75	-0.125	-0.047408344	0.918546782	0.929%	0.929%	3	-178	0.1875	-0.11252	1.264767	1.279%	1.279%	-4	-86	-0.25	-0.
7	28	1732	1.617%	4	65	0.142857	0.037528868	1.101518404	1.781%	1.781%	2	25	0.071429	0.014434	1.154305	1.866%	1.866%	-4	79	-0.14286	0.0

Best Estimate Incorporating

Baseline for low default portfolio

Year ↓	$\widetilde{DR}^1$	$\widetilde{DR}^2$	$\widetilde{DR}^3$
1	0.400%	0.413%	0.425%
2	2.290%	2.339%	1.818%
3	0.604%	0.691%	1.006%
4	0.376%	0.207%	0.107%
5	0.100%	0.272%	0.267%
6	0.929%	1.279%	1.046%
7	1.781%	1.866%	1.578%

$DR_{long-run}$ 0.756%

$\widetilde{DR}_{long-run}$ 0.892%

21.1 Margin of Conservatism type B

PD - Adjusment Amount and Best Estimate (CAT B)

Judgemental decisions with regards to category B :

10% more defaults
negatively affects customer base by ***20%*** } **B1**

add ***15 bps*** to default central tendency } **B2**

β^{B1}	10%
α^{B1}	-20%
B2 (addon)	0.0015

Year ↓	DR	$\Sigma\beta^{A}$	$\Sigma\alpha^{A}$	$\widetilde{DR}^{A}$	β^{B1}	α^{B1}	$\widetilde{DR}^{A\to B1}$
1	0.33%	50.00%	16.68%	0.425%	10%	-20%	0.547%
2	1.73%	6.90%	1.91%	1.818%	10%	-20%	2.474%
3	0.48%	112.50%	2.24%	1.006%	10%	-20%	1.309%
4	0.11%	0.00%	6.36%	0.107%	10%	-20%	0.145%
5	0.00%	150.00%	-6.72%	0.267%	10%	-20%	0.353%
6	1.01%	-18.75%	-21.43%	1.046%	10%	-20%	1.576%
7	1.62%	7.14%	9.76%	1.578%	10%	-20%	2.110%
Long Run	**0.756%**			**0.892%**			**1.216%**

Year ↓	DR	$\widetilde{mPD}^{A,B}$
1	0.33%	0.597%
2	1.73%	3.134%
3	0.48%	0.875%
4	0.11%	0.205%
5	0.00%	0.000%
6	1.01%	1.829%
7	1.62%	2.923%
Long Run	**0.756%**	**1.366%**

$\widetilde{AA}_{long-run}$	**1.6095**
$\widetilde{RMoC}^{A,B1}{}_{long-run}$	**0.000**
$\widetilde{DR}^{A,B1,B2}{}_{long-run}$	**1.366%**
$\widetilde{RMoC}^{B2}{}_{long-run}$	**0.198518702**

(indicates that the conservatism is embedded in $\widetilde{AA}_{long-run}$)

$$\widetilde{\mathrm{mRP}}_{\mathrm{MoC}} = \mathrm{mRP} \cdot \mathrm{AA}_{A,B}^{long-run} + \mathrm{mRP} \cdot \left(\mathrm{RMoC}_{A,B}^{long-run} + \mathrm{RMoC}_{C}^{long-run}\right)$$ **1.366%**

21.1 Margin of Conservatism type C – rank ordering error

PEAKS2TAILS

PD - Adjusment Amount and Best Estimate (CAT C) - rank ordering error

customer	Default,P	PD	Filter
10012	0	0.617117684	1
10017	0	0.002701466	1
10030	0	0.005507952	1
10039	0	0.563053909	1
10069	1	0.703749817	1
10071	1	0.459979846	1
10096	1	0.482475534	1
10128	0	0.472635457	1
10129	0	0.001227912	1
10140	0	0.401956613	1
10142	0	0.014026414	1
10169	1	0.519919605	1
10197	0	0.978621273	1
10200	0	0.396988654	1
10218	0	0.183129219	1
10234	1	0.562037514	1
10351	1	0.819767195	1
10423	1	0.225416527	1
10428	1	0.491069925	1
10449	0	0.002250344	1
10455	1	0.575377203	1
10474	0	0.164718135	1
10525	0	0.057399096	1
10564	1	0.774208705	1
10567	0	0.011041302	1
10746	0	0.392524507	1
10775	1	0.709469894	1
10776	0	0.020793913	1
10797	0	0.003853446	1
10798	0	0.587603516	1
10800	1	0.449075554	1
10802	1	0.375832253	1
10805	1	0.211446685	1
10824	1	0.171142861	1
10839	0	0.260857569	1
10870	0	0.517403183	1
10884	0	0.260350625	1
10887	0	0.107465806	1
10893	0	0.030946581	1
10927	1	0.784017629	1
10935	0	0.000205004	1
11045	1	0.106343754	1
11057	1	0.420140468	1
11080	0	0.002889144	1

$$\epsilon_{c_1} = \frac{2 \cdot \sum^{n^{dis}} |\delta \hat{p}|}{\sqrt{[n(n-1) - \sum_i t_i(t_i-1)][n(n-1) - \sum_j u_j(u_j-1)]}} = 2.159\%$$

Total Pairs	24976
Concordan	9104
Discordant	1840
Neither	14032
Kendall's t	0.439366

	Y	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	1	1	1	1
Y	X	0.617118	0.002701	0.005508	0.563054	0.70375	0.45998	0.482475534	0.472635	0.001228	0.401957	0.014026	0.51992	0.978621	0.396989	0.183129	0.562038	0.819767	0.225417	0.49107
0	0.6171177						0.157138	0.13464215					0.097198				0.05508		0.391701	0.126048
0	0.0027015																			
0	0.005508																			
0	0.5630539						0.103074	0.080578375					0.043134				0.001016		0.337637	0.071984
1	0.7037498													0.274871						
1	0.4599798	0.157138			0.103074				0.012656					0.518641						
1	0.4824755	0.134642			0.080578									0.496146						
0	0.4726355						0.012656												0.247219	
0	0.0012279																			
0	0.4019566																		0.17654	
0	0.0140264																			
1	0.5199196	0.097198			0.043134									0.458702						
0	0.9786213					0.274871	0.518641	0.496145739					0.458702				0.416584	0.158854	0.753205	0.487551
0	0.3969887																		0.171572	
0	0.1831292																			
1	0.5620375	0.05508			0.001016									0.416584						
1	0.8197672													0.158854						
1	0.2254165	0.391701			0.337637				0.247219		0.17654			0.753205	0.171572					
1	0.4910699	0.126048			0.071984									0.487551						
0	0.0022503																			
1	0.5753772	0.04174												0.403244						
0	0.1647181																			
0	0.0573991																			
1	0.7742087													0.204413						
0	0.0110413																			
0	0.3925245																		0.167108	
1	0.7094699													0.269151						
0	0.0207939																			
0	0.0038534																			
0	0.5876035						0.127624	0.105127982					0.067684				0.025566		0.362187	0.096534
1	0.4490756	0.168042			0.113978				0.02356					0.529546						
1	0.3758323	0.241285			0.187222				0.096803		0.026124			0.602789	0.021156					
1	0.2114467	0.405671			0.351607				0.261189		0.19051			0.767175	0.185542					
1	0.1711429	0.445975			0.391911				0.301493		0.230814			0.807478	0.225846	0.011986				
0	0.2608576																		0.035441	
0	0.5174032						0.057423	0.034927649											0.291987	0.026333

21.1 Margin of Conservatism type C – general estimation error

PEAKS2TAILS

PD - Adjusment Amount and Best Estimate (CAT C) - general estimation error

customer	PD	Grade
10012	0.617118	11
10017	0.002701	2
10030	0.005508	3
10039	0.563054	11
10069	0.70375	12
10071	0.45998	11
10096	0.482476	11
10128	0.472635	11
10129	0.001228	1
10140	0.401957	11
10142	0.014026	5
10169	0.51992	11
10197	0.978621	13
10200	0.396989	11
10218	0.183129	9
10234	0.562038	11
10351	0.819767	12
10423	0.225417	10
10428	0.49107	11
10449	0.00225	1
10455	0.575377	11
10474	0.164718	9
10525	0.057399	7
10564	0.774209	12
10567	0.011041	4
10746	0.392525	11
10775	0.70947	12
10776	0.020794	5
10797	0.003853	2
10798	0.587604	11
10800	0.449076	11
10802	0.375832	11
10805	0.211447	10
10824	0.171143	9
10839	0.260858	10
10870	0.517403	11
10884	0.260351	10

Row Labels	Average of PD	Count of customer
1	0.000910084	39
2	0.00312056	20
3	0.005589166	14
4	0.009126357	13
5	0.016181203	16
6	0.031831109	27
7	0.051780923	40
8	0.09269071	56
9	0.161248934	87
10	0.273802867	167
11	0.472243437	235
12	0.749484115	140
13	0.971001328	46
Grand Total	**0.365555369**	**900**

Grade	Grade PD	Obligors	w^2	σ^2
1	0.09%	39	0.001877778	0.002%
2	0.31%	20	0.000493827	0.016%
3	0.56%	14	0.000241975	0.040%
4	0.91%	13	0.000208642	0.070%
5	1.62%	16	0.000316049	0.099%
6	3.18%	27	0.0009	0.114%
7	5.18%	40	0.001975309	0.123%
8	9.27%	56	0.003871605	0.150%
9	16.12%	87	0.009344444	0.155%
10	27.38%	167	0.034430864	0.119%
11	47.22%	235	0.068179012	0.106%
12	74.95%	140	0.024197531	0.134%
13	97.10%	46	0.002612346	0.061%
	Total	900		

Portfolio PD	**36.556%**	**DR =**	**0.756%**
SE (ϵ_{C2})	**1.311%**		
$\widetilde{RMoC}^{C1}{}_{long-run}$	2.85676632		
$\widetilde{RMoC}^{C2}{}_{long-run}$	1.73479939		

$$\widetilde{\mathrm{mRP}}_{\mathrm{MoC}} = \mathrm{mRP} \cdot \mathrm{AA}_{A,B}^{long-run} + \mathrm{mRP} \cdot \left(\mathrm{RMoC}_{A,B}^{long-run} + \mathrm{RMoC}_{C}^{long-run}\right)$$

3.469%

22.1 ML Discriminant Analysis

PEAKS2TAILS

Set Up

Class Split

Training Set

OBS#	DURATION	AMOUNT	EMPLOYMENT	AGE	JOB	RESPONSE
1	16	1169	4	67	2	1
2	58	5951	2	22	2	0
3	22	2096	3	49	1	1
4	52	7882	3	45	2	1
5	34	4870	2	53	2	0
6	46	9055	2	35	1	1
7	34	2835	4	53	2	1
8	46	6948	2	35	3	1
9	22	3059	3	61	1	1
10	40	5234	0	28	3	0
11	22	1295	1	25	2	0
12	58	4308	1	24	2	0
13	22	1567	2	22	2	1
14	34	1199	4	60	1	0
15	25	1403	2	28	2	1
16	34	1282	2	32	1	0
17	34	2424	4	53	2	1
18	40	8072	1	25	2	1
19	34	12579	4	44	3	0
20	34	3430	4	31	2	1
21	19	2134	2	48	2	1
22	16	2647	2	44	2	1
23	20	2241	1	48	1	1
24	22	1804	1	44	2	1
25	20	2069	2	26	2	1
26	16	1374	2	36	1	1
27	16	426	4	39	1	1
28	22	409	2	42	2	1
29	17	2415	2	34	2	1
30	70	6836	4	63	2	0
31	28	1913	1	36	2	1
32	34	4020	2	27	2	1

Total 207

Default Class (RESPONSE = 0)

OBS#	DURATION	AMOUNT	EMPLOYMENT	AGE	JOB
2	58	5951	2	22	2
5	34	4870	2	53	2
10	40	5234	0	28	3
11	22	1295	1	25	2
12	58	4308	1	24	2
14	34	1199	4	60	1
16	34	1282	2	32	1
19	34	12579	4	44	3
30	70	6836	4	63	2
36	55	4746	1	25	1
38	28	2100	2	37	2
45	58	6143	4	58	1
55	46	2225	4	57	2
57	22	6468	0	52	3
60	46	6229	1	23	1
63	46	1953	4	61	3
64	58	14421	2	25	2
69	46	1819	2	37	2
75	46	1977	4	40	3
77	52	3965	1	34	2
81	34	5943	1	44	2
88	46	12612	2	47	2
90	22	1108	3	28	2
93	22	797	4	33	1
96	64	15945	1	58	2
106	34	11938	2	39	3
107	28	6458	4	39	3
114	46	7855	2	25	2
117	52	7174	3	30	3
119	43	4281	2	23	2
121	31	1835	2	25	2
125	28	1924	1	27	2

Total 493

Non-Default Class (RESPONSE = 1)

OBS#	DURATION	AMOUNT	EMPLOYMENT	AGE	JOB
1	16	1169	4	67	2
3	22	2096	3	49	1
4	52	7882	3	45	2
6	46	9055	2	35	1
7	34	2835	4	53	2
8	46	6948	2	35	3
9	22	3059	3	61	1
13	22	1567	2	22	2
15	25	1403	2	28	2
17	34	2424	4	53	2
18	40	8072	1	25	2
20	34	3430	4	31	2
21	19	2134	2	48	2
22	16	2647	2	44	2
23	20	2241	1	48	1
24	22	1804	1	44	2
25	20	2069	2	26	2
26	16	1374	2	36	1
27	16	426	4	39	1
28	22	409	2	42	2
29	17	2415	2	34	2
31	28	1913	1	36	2
32	34	4020	2	27	2
33	28	5866	2	30	2
34	22	1264	4	57	1
35	22	1474	1	33	3
37	58	6110	2	31	2
39	20	1225	2	37	2
40	19	458	2	24	2
41	40	2333	4	30	3
42	22	1158	2	26	2
43	28	6204	2	44	1

Default Class

	DURATION	AMOUNT	AGE	
Means	**34.56521739**	**3802.048309**	**34**	|∑|
Covariance	DURATION	AMOUNT	AGE	Inv
DURATION	171.6853602	23442.93888	9.652174	0.00
AMOUNT	23442.93888	11503246.11	6448.541	-1.6
AGE	9.652173913	6448.541063	127.2271	0.00

Non-Default Class

	DURATION	AMOUNT	AGE	
Means	**29.01014199**	**2922.125761**	**35.8783**	|∑|
Covariance	DURATION	AMOUNT	AGE	Inv
DURATION	132.586104	18810.97236	-10.2422	0.01
AMOUNT	18810.97236	5471964.268	-1177.74	-5.0
AGE	-10.24217339	-1177.735202	127.2185	0.00

*from sheet 13

	Category (X)	No-Default	Default	P(X | D)	P (X
EMPLOYMENT	0	25	19	0.091787	0.05
	1	71	47	0.227053	0.14
	2	172	72	0.347826	0.34
	3	100	26	0.125604	0.20
	4	125	43	0.207729	0.25
Job	0	8	5	0.024155	0.01
	1	99	38	0.183575	0.20
	2	320	125	0.603865	0.64
	3	66	39	0.188406	0.13

22.2 ML Fisher's LDA

PEAKS2TAILS

Fisher's LDA

Only works with continuous features
Assumes homogeneous variances for different classes
Maximizes the ratio of between group and within group variances

$$S = \frac{\sigma^2_{\text{between}}}{\sigma^2_{\text{within}}} = \frac{(\vec{w}\cdot\vec{\mu}_1 - \vec{w}\cdot\vec{\mu}_0)^2}{\vec{w}^T\Sigma_1\vec{w} + \vec{w}^T\Sigma_0\vec{w}} = \frac{(\vec{w}\cdot(\vec{\mu}_1 - \vec{\mu}_0))^2}{\vec{w}^T(\Sigma_0 + \Sigma_1)\vec{w}}$$

$$\vec{w} \propto (\Sigma_0 + \Sigma_1)^{-1}(\vec{\mu}_1 - \vec{\mu}_0)$$

The threshold condition is found explicity $\vec{w}\cdot\vec{x} > c$

$$c = \vec{w}\cdot\frac{1}{2}(\vec{\mu}_0 + \vec{\mu}_1) = \frac{1}{2}\vec{\mu}_1^T\Sigma_1^{-1}\vec{\mu}_1 - \frac{1}{2}\vec{\mu}_0^T\Sigma_0^{-1}\vec{\mu}_0$$

	Default Class : Σ_0			Non-Default Class : Σ_1			$\Sigma_0 + \Sigma_1$		
	171.68536	23442.94	9.652174	132.586104	18810.97	-10.2422	304.2715	42253.91	-0.59
	23442.939	11503246	6448.541	18810.9724	5471964	-1177.74	42253.91	16975210	5270.806
	9.6521739	6448.541	127.2271	-10.242173	-1177.74	127.2185	-0.59	5270.806	254.4455
	DURTION	AMOUNT	AGE	DURTION	AMOUNT	AGE	$\mu_1 - \mu_0$		
Means	34.565217	3802.048	34	29.010142	2922.126	35.8783	-5.55508	-879.923	1.878296
							$\mu_1 + \mu_0$		
	-0.016376						63.57536	6724.174	69.8783
w	-1.34E-05								
	0.0076224								
c	-0.299415								

Validation Set

OBS#	DURATION	AMOUNT	EMPLOYMENT	AGE	JOB	RESPONSE	Score	Predicted Class
701	22	1123	2	29	1	0	-0.15431	1
702	58	6331	4	46	2	0	-0.68425	0
703	34	1377	4	47	2	1	-0.21703	1
704	40	2503	4	41	2	1	-0.37615	0
705	37	2528	1	32	2	1	-0.39596	0
706	25	5324	4	35	2	1	-0.21417	1
707	58	6560	3	24	2	0	-0.85502	0
708	22	2969	1	25	2	0	-0.20961	1
709	19	1206	4	25	2	1	-0.13679	1
710	19	2118	2	37	1	1	-0.05758	1
711	28	629	4	32	3	1	-0.22306	1
712	16	1198	4	35	2	0	-0.01133	1
713	31	2476	4	46	3	1	-0.1903	1
714	19	1138	2	25	1	1	-0.13587	1
715	70	14027	3	27	3	0	-1.12903	0
716	40	7596	4	63	2	1	-0.27691	1
717	40	3077	4	40	2	1	-0.39149	0
718	28	1505	2	32	3	1	-0.23483	1
719	34	3148	2	31	2	1	-0.36279	0
720	30	6148	4	31	2	1	-0.33761	0

TP	TN	FP	FN	ACC	F1 Score
51	131	76	42	60.67%	46.36%

One can vary the cut-off score , c and extract ROC curve

22.3 ML Support Vector Machine

PEAKS2TAILS

Soft Margin - SVM

Hard Margin

The modified Objective function : **Min** $0.5 \times \|w\|^2 + C \times \sum \xi_i$ → Penalize misclassification

S.t $y_i (wx_i+b) >= 1 - \xi_i$, for all points

$y_i = -1, 1$

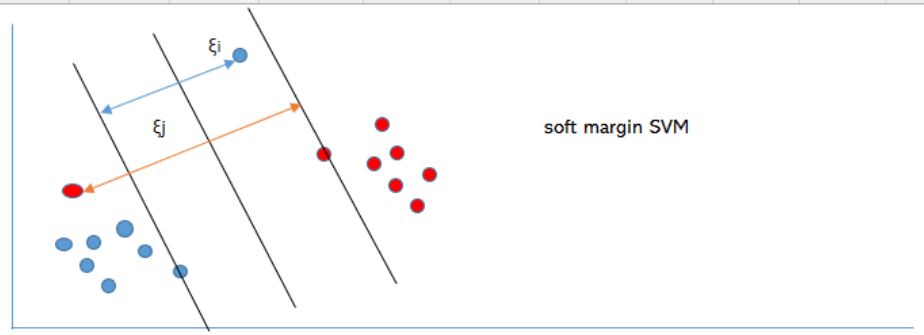
$\xi_i > 0$ is a slack variable

C is called the slack penalty also Regularization parameter

C can be found out from k-crossfold validation, but we will assume a value

Since SVM works on euclidean distance, we need numerical data.

One can use dummy codes for categorical variable



soft margin SVM

Training set has been trimmed down to 50 observation as Excel does not support more than 200 decision variables

b	w1	w2	w3
0.999979	1.21068E-07	-1.9E-09	9.34E-07

Minimize **60.00016**

C 5

OBS#	DURATION	AMOUNT	AGE	RESPONSE	yi	ξi			yi (wxi+b)		1	Decision Variables
												0.999979
												1.21E-07
												-1.91E-09
												9.34E-07
1	34	2835	53	1	1	0	>=	0	1.000026981	>=	1	0
2	46	6948	35	0	-1	1.000002	>=	0	-1.89252E-06	>=	-1.893E-06	1.000002
3	22	3059	61	1	1	0	>=	0	1.000032571	>=	1	0
4	40	5234	28	1	1	0	>=	0	0.99999979	>=	1	0
5	22	1295	25	0	-1	1.000001	>=	0	-1.15991E-06	>=	-1.16E-06	1.000001
6	58	4308	24	1	1	0	>=	0	1	>=	1	0
7	22	1567	22	1	1	0	>=	0	0.999999	>=	1	0
8	34	1199	60	1	1	0	>=	0	1.000036637	>=	1	0
9	25	1403	28	1	1	0	>=	0	1.000005278	>=	1	0
10	34	1282	32	0	-1	1.000005	>=	0	-5.16683E-06	>=	-5.167E-06	1.000005
11	34	2424	53	0	-1	1.000014	>=	0	-1.38825E-05	>=	-1.388E-05	1.000014
12	40	8072	25	0	-1	0.999996	>=	0	4.21092E-06	>=	4.2109E-06	0.999996
13	34	12579	44	1	1	0	>=	0	1	>=	1	0
14	34	3430	31	0	-1	1.000003	>=	0	-2.54028E-06	>=	-2.764E-06	1.000003
15	19	2134	48	1	1	0	>=	0	1.000021833	>=	1	0
16	16	2647	44	0	-1	1.000008	>=	0	-8.37846E-06	>=	-8.378E-06	1.000008
17	20	2241	48	1	1	0	>=	0	1.00002175	>=	1	0
18	22	1804	44	1	1	0	>=	0	1.000019091	>=	1	0
19	20	2069	26	0	-1	1.000001	>=	0	-3.75834E-07	>=	-1.16E-06	1.000001
20	16	1374	36	1	1	0	>=	0	1.000011714	>=	1	0
21	16	426	39	1	1	0	>=	0	1.000016323	>=	1	0
22	22	409	42	1	1	0	>=	0	1.000019883	>=	1	0

22.4 ML K Nearest Neighbour

PEAKS2TAILS

kNN Classifier

Core of kNN is Euclidean distance which identifies neighbors.
Two major challenges are categorical data and numerical data with different scales.
Categorical data are dummy coded (0,1) and numerical values are scaled (e.g. MinMax Scaling)

Algorithm :

An Obligor is a point in the feature space.
k-Nearest Neighbors to that obligor are identified in the feature space and classification is done via voting.
If k is too small, the prediction will be very sensitive to local conditions and unstable and sensitive to Outliers
If k is too large, it loses sensitivity as one includes far off points into the voting
It's a non-parametric model and also is a lazy learner. There is no underlying formula and there is nothing to train.

Model Performance on validation Set

TP	14
TN	192
FP	15
FN	79
TPR	0.150538
TNR	0.927536

ACC	68.67%
F1 Score	22.95%

Odd Number to avoid ties

CALCULATE

(Click CALCULATE button to recalculate)

Data

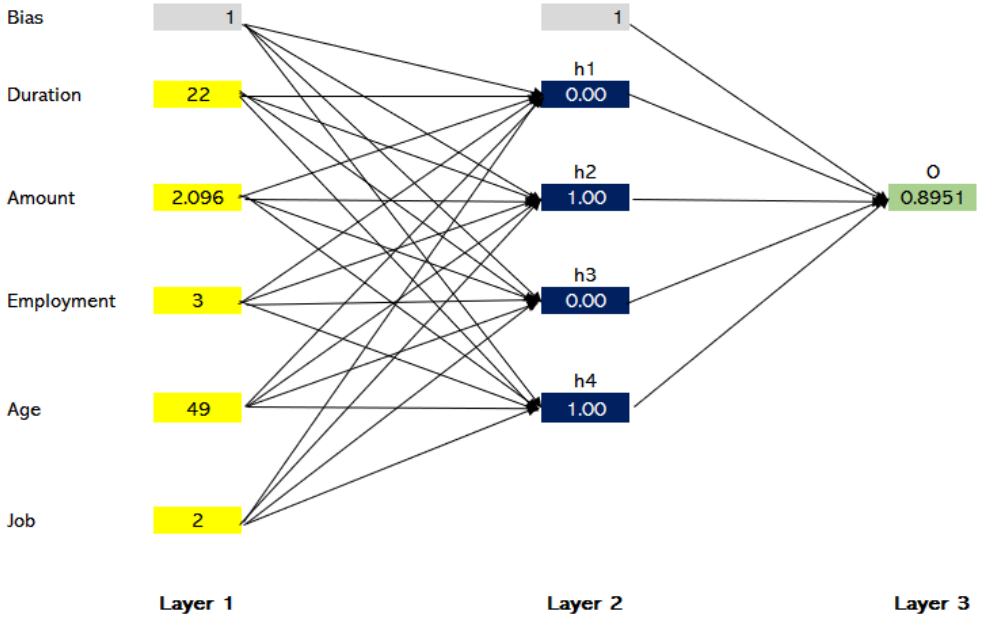
Training set - First 700 Observations
Validation Set - Last 300 Observations

OBS#	DURATION	AMOUNT	EMPLOYMENT	AGE	JOB	RESPONSE
1	16	1169	4	67	2	1
2	58	5951	2	22	2	0
3	22	2096	3	49	1	1
4	52	7882	3	45	2	1
5	34	4870	2	53	2	0
6	46	9055	2	35	1	1
7	34	2835	4	53	2	1
8	46	6948	2	35	3	1
9	22	3059	3	61	1	1
10	40	5234	0	28	3	0
11	22	1295	1	25	2	0
12	58	4308	1	24	2	0
13	22	1567	2	22	2	1
14	34	1199	4	60	1	0
15	25	1403	2	28	2	1
16	34	1282	2	32	1	0
17	34	2424	4	53	2	1
18	40	8072	1	25	2	1
19	34	12579	4	44	3	0
20	34	3430	4	31	2	1
21	19	2134	2	48	2	1
22	16	2647	2	44	2	1
23	20	2241	1	48	1	1

Data (Standardized using MinMax Scaling) k 11

OBS#	DURATION	AMOUNT	Emp_0	Emp_1	Emp_2	Emp_3	Emp_4	AGE	Job_0	Job_1	Job_2	Job_3	Distance from Dummy from each obs in the training set	Actual Response	k-Nearest Responses For Dummy Variable	PREDICTED Class of Dummy
Entering	0.602941176	0.23803235	1	0	0	0	0	0.142857	0	0	1	0				
1	0.029411765	0.05056674	0	0	0	0	1	0.857143	0	0	1	0	1.69537118	1	1	0
2	0.647058824	0.31368989	0	0	1	0	0	0.053571	0	0	1	0	1.4197332	0	0	
3	0.117647059	0.10157368	0	0	0	1	0	0.535714	0	1	0	0	2.09963523	1	0	
4	0.558823529	0.41994057	0	0	0	1	0	0.464286	0	0	1	0	1.46231094	1	1	
5	0.294117647	0.25420931	0	0	1	0	0	0.607143	0	0	1	0	1.52026145	0	0	
6	0.470588235	0.48448333	0	0	1	0	0	0.285714	0	1	0	0	2.02451563	1	0	
7	0.294117647	0.14223616	0	0	0	0	1	0.607143	0	0	1	0	1.52319077	1	0	
8	0.470588235	0.36854848	0	0	1	0	0	0.285714	0	0	0	1	2.01369311	1	0	
9	0.117647059	0.15456146	0	0	0	1	0	0.75	0	1	0	0	2.14734725	1	1	
10	0.382352941	0.27423792	1	0	0	0	0	0.160714	0	0	0	1	1.43188299	0	1	
11	0.117647059	0.05749972	0	1	0	0	0	0.107143	0	0	1	0	1.50644546	0	0	
12	0.647058824	0.22328601	0	1	0	0	0	0.089286	0	0	1	0	1.41599213	0		
13	0.117647059	0.07246616	0	0	1	0	0	0.053571	0	0	1	0	1.50694873	1		
14	0.294117647	0.05221745	0	0	0	0	1	0.732143	0	1	0	0	2.1159293	0		
15	0.161764706	0.06344228	0	0	1	0	0	0.160714	0	0	1	0	1.49178995	1		
16	0.294117647	0.05678442	0	0	1	0	0	0.232143	0	1	0	0	2.03376368	0		
17	0.294117647	0.11962144	0	0	0	0	1	0.607143	0	0	1	0	1.5247801	1		
18	0.382352941	0.43039507	0	1	0	0	0	0.107143	0	0	1	0	1.44462386	1		
19	0.294117647	0.67838671	0	0	0	0	1	0.446429	0	0	0	1	2.09318884	0		
20	0.294117647	0.17497524	0	0	0	0	1	0.214286	0	0	1	0	1.45067233	1		
21	0.073529412	0.10366458	0	0	1	0	0	0.517857	0	0	1	0	1.56171589	1		
22	0.029411765	0.13189171	0	0	1	0	0	0.446429	0	0	1	0	1.55960169	1		
23	0.088235294	0.10955211	0	1	0	0	0	0.517857	0	1	0	0	2.10286812	1		

22.5 ML Neural Networks



Lets follow the vectorized form to handle the weights in a compact form

h = σ(WX + b) ,
O = σ(Ah + d) ,

b	W					d			A	
-2.362	-2.812	-0.746	18.43102308	-0.866	2.9157218	-0.35834	6.081	1.2859	-3.972434367	1.218
11.606	-2.146	-2.619	1.720634432	0.8634	0.088926					
-9.151	-0.439	-11.81	-1.25454585	-2.019	-1.461059					
-0.603	-1.545	1.4205	5.309894588	1.8856	-1.27419					
8.1155										

Minimize 392.291

TP	193
TN	26
FP	67
FN	14
Sensitivity (TPR)	0.93237
Specificity(TNR)	0.27957
FPR = 1 - TNR	0.72043
TPR - FPR	0.211937
Accuracy (ACC)	0.73
F1 Score	0.82655

#	Duration	Amount	Employment	Age	Job	z1	z2	z3	z4	h1	h2	h3	h4	Z	p	y	log loss	predicted
1	16	1.169	4	67	2	-26.72	39.121	-173.2	121.36	2E-12	1	6E-76	1	2.1456	0.895256943	1	0.1106445	1
2	58	5.951	2	22	2	-146.3	-105.8	-154.7	-32.2	3E-64	1E-46	6E-68	1E-14	-0.358	0.411362507	0	0.5299447	0
3	22	2.096	3	49	1	-50.03	6.4663	-147.7	75.435	2E-22	0.9984	7E-65	1	2.1436	0.895069592	1	0.1108538	1
4	52	7.882	3	45	2	-132.3	-76.43	-222.6	28.49	3E-58	6E-34	2E-97	1	0.8597	0.702600769	1	0.3529664	1
5	34	4.87	2	53	2	-104.8	-24.73	-194	61.795	3E-46	2E-11	6E-85	1	0.8597	0.702600769	0	1.2126798	1
6	46	9.055	2	35	1	-129	-77.07	-210.9	16.535	9E-57	3E-34	3E-92	1	0.8597	0.702600753	1	0.3529665	1
7	34	2.835	4	53	2	-66.45	-15.96	-172.5	69.524	1E-29	1E-07	1E-75	1	0.8597	0.702600801	1	0.3529664	1
8	46	6.948	2	35	3	-121.6	-71.37	-188.9	10.993	2E-53	1E-31	9E-83	1	0.8597	0.70259649	1	0.3529725	1
9	22	3.059	3	61	1	-61.15	14.305	-183.3	99.43	3E-27	1	2E-80	1	2.1456	0.895256869	1	0.1106446	1
10	40	5.234	0	28	3	-134.3	-63.49	-149.4	-5.991	5E-59	3E-28	1E-65	0.0025	-0.355	0.412098612	0	0.5311961	0

Validation

#	Duration	Amount	Emp
701	22	1.123	
702	58	6.331	
703	34	1.377	
704	40	2.503	
705	37	2.528	
706	25	5.324	
707	58	6.56	
708	22	2.969	
709	19	1.206	
710	19	2.118	

22.6 ML Naive Bayes

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	
Age Level	**-0.527414536**	**-0.286252**	**-0.2098795**	**-0.03493805**	**0.07042246**	**0.0784**	**0.08629581**	**0.424594278**	**0.43506558**	**0.6357363**	**0.76356964**	**Grand Total**
0	110	7	17	153	30	63	127	57	108	22	5	699
1	80	4	9	68	12	25	50	16	30	5	1	300
Grand Total	190	11	26	221	42	88	177	73	138	27	6	999
P (Age Level | 1)	0.266666667	0.0133333	0.03	0.22666667	0.04	0.0833	0.16666667	0.053333333	0.1	0.0166667	0.00333333	

Sensitivity = TP / (TP + FN)

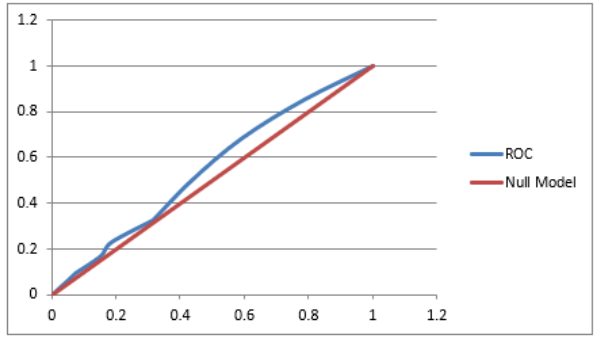
1- Specificity = FP / (TN + FP)

		Predicted Class	
		Yes	No
Actual Class	Yes	TP	FN
	No	FP	TN

	D1	D2	D3	D4	D5	D6	D7	D8	D9	
Duration Level	**-1.220561717**	**-0.845868**	**-0.69171759**	**-0.52309488**	**-0.11993126**	**0**	**0.00944567**	**0.06469179**	**0.88217539**	**Grand Total**
0	22	1	7	58	31		167	261	152	699
1	32	1	6	42	15	1	71	105	27	300
Grand Total	54	2	13	100	46	1	238	366	179	999
P(Duration Level | 1)	0.106666667	0.0033333	0.02	0.14	0.05	0.0033	0.23666667	0.35	0.09	

Status	Age WoE	Duration WoE	P (Ai | 1) x P (Di | 1)	P (Ai | 1) x P (Di | 1)	Status
1	-0.527414536	-1.220561717	0.0284444	0.09333333	1
0	0.424594278	0.06469179	0.0186667	0.09333333	0
0	0.078390634	-0.523094875	0.0116667	0.09333333	0
1	0.070422464	0.009445668	0.0094667	0.09333333	0
0	0.086295813	-0.523094875	0.0233333	0.09333333	0
0	0.070422464	0.009445668	0.0094667	0.09333333	0
0	0.086295813	-0.523094875	0.0233333	0.09333333	0
0	0.635736273	0.06469179	0.0058333	0.09333333	0
1	-0.034938051	-0.119931264	0.0113333	0.09333333	1
1	-0.527414536	0.06469179	0.0933333	0.09333333	1
1	-0.527414536	-1.220561717	0.0284444	0.09333333	1
0	-0.527414536	0.06469179	0.0933333	0.09333333	1
1	-0.209879501	0.009445668	0.0071	0.09333333	0
0	-0.034938051	0.06469179	0.0793333	0.09333333	0
1	0.086295813	0.009445668	0.0394444	0.09333333	1
0	0.070422464	0.009445668	0.0094667	0.09333333	0
0	-0.527414536	-0.119931264	0.0133333	0.09333333	0
1	0.078390634	0.009445668	0.0197222	0.09333333	0
0	0.086295813	0.009445668	0.0394444	0.09333333	1
0	0.424594278	0.882175387	0.0048	0.09333333	0
0	0.078390634	0.882175387	0.0075	0.09333333	1
0	0.424594278	0.882175387	0.0048	0.09333333	0
0	0.078390634	0.06469179	0.0291667	0.09333333	0
0	-0.034938051	0.882175387	0.0204	0.09333333	1
0	0.435065578	0.882175387	0.009	0.09333333	0
0	0.435065578	0.882175387	0.009	0.09333333	0
0	0.078390634	0.06469179	0.0291667	0.09333333	0
0	0.086295813	0.882175387	0.015	0.09333333	0
1	0.635736273	-0.691717588	0.0003333	0.09333333	0
0	0.435065578	0.06469179	0.035	0.09333333	0
0	-0.034938051	0.009445668	0.0536444	0.09333333	1

Threshold	TP	FP	FN	TN	1- Specificity	Sensitivity
0.1	0	0	300	699	0	0
0.09	29	52	271	647	0.07439199	0.096666667
0.08	29	52	271	647	0.07439199	0.096666667
0.07	51	106	249	593	0.15164521	0.17
0.06	69	129	231	570	0.18454936	0.23
0.05	98	220	202	479	0.31473534	0.326666667
0.04	98	220	202	479	0.31473534	0.326666667
0.03	150	304	150	395	0.43490701	0.5
0.02	205	413	95	286	0.59084406	0.683333333
0.01	256	550	44	149	0.78683834	0.853333333
0	300	699	0	0	1	1



23.1 Multinominal Logit

PEAKS2TAILS

1. Data & Model

Data consists of Employees going to office by Car, Bus, Bike (Y) based on distance between Home and Office (X1) and Gender (X2)

X1 = Distance of Home from Office (miles)
X2 = Binary (0 - Female , 1 = Male)
Y = Nominal (A = Car, B = Bike, C = Bus)

a1	b1	c1	a2	b2	c2
-24.588	4.42398	0.00936	-10.272	2.35015	0.71269929

$\ln(P(Y=A)/P(Y=C)) = a1 + b1X1 + c1X2 + \epsilon 1$
$\ln(P(Y=B)/P(Y=C)) = a2 + b2X1 + c2X2 + \epsilon 1$

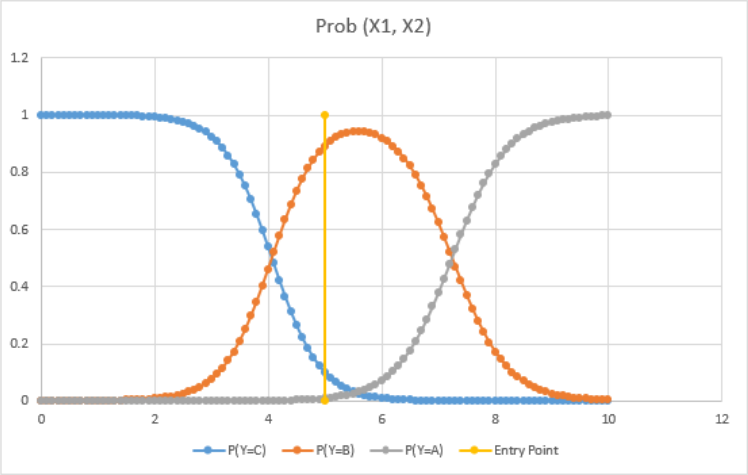
OBS	X1	X2	Y	P(Y=C)	P(Y=A)	P(Y=B)	P	ln(P)
1	4	1	B	0.53939	0.00055	0.46006	0.46006	-0.7764
2	3	1	B	0.92478	1.1E-05	0.07521	0.07521	-2.5874
3	3	1	C	0.92478	1.1E-05	0.07521	0.92478	-0.0782
4	7	1	B	0.00063	0.37679	0.62258	0.62258	-0.4739
5	4	1	C	0.53939	0.00055	0.46006	0.53939	-0.6173
6	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
7	8	1	B	1.7E-05	0.828	0.17198	0.17198	-1.7604
8	1	1	C	0.99926	1.8E-09	0.00074	0.99926	-0.0007
9	3	1	C	0.92478	1.1E-05	0.07521	0.92478	-0.0782
10	5	1	B	0.0997	0.00853	0.89177	0.89177	-0.1145
11	5	1	B	0.0997	0.00853	0.89177	0.89177	-0.1145
12	2	1	C	0.9923	1.5E-07	0.0077	0.9923	-0.0077
13	4	1	B	0.53939	0.00055	0.46006	0.46006	-0.7764
14	3	1	C	0.92478	1.1E-05	0.07521	0.92478	-0.0782
15	5	1	B	0.0997	0.00853	0.89177	0.89177	-0.1145
16	5	1	C	0.0997	0.00853	0.89177	0.0997	-2.3056
17	6	1	B	0.00981	0.07	0.92019	0.92019	-0.0832
18	4	1	B	0.53939	0.00055	0.46006	0.46006	-0.7764
19	4	1	C	0.53939	0.00055	0.46006	0.53939	-0.6173
20	5	1	B	0.0997	0.00853	0.89177	0.89177	-0.1145
21	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
22	7	1	B	0.00063	0.37679	0.62258	0.62258	-0.4739
23	4	1	B	0.53939	0.00055	0.46006	0.46006	-0.7764
24	6	1	B	0.00981	0.07	0.92019	0.92019	-0.0832
25	2	1	C	0.9923	1.5E-07	0.0077	0.9923	-0.0077
26	3	1	C	0.92478	1.1E-05	0.07521	0.92478	-0.0782
27	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
28	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
29	1	1	C	0.99926	1.8E-09	0.00074	0.99926	-0.0007
30	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
31	7	1	B	0.00063	0.37679	0.62258	0.62258	-0.4739
32	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
33	6	1	B	0.00981	0.07	0.92019	0.92019	-0.0832
34	8	1	A	1.7E-05	0.828	0.17198	0.828	-0.1887
35	2	1	C	0.9923	1.5E-07	0.0077	0.9923	-0.0077

Total Ln (Likelihood)
-34.360824 → Maximize using Excel Solver

Entering Employee

X1	X2
5	1

P(Y=C)	0.0997
P(Y=A)	0.00853
P(Y=B)	0.89177
Predicted	B
Total	1



Yellow vertical line shows the position of the entering data
It's easy to see the probability distribution for all three states for a given point
We choose the state that has highest probability

Probability Distribution of different outcomes (A, B , C) based on X1 and X2

X1	P(Y=C)	P(Y=B)	P(Y=A)
0	0.99993	7.05109E-05	2.11625E-11
0.1	0.99991	8.91895E-05	3.29375E-11
0.2	0.99989	0.000112816	5.12639E-11
0.3	0.99986	0.000142699	7.97865E-11
0.4	0.99982	0.000180497	1.24178E-10
0.5	0.99977	0.000228305	1.93266E-10
0.6	0.99971	0.000288772	3.00788E-10
0.7	0.99963	0.000365248	4.68121E-10
0.8	0.99954	0.000461967	7.28529E-10
0.9	0.99942	0.000584284	1.13377E-09
1	0.99926	0.000738962	1.76437E-09
1.1	0.99907	0.000934551	2.74559E-09
1.2	0.99882	0.001181846	4.27227E-09
1.3	0.99851	0.001494482	6.64744E-09
1.4	0.99811	0.001889663	1.03422E-08
1.5	0.99761	0.002389091	1.60889E-08
1.6	0.99698	0.003020116	2.50256E-08
1.7	0.99618	0.003817173	3.89195E-08
1.8	0.99518	0.004823569	6.05146E-08
1.9	0.99391	0.006093678	9.40668E-08
2	0.9923	0.007695637	1.46173E-07
2.1	0.99029	0.009714617	2.27046E-07
2.2	0.98774	0.01225674	3.52475E-07
2.3	0.98455	0.015453704	5.4683E-07
2.4	0.98053	0.019468105	8.47635E-07
2.5	0.9755	0.024499371	1.31252E-06
2.6	0.96921	0.030790062	2.02968E-06
2.7	0.96136	0.038632026	3.1335E-06
2.8	0.95162	0.048371562	4.82767E-06
2.9	0.93958	0.060412178	7.41886E-06
3	0.92478	0.075212945	1.1365E-05
3.1	0.9067	0.093279646	1.73433E-05
3.2	0.88483	0.115145368	2.63424E-05
3.3	0.85862	0.141336965	3.97861E-05
3.4	0.82762	0.172324669	5.96882E-05

23.2 Multi-ordinal Logit

PEAKS2TAILS

1. Data & Model

Data consists of Students getting Grade (A = Excellent, B = Good, C = Poor) which is regressed over X1 (Hours of studying) and X2 (Gender)

X1 = Hours Studied
X2 = Binary (0 - Female , 1 = Malae)
Y = Ordinal (A = Excellent, B = Good, C = Poor)

Thresholds
θ_B
θ_C

P (Y = A) = $1 - \sigma(\theta_B - aX1 - bX2)$
P (Y = B OR C) = $\sigma(\theta_B - aX1 - bX2)$
P (Y = C) = $\sigma(\theta_C - aX1 - bX2)$

$\sigma(L) = 1 / (1 + \exp(-L))$

a	b	θ_B	θ_C
2.215327	-0.1542225	15.572963	9.10325

θ_B > θ_C

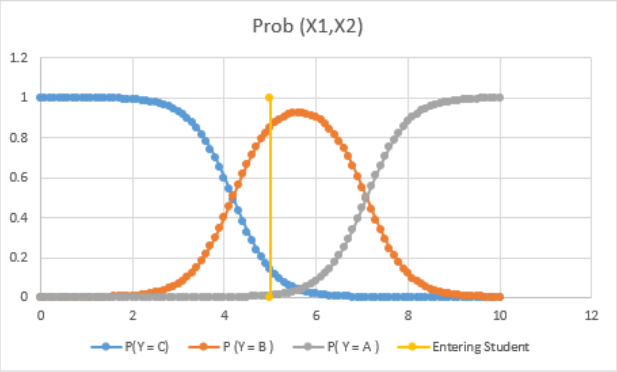
Sum LN (Likelihood)
-35.06480265 → Maximize using Excel Solver

OBS	X1	X2	Y	P(Y = C)	P (Y = B)	P(Y = A)	P	ln (P)
1	4	1	B	0.597765	0.4011935	0.0010417	0.40119	-0.913311343
2	3	1	B	0.931597	0.068289	0.0001138	0.06829	-2.684006865
3	3	1	C	0.931597	0.068289	0.0001138	0.9316	-0.070854695
4	7	1	B	0.001927	0.5528263	0.4452466	0.55283	-0.592711468
5	4	1	C	0.597765	0.4011935	0.0010417	0.59776	-0.514557937
6	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363
7	8	1	B	0.000211	0.1194727	0.8803166	0.11947	-2.124667133
8	1	1	C	0.999127	0.0008721	1.355E-06	0.99913	-0.00087387
9	3	1	C	0.931597	0.068289	0.0001138	0.9316	-0.070854695
10	5	1	B	0.139534	0.8510002	0.0094659	0.851	-0.161342865
11	5	1	B	0.139534	0.8510002	0.0094659	0.851	-0.161342865
12	2	1	C	0.992052	0.0079359	1.242E-05	0.99205	-0.007980075
13	4	1	B	0.597765	0.4011935	0.0010417	0.40119	-0.913311343
14	3	1	C	0.931597	0.068289	0.0001138	0.9316	-0.070854695
15	5	1	B	0.139534	0.8510002	0.0094659	0.851	-0.161342865
16	5	1	C	0.139534	0.8510002	0.0094659	0.13953	-1.969447919
17	6	1	B	0.017387	0.9020871	0.0805259	0.90209	-0.103044217
18	4	1	B	0.597765	0.4011935	0.0010417	0.40119	-0.913311343
19	4	1	C	0.597765	0.4011935	0.0010417	0.59776	-0.514557937
20	5	1	B	0.139534	0.8510002	0.0094659	0.851	-0.161342865
21	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363
22	7	1	B	0.001927	0.5528263	0.4452466	0.55283	-0.592711468
23	4	1	B	0.597765	0.4011935	0.0010417	0.40119	-0.913311343
24	6	1	B	0.017387	0.9020871	0.0805259	0.90209	-0.103044217
25	2	1	C	0.992052	0.0079359	1.242E-05	0.99205	-0.007980075
26	3	1	C	0.931597	0.068289	0.0001138	0.9316	-0.070854695
27	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363
28	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363
29	1	1	C	0.999127	0.0008721	1.355E-06	0.99913	-0.00087387
30	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363
31	7	1	B	0.001927	0.5528263	0.4452466	0.55283	-0.592711468
32	8	1	A	0.000211	0.1194727	0.8803166	0.88032	-0.12747363

Entering Student				
	X1	5	P (Y = C)	0.1395
	X2	1	P (Y = B)	0.851
			P (Y = A)	0.0095
Predicted Grade	B			

Alternatively		Score	Grade
Score (w.X)	10.922414	<=θ_C	C
		>θ_C AND <=θ_B	B
Predicted Grade	B	>θ_B	A

X1	P(Y = C)	P (Y = B)	P(Y = A)
0	0.9999046	9.524E-05	1.47833E-07
0.1	0.999881	0.00011885	1.84493E-07
0.2	0.9998514	0.00014833	2.30245E-07
0.3	0.9998146	0.0001851	2.87343E-07
0.4	0.9997686	0.00023099	3.58601E-07
0.5	0.9997113	0.00028826	4.4753E-07
0.6	0.9996397	0.00035972	5.58512E-07
0.7	0.9995504	0.00044889	6.97016E-07
0.8	0.999439	0.00056014	8.69868E-07
0.9	0.9993	0.00069895	1.08558E-06
1	0.9991265	0.00087213	1.3548E-06
1.1	0.9989101	0.00108818	1.69077E-06
1.2	0.9986402	0.00135766	2.11006E-06
1.3	0.9983036	0.00169378	2.63333E-06
1.4	0.9978838	0.00211292	3.28636E-06



Yellow vertical line shows the X1 cordinate of the entering student
It's easy to discern the three probabilities for a given point and choose the one that's highest

Another direct way is to compute the scores (w.x) and compare with cut-offs

23.3 Gibbs Sampling

PEAKS2TAILS

Gibbs Sampling

Data (X)

2	6	2	11	5	5	7	4	2	3

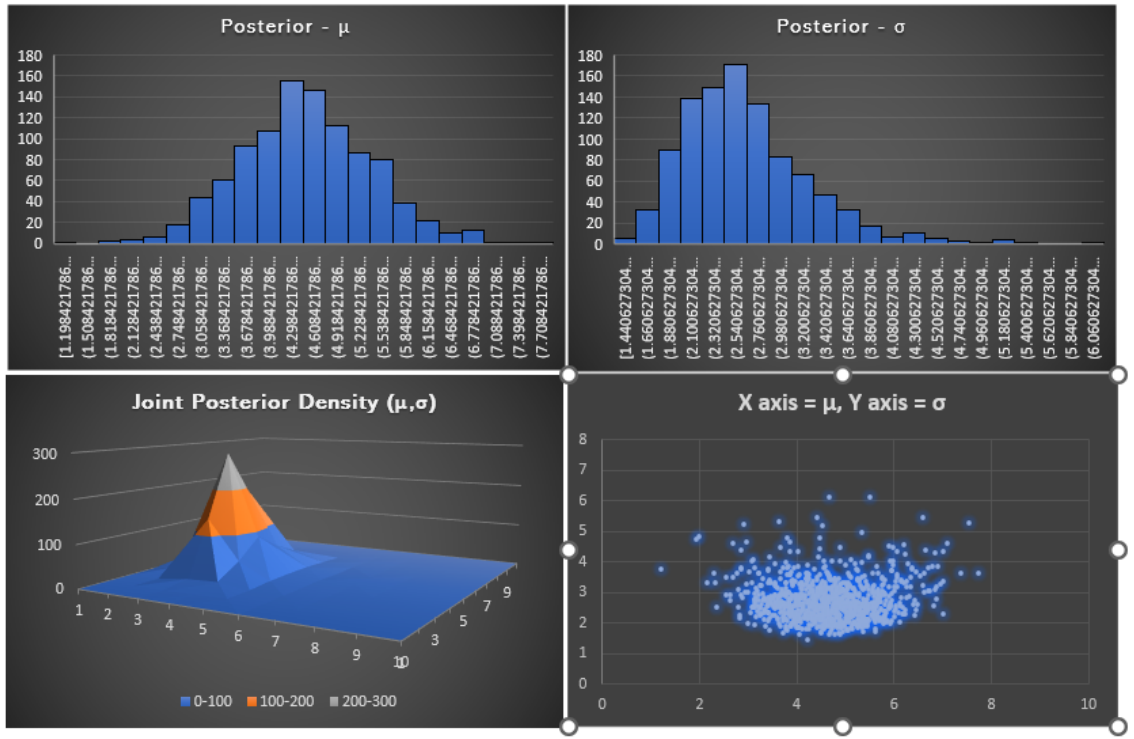
Data synthesized using EXCEL norm.inv function with μ=5 and σ=3

$\bar{x}$ = 4.7

n = 10

Conditional Posterior for μ : $N\left(\bar{x}, \sigma^2/n\right)$

Conditional Posterior for σ : $\dfrac{1}{\sqrt{Gamma\left(\frac{n}{2}+1, \frac{2}{\sum_{i=1}^{i=n}\{(x_i-\mu)^2\}}\right)}}$



Seed Values

μ = 1

σ = 20

Bayesian Estimates

	Mean
μ	4.651077
σ	2.745692

$\mathcal{N}\left(\bar{x}, \dfrac{\sigma^2}{n}\right)$

$1/\sqrt{Gamma\left(\dfrac{n}{2}+1, \dfrac{2}{\sum_{i=1}^{i=n}\{(x_i-\mu)^2\}}\right)}$

(1200 Simulations. Discard the first 200 as Burn-in)

Iteration	μ	x_1 - μ	x_2 - μ	x_3 - μ	x_4 - μ	x_5 - μ	x_6 - μ	x_7 - μ	x_8 - μ	x_9 - μ	x_{10} - μ	$\sum (x_i - \mu)^2$	σ
0	17.70126	-15.7013	-11.7013	-15.7013	-6.70126	-12.7013	-12.7013	-10.7013	-13.7013	-15.7013	-14.7013	1762.428142	11.8479514

σ									
0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10

23.4 Metrapolis Hastings

PEAKS2TAILS

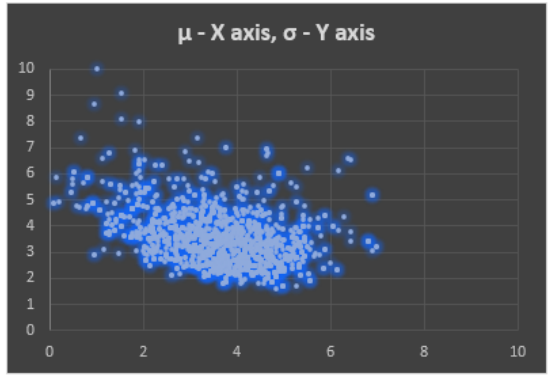
Metropolis- Hastings Sampling

Metropilos - Hastings algorithm is another popular MCMC technique which works almost universally. Once does not have to formulate conditional posterior for parameters. In every iteration we attempt to make a jump to a proposed state based on a proposal distribution (e.g. Normal). We calculate the ratio of *likelihood x prior* in proposed state to the current state (call it p). If p > 1 , we move to the proposed state, if p < 1, we may accept the proposed state with the probability p. This way we roam in the posterior space of the parameter values , spending more time in places where *likelihood x prior* is higher. After , sufficient number of such iterations we infact generate the sample from posterior.

Let's use a Normal Prior for $\mu \sim N(0, b^2)$ and continue using a flat prior for σ. Note : since we have only 10 data points, the posterior for μ will be affected by the prior. Posterior $\propto$ Likelihood x Prior

Posterior $\propto \sigma^{-n} \exp\left(-\frac{1}{2\sigma^2}\sum(y_i-\mu)^2\right) \times \exp\left(-\frac{\mu^2}{2b^2}\right)$, Let's also assume a proposal density = $N(0, q^2)$

If we choose too small a q, most of the proposal will be accepted and the chain will move slowly. If we choose a large q, most proposals will be rejected. In Metropolis Hastings, we draw the Trace plots and calculate the acceptance rate to tune our priors and proposal distributions.



Data (Y) (Alter b and q and press F9 to regenerate posterior sample and plots)

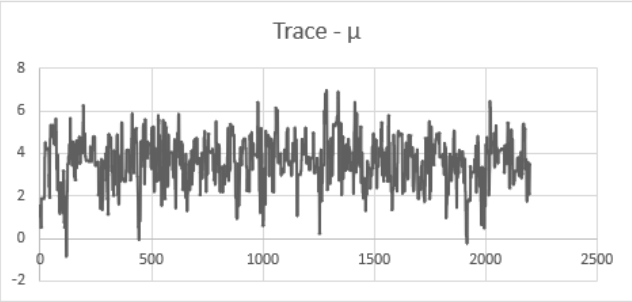
2	6	2	11	5	5	7	4	2	3

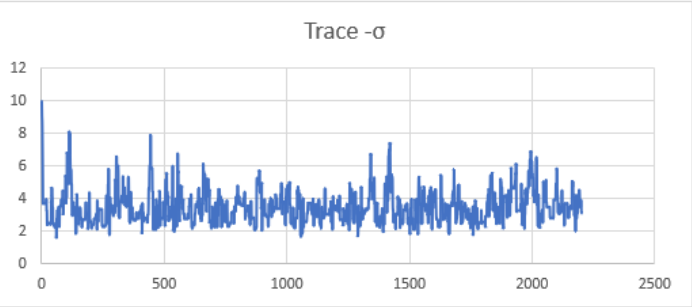
b = 2 μ_{seed} = 1 Acceptance Rate 39.59%

q = 1.3 σ_{seed} = 10

** σ is floored to 0.1 during all iterations so that negative values can be supressed for a proposal state

Should be Ideally between 40% to 50%





Metropolis - Hastings Sampling

	Current State		Likelihood at Current State (*directly calculated using EXCEL's NORM.DIST function*)												Proposed State		Likelihood at Proposed State (*directly calculated using EXCEL's NORM.DIST functio*									
Iteration	μ	σ	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	Total	Current State Prior	μ	σ	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
0	1	10	0.039695	0.035207	0.039695	0.024197	0.036827	0.036827	0.033322	0.038139	0.039695	0.039104	3.59139E-15	0.176032663	1.535386	9.098382	0.04379	0.038874	0.04379	0.025525	0.040781	0.040781	0.036611	0.042268	0.04379	0.043283
1	1.535386	9.098382	0.04379	0.038874	0.04379	0.025525	0.040781	0.0407811	0.036611	0.042268	0.04379	0.043283	9.28163E-15	0.148560629	0.947127	8.683894	0.045604	0.038786	0.045604	0.023506	0.0412	0.0412	0.036033	0.043187	0.045604	0.044675
2	0.947127	8.683894	0.045604	0.038786	0.045604	0.023506	0.0412	0.0411999	0.036033	0.043187	0.045604	0.044675	1.02043E-14	0.178312622	0.664728	7.376747	0.053202	0.041635	0.053202	0.020267	0.045504	0.045504	0.037401	0.048826	0.053202	0.051438
3	0.664728	7.376747	0.053202	0.041635	0.053202	0.020267	0.045504	0.0455036	0.037401	0.048826	0.053202	0.051438	2.47146E-14	0.188752503	0.501057	5.61931	0.068513	0.043983	0.068513	0.012394	0.051527	0.051527	0.036373	0.058484	0.068513	0.064311
4	0.501057	5.61931	0.068513	0.043983	0.068513	0.012394	0.051527	0.0515272	0.036373	0.058484	0.068513	0.064311	6.36779E-14	0.193308499	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.075277	0.041333	0.091114	0.10738	0.102567
5	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.0752768	0.041333	0.091114	0.10738	0.102567	8.17995E-13	0.128971594	3.117183	1.966095	0.172661	0.069255	0.172661	6.56E-05	0.128282	0.128282	0.028866	0.183453	0.172661	0.202551
6	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.0752768	0.041333	0.091114	0.10738	0.102567	8.17995E-13	0.128971594	0.708217	3.055189	0.119413	0.029135	0.119413	0.000449	0.048682	0.048682	0.015665	0.073079	0.119413	0.098556
7	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.0752768	0.041333	0.091114	0.10738	0.102567	8.17995E-13	0.128971594	0.903205	2.47193	0.14626	0.019263	0.14626	3.85E-05	0.040872	0.040872	0.007708	0.073632	0.14626	0.112625
8	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.0752768	0.041333	0.091114	0.10738	0.102567	8.17995E-13	0.128971594	-0.22778	4.622792	0.076838	0.034826	0.076838	0.004519	0.045531	0.045531	0.02542	0.056805	0.076838	0.06763
9	1.867785	3.712868	0.10738	0.057841	0.10738	0.005218	0.075277	0.0752768	0.041333	0.091114	0.10738	0.102567	8.17995E-13	0.128971594	1.590947	2.668406	0.147759	0.038178	0.147759	0.000298	0.066106	0.066106	0.01916	0.099465	0.147759	0.13005

23.5 Kalman Regression

2. Linear Regression (Univariate)

Observation model $y_t = \boldsymbol{X}_t \boldsymbol{\beta}_t + \epsilon_t\,, \qquad \epsilon_t \sim N(0, v_\epsilon)$

State Transition $\boldsymbol{\beta_t} = \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \qquad \eta_t \sim N(0, \boldsymbol{W}_t)$

$$\beta_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_{a,t} \\ \mu_{b,t} \end{pmatrix}, \begin{pmatrix} \Sigma_{a,t} & \Sigma_{ab,t} \\ \Sigma_{ab,t} & \Sigma_{b,t} \end{pmatrix} \right)$$

$$F_t = X_t = (1 \quad x_t)$$

$$\eta_t \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} w_1 & w_{12} \\ w_{21} & w_2 \end{pmatrix} \right)$$

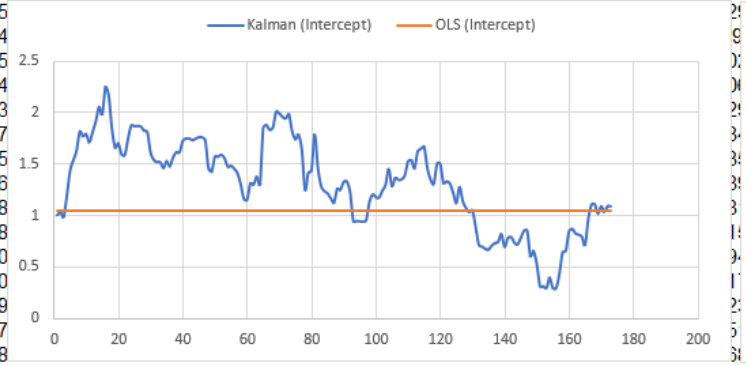
seed values

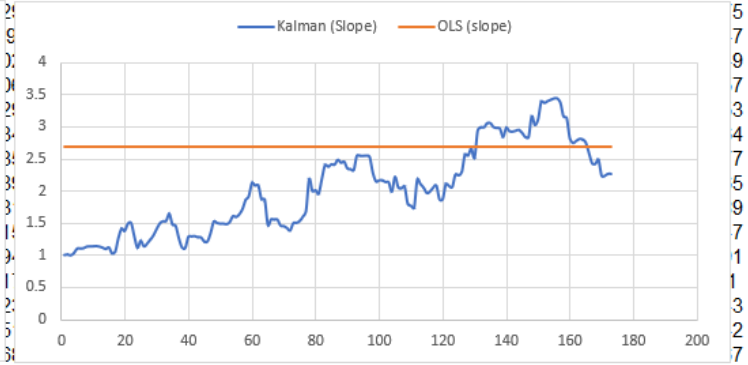
$\mu_{a,0}$	1	$\Sigma_{a,0}$	0.01
$\mu_{b,0}$	1	$\Sigma_{ab,0}$	0
		$\Sigma_{b,0}$	0.01

parameters

v_ϵ	0.894955
w_1	0.024868
w_{12}	0.002687
w_2	0.003048

a	b		Current					Prediction								Kalman Gain		Error	Correction			
1.046297	2.693727																					
#	y	x	μ_a	μ_b	Σ_a	Σ_{ab}	Σ_b	μ_a	μ_b	Σ_a	Σ_{ab}	Σ_b	$\mathbb{E}[y_t\|x_t]$	$\mathbb{V}[y_t\|x_t]$	Positive Variance	K_{at}	K_{bt}	e_t	μ_a	μ_b	Σ_a	Σ_{ab}
1	2.294901	0.448436	1	1	0.01	0	0.01	1	1	0.034868	0.002687	0.013048	1.44843561	0.93485755	0.934857552	0.038587034	0.009133	0.846466	1.032663	1.007731	0.033498	0.002073
2	1.200026	0.838799	1.032663	1.007731	0.033498	0.002073	0.012948	1.032663	1.007731	0.058367	0.00476	0.015995	1.87794593	0.97256116	0.972561163	0.064118588	0.01868942	-0.67792	0.989195	0.995061	0.054535	0.001546
3	3.949977	0.522509	0.989195	0.995061	0.054535	0.001546	0.015489	0.989195	0.995061	0.079404	0.004233	0.018536	1.50912317	0.98384353	0.983843527	0.082955944	0.01414691	2.440854	1.191679	1.029591	0.072757	0.00076
4	4.88545															0.098585299	0.02867946	2.425721	1.430819	1.09916	0.087902	-0.00853
5	2.79784															0.109692994	0.00339237	0.92736	1.532544	1.102306	0.10042	-0.01078
6	2.8705															0.119009632	0.0050185	0.786803	1.626181	1.106254	0.110419	-0.01553
7	4.21834															0.118979874	0.01539688	1.527632	1.807939	1.129775	0.119388	-0.02814
8	1.79833															0.133644651	-0.01583651	-0.31492	1.765851	1.134762	0.124575	-0.03077
9	2.88077															0.122682759	0.00407046	0.153352	1.784665	1.135386	0.131223	-0.04353
10	2.04505															0.123671608	-0.00779965	-0.59115	1.711556	1.139997	0.136469	-0.05556
11	3.48886															0.116479398	-0.00995307	0.818671	1.806914	1.131849	0.142019	-0.06911
12	3.66268															0.113082601	-0.01752087	0.925383	1.911559	1.115635	0.146851	-0.0829
13	4.34888															0.098928162	-0.01535013	1.394097	2.049475	1.094236	0.153501	-0.09725
14	2.20440															0.119111932	-0.04278862	-0.55135	1.983802	1.117828	0.153077	-0.11088
15	5.50900															0.096590793	-0.03581681	2.645445	2.239327	1.023076	0.156882	-0.12477
16	2.14319															0.078523643	-0.02883215	-0.99724	2.16102	1.051829	0.163959	-0.13776
17	0.30107															0.150786117	-0.10151757	-2.12975	1.839883	1.268036	0.146643	-0.14589
18	0.79248															0.114084578	-0.08919849	-1.58756	1.658767	1.409644	0.139171	-0.15698
19	2.603467	0.431263	1.658767	1.409644	0.139171	-0.15698	0.14537	1.658767	1.409644	0.164039	-0.15429	0.148418	2.2666951	0.95351584	0.95351584	0.102250908	-0.09468838	0.336772	1.693202	1.377756	0.132656	-0.16783
20	1.151676	0.45	1.693202	1.377756	0.132656	-0.16783	0.161283	1.693202	1.377756	0.157525	-0.16514	0.16433	2.31319247	0.93712966	0.937129663	0.088793502	-0.09731077	-1.16152	1.590067	1.490784	0.127467	-0.17867
21	2.368586	0.6	1.590067	1.490784	0.127467	-0.17867	0.178125	1.590067	1.490784	0.152336	-0.17598	0.181172	2.48453755	0.9013372	0.901337205	0.051864943	-0.07464095	-0.11595	1.584054	1.499438	0.1313	-0.1886
22	4.254728	0.448964	1.584054	1.499438	0.1313	-0.1886	0.194762	1.584054	1.499438	0.156168	-0.18591	0.19781	2.25724704	0.92405773	0.924057728	0.078673725	-0.1050857	1.997481	1.741203	1.289532	0.124345	-0.2002
23	2.875928	0.1	1.741203	1.289532	0.124345	-0.2002	0.21371	1.741203	1.289532	0.149213	-0.19752	0.216757	1.87015598	1.00683311	1.006833109	0.128583045	-0.17464624	1.005772	1.870528	1.113877	0.095532	-0.20288
24	1.194758	0.576058	1.870528	1.113877	0.095532	-0.20288	0.223083	1.870528	1.113877	0.1204	-0.2002	0.22613	2.51218654	0.85974548	0.859745483	0.00590308	-0.08134065	-1.31743	1.862751	1.221038	0.103405	-0.20999
25	3.8305	0.608765	1.862751	1.221038	0.103405	-0.20999	0.237407	1.862751	1.221038	0.128274	-0.2073	0.240454	2.60607673	0.85994679	0.859946791	0.002415101	-0.07084061	1.224423	1.865708	1.134299	0.113279	-0.21643
26	1.950044	0.603625	1.865708	1.134299	0.113279	-0.21643	0.251129	1.865708	1.134299	0.138147	-0.21374	0.254177	2.5503998	0.86767623	0.867676234	0.010519589	-0.06951156	-0.60036	1.859393	1.176031	0.121836	-0.22359
27	1.659426	0.1	1.859393	1.176031	0.121836	-0.22359	0.266199	1.859393	1.176031	0.146705	-0.2209	0.269246	1.97699587	1.00017271	1.000172708	0.12459327	-0.19394134	-0.31757	1.819826	1.237621	0.085585	-0.22701
28	1.79763	0.4	1.819826	1.237621	0.085585	-0.22701	0.277221	1.819826	1.237621	0.110453	-0.22432	0.280268	2.31487396	0.87079195	0.870791947	0.023798354	-0.12886796	-0.51724	1.807516	1.304277	0.078916	-0.23694
29	4.373435	0.9973	1.807516	1.304277	0.078916	-0.23694	0.296851	1.807516	1.304277	0.103785	-0.23425	0.299898	3.10827142	0.82978186	0.829781864	-0.15646873	0.07813672	1.265164	1.609558	1.403133	0.138328	-0.1998
30	1.247567	0.115016	1.609558	1.403133	0.138328	-0.1998	0.239974	1.609558	1.403133	0.163196	-0.19712	0.243022	1.77094002	1.0160234	1.016023398	0.138308395	-0.16649644	-0.52337	1.537171	1.490272	0.107806	-0.20349
31	1.716739	0.298727	1.537171	1.490272	0.107806	-0.20349	0.250811	1.537171	1.490272	0.132674	-0.2008	0.253859	1.98235477	0.93031488	0.930314879	0.078134656	-0.1343254	-0.26562	1.516417	1.525951	0.095335	-0.21195



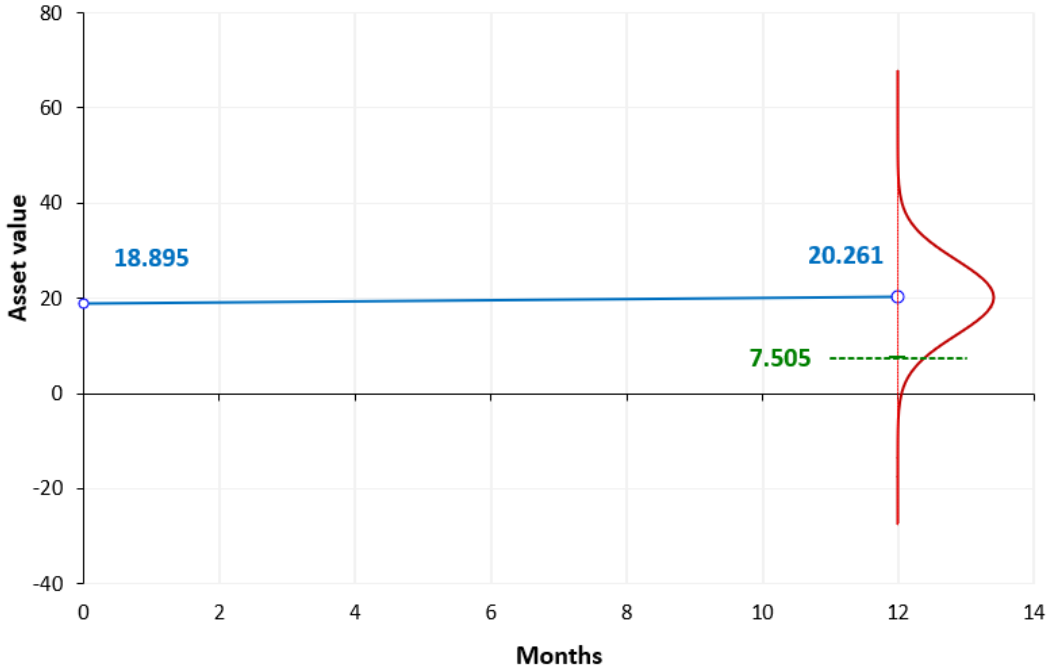


24.1 Merton & KMV Model

PARAMETERS	
Time horizon (years)	1
Risk free borrowing + lending rate	12.00%
Book value of liabilities	10
Current liabilities: short term (< 1 year)	8.00
Longer term liabilities	4.00
Market value of equity (observed)	**10.00**
Volatility of equity (observed)	**70.0%**
Expected return on assets	7.0%

VARIABLES (shaded areas should = 0.0)	
Market value of equity (calculated)	**10.00**
Volatility of equity (calculated)	**70.00%**
Difference (MV equity)	0.00
Difference (Volatility equity)	0.00
d_1	1.6081
$N(d_1)$	0.9461
d_2	1.2165
$N(d_2)$	0.8881
Risk-neutral prob of default $N(-d_2)$	**11.19%**

KMV PD



ESTIMATED VARIABLES			
Current MV of assets	18.895	Future MV of assets	20.265
Volatility of assets (%)	39.16%	Volatility of assets (%)	39.16%
PV Volatility of assets (R)	7.40	FV Volatility of assets (R)	7.94

Base	6.06%	
orderly	6.80%	12.20%
disorderly	11.19%	84.66%

24.2 Credit Risk Plus Model

PEAKS2TAILS

Credit Risk + (General Sector Analysis)

The underlying factor of a sector is responsible for variablility in the obligor's year-on-year default rate.

Rating	μ_A	σ_A
A	1.50%	0.75%
B	1.65%	0.80%
C	3.00%	1.50%
D	5.00%	2.50%
E	7.50%	3.75%
F	10.00%	5.00%
G	15.00%	7.50%

unit loss 100

#	obligor	rating	Exposure	Band
1	Reliance	A	212	2
2	Tata	A	330	3
3	Bajaj	B	200	2
		Total	742	

Sector Allocation

μ_k	2.700%	1.200%	0.7500%
σ_k	1.32500%	0.600%	0.37500%
#	Manufacturing	Telecom	FMCG
1	30%	60%	10%
2	40%	20%	40%
3	100%	0%	0%

Sector default rates are Gamma distributed (αk , βk)

#	Name	μ_k	σ_k	α_k	β_k
1	Manufacturing	2.700%	1.32500%	4.15237	0.006502
2	Telecom	1.200%	0.600%	4	0.003
3	FMCG	0.7500%	0.37500%	4	0.001875

2000

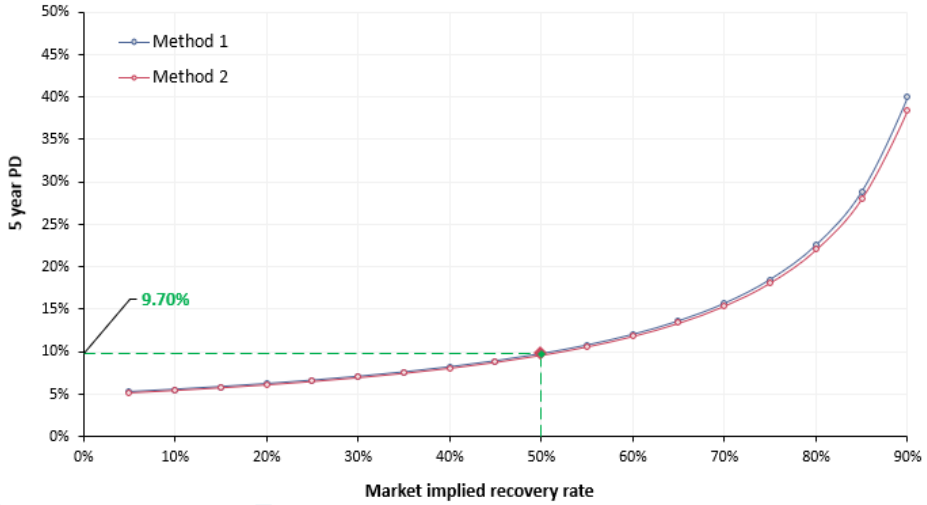
Simulation	Default Events Manu	Tele	FMCG	PD(1)	PD(2)	PD(3)	Band (2)	Band (3)	Loss
1	0.01605	0.00797	0.00505	0.97%	0.96%	0.98%	1.95%	0.96%	0
2	0.04018	0.01258	0.0105	1.82%	2.05%	2.46%	4.28%	2.05%	0
3	0.02593	0.01185	0.00607	1.44%	1.36%	1.58%	3.03%	1.36%	0
4	0.04575	0.01243	0.0098	1.89%	2.11%	2.80%	4.69%	2.11%	0
5	0.01756	0.00866	0.0182	1.31%	2.06%	1.07%	2.38%	2.06%	0
6	0.01133	0.01299	0.00834	1.33%	1.24%	0.69%	2.02%	1.24%	0
7	0.03742	0.03112	0.00953	3.15%	2.37%	2.29%	5.43%	2.37%	0
8	0.04341	0.00442	0.00326	1.12%	1.34%	2.65%	3.77%	1.34%	0
9	0.05035	0.00678	0.00888	1.53%	2.00%	3.08%	4.60%	2.00%	0
10	0.01444	0.01213	0.00903	1.33%	1.35%	0.88%	2.21%	1.35%	0
11	0.03567	0.0203	0.01233	2.36%	2.29%	2.18%	4.54%	2.29%	0
12	0.02662	0.0082	0.00128	1.08%	0.90%	1.63%	2.71%	0.90%	0
13	0.03611	0.01332	0.00251	1.65%	1.34%	2.21%	3.86%	1.34%	0
14	0.04026	0.00869	0.00148	1.35%	1.23%	2.46%	3.81%	1.23%	0
15	0.02508	0.0037	0.00568	0.81%	1.10%	1.53%	2.34%	1.10%	0
16	0.02464	0.00705	0.00644	1.07%	1.24%	1.51%	2.57%	1.24%	0
17	0.01165	0.01733	0.00688	1.63%	1.24%	0.71%	2.34%	1.24%	3
18	0.05281	0.01598	0.00636	2.21%	2.08%	3.23%	5.43%	2.08%	0
19	0.01559	0.01689	0.01091	1.74%	1.64%	0.95%	2.70%	1.64%	0
20	0.01001	0.01712	0.00196	1.49%	0.81%	0.61%	2.10%	0.81%	0
21	0.03279	0.00876	0.0049	1.30%	1.34%	2.00%	3.31%	1.34%	0
22	0.05244	0.00443	0.00152	1.24%	1.40%	3.20%	4.44%	1.40%	0
23	0.01069	0.01035	0.00752	1.11%	1.10%	0.65%	1.76%	1.10%	0
24	0.01595	0.01341	0.00157	1.30%	0.82%	0.97%	2.28%	0.82%	0
25	0.01165	0.01107	0.01653	1.35%	1.86%	0.71%	2.07%	1.86%	0

24.3 Reduced Form Models

RR	Method 1	Method 2
5%	5.2%	5.2%
10%	5.5%	5.5%
15%	5.8%	5.8%
20%	6.2%	6.1%
25%	6.6%	6.5%
30%	7.0%	7.0%
35%	7.5%	7.5%
40%	8.1%	8.1%
45%	8.9%	8.8%
50%	9.7%	9.6%
55%	10.7%	10.6%
60%	12.0%	11.8%
65%	13.6%	13.4%
70%	15.6%	15.4%
75%	18.5%	18.1%
80%	22.5%	22.0%
85%	28.82%	28.03%
90%	39.95%	38.47%
92%	47.14%	45.12%
95%	63.94%	60.48%
97%	81.73%	76.85%

T (years)	Spread over rfr (bps)	RR
5.00	102	50%

	Implied 5 year PD	Implied 1y PD
Method 1	9.697%	2.019%
Method 2	9.604%	1.999%



	5 year PD		1y PDs
PD	0.50%	Upper	0.1%
	9.61%	Lower	2.0%

Time to maturity	5
LGD	50%

Spread (bps)	5	Upper
	102	Lower

24.4 Credit Metrics

PEAKS2TAILS

Mean, μ 0

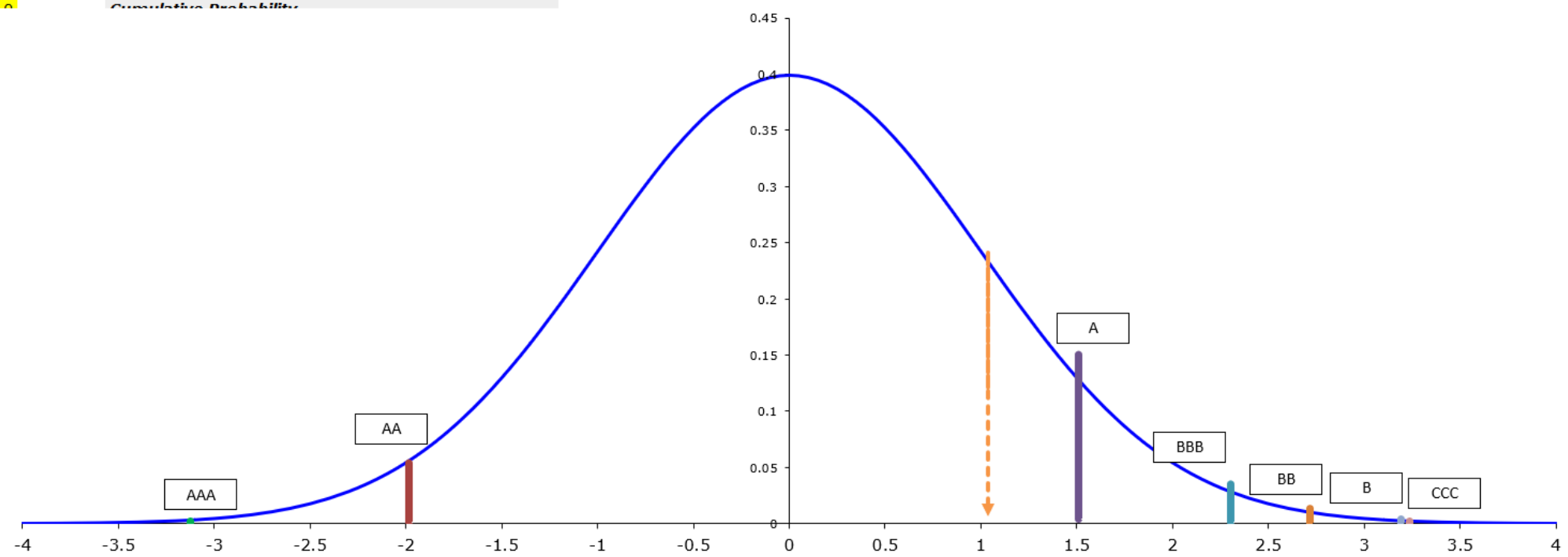
Standard Deviation, σ

Graph Limits

z_{min} -4

z_{max} 4

z	x	f(x)
-4	-4	0.0001
-3.8	-3.8	0.0002
-3.6	-3.6	0.0006
-3.4	-3.4	0.0012
-3.2	-3.2	0.0023
-3	-3	0.0044
-2.8	-2.8	0.0079
-2.6	-2.6	0.0135
-2.4	-2.4	0.0223
-2.2	-2.2	0.0354
-2	-2	0.0539
-1.8	-1.8	0.078
-1.6	-1.6	0.1109
-1.4	-1.4	0.1497
-1.2	-1.2	0.1941
-1	-1	0.2419
-0.8	-0.8	0.2896
-0.6	-0.6	0.3332
-0.4	-0.4	0.368
-0.2	-0.2	0.3910
1.28E-15	1.28E-15	0.3989
0.2	0.2	0.3910
0.4	0.4	0.368
0.6	0.6	0.3332
0.8	0.8	0.2896
1	1	0.2419
1.2	1.2	0.1941



24.5 Default Correlation

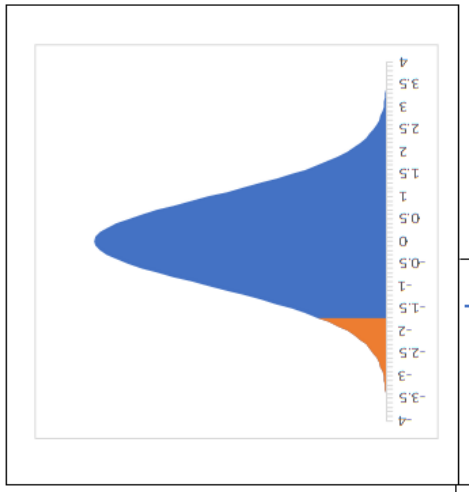
PEAKS2TAILS
http://peaks2tails.com

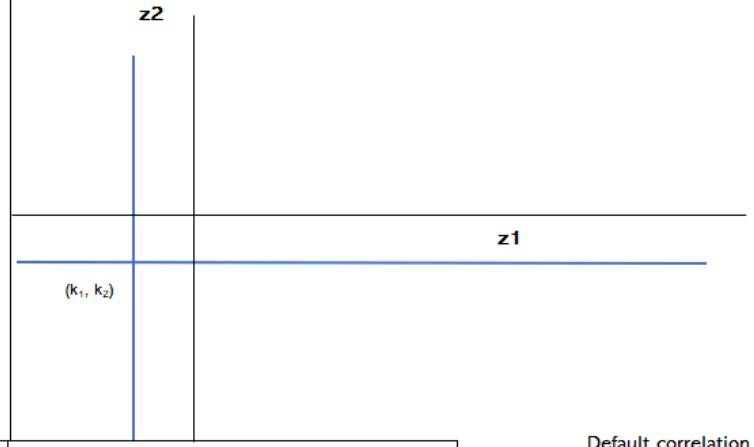
Combining Default Probability, Asset Correlation and Default Correlation

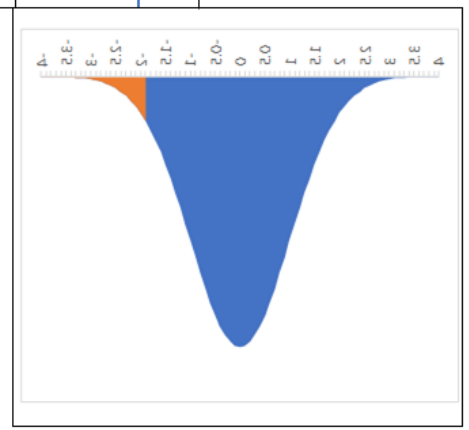
		PD	5%	3%
	z <=	k	-1.64485	-1.88079

ρ^A
40%

z1	z2	f(z1)	f(z2)	f(z1)_show	f(z2)_show
-4	-4	0.000134	0.000134	0.000134	0.000134
-3.9	-3.9	0.000199	0.000199	0.000199	0.000199
-3.8	-3.8	0.000292	0.000292	0.000292	0.000292
-3.7	-3.7	0.000425	0.000425	0.000425	0.000425
-3.6	-3.6	0.000612	0.000612	0.000612	0.000612
-3.5	-3.5	0.000873	0.000873	0.000873	0.000873
-3.4	-3.4	0.001232	0.001232	0.001232	0.001232
-3.3	-3.3	0.001723	0.001723	0.001723	0.001723
-3.2	-3.2	0.002384	0.002384	0.002384	0.002384
-3.1	-3.1	0.003267	0.003267	0.003267	0.003267
-3	-3	0.004432	0.004432	0.004432	0.004432
-2.9	-2.9	0.005953	0.005953	0.005953	0.005953
-2.8	-2.8	0.007915	0.007915	0.007915	0.007915
-2.7	-2.7	0.010421	0.010421	0.010421	0.010421
-2.6	-2.6	0.013583	0.013583	0.013583	0.013583
-2.5	-2.5	0.017528	0.017528	0.017528	0.017528
-2.4	-2.4	0.022395	0.022395	0.022395	0.022395
-2.3	-2.3	0.028327	0.028327	0.028327	0.028327
-2.2	-2.2	0.035475	0.035475	0.035475	0.035475
-2.1	-2.1	0.043984	0.043984	0.043984	0.043984
-2	-2	0.053991	0.053991	0.053991	0.053991
-1.9	-1.9	0.065616	0.065616	0.065616	0.065616
-1.8	-1.8	0.07895	0.07895	0.07895	
-1.7	-1.7	0.094049	0.094049	0.094049	
-1.6	-1.6	0.110921	0.110921		
-1.5	-1.5	0.129518	0.129518		
-1.4	-1.4	0.149727	0.149727		
-1.3	-1.3	0.171369	0.171369		
-1.2	-1.2	0.194186	0.194186		
-1.1	-1.1	0.217852	0.217852		







Identify regions

No one defaults	P(Z1>k1, Z2 >k2)
1 defaults, 2 does not	P(Z1<=k1, Z2 >k2)
2 defaults, 1 does not	P(Z1>k1, Z2 <=k2)
both default	P(Z1<=k1, Z2 <=k2)

JDP N_2 **(k1, k2, ρ)** 0.638%

Default Correlation ρ^D **13.1%**

Default correlation is simply the correlation between default events
This default correlation is related to marginal PDs and Asset correla

$$\rho^D = \frac{JDP - PD_1 PD_2}{\sqrt{PD_1(1-PD_1)PD_2(1-PD_2)}}$$

Asset correlation is used in ASRF framework for credit risk capital c

$$WCDR = N\left(\frac{G(PD) - \rho G(1\%)}{\sqrt{1-\rho^2}}\right), \quad G = N^{-1}$$

24.6 Asset Correlation

PEAKS2TAILS

Group 1

PD 2.53%

12 month Periods	Firms at the beginning of a Month	Firms defaulted within 12 months	EDF
1	1015	23	2.27%
2	1020	25	2.45%
3	1031	26	2.52%
4	1042	25	2.40%
5	1054	29	2.75%
6	1062	32	3.01%
7	1075	23	2.14%
8	1081	27	2.50%
9	1093	27	2.47%
10	1100	33	3.00%
11	1112	32	2.88%
12	1122	23	2.05%
13	1133	24	2.12%
14	1140	28	2.46%
15	1154	26	2.25%
16	1165	26	2.23%
17	1170	31	2.65%
18	1185	32	2.70%
19	1193	30	2.51%
20	1205	27	2.24%
21	1215	28	2.30%
22	1223	36	2.94%
23	1235	30	2.43%
24	1243	36	2.90%
25	1252	29	2.32%
26	1261	38	3.01%
27	1270	31	2.44%
28	1283	35	2.73%
29	1292	27	2.09%
30	1305	36	2.76%
31	1315	39	2.97%

Group 2

PD 5.50%

12 month Periods	Firms at the beginning of a Month	Firms defaulted within 12 months	EDF	Joint Default Probabilities	Nc x Nd	w
1	5011	262	5.23%	0.001184783	5086165	0.0041
2	5039	277	5.50%	0.001347334	5139780	0.0042
3	5053	278	5.50%	0.001387427	5209643	0.0042
4	5074	302	5.95%	0.001428002	5287108	0.0043
5	5095	277	5.44%	0.001495867	5370130	0.0044
6	5129	290	5.65%	0.001703691	5446998	0.0044
7	5143	280	5.44%	0.001164826	5528725	0.0045
8	5154	304	5.90%	0.001473219	5571474	0.0045
9	5172	263	5.09%	0.001256148	5652996	0.0046
10	5207	290	5.57%	0.001670828	5727700	0.0047
11	5228	271	5.18%	0.001491691	5813536	0.0047
12	5248	295	5.62%	0.001152294	5888256	0.0048
13	5270	275	5.22%	0.001105359	5970910	0.0049
14	5290	316	5.97%	0.001467184	6030600	0.0049
15	5309	292	5.50%	0.001239189	6126586	0.005
16	5324	302	5.67%	0.001265949	6202460	0.0051
17	5330	320	6.00%	0.001590738	6236100	0.0051
18	5367	281	5.24%	0.00141386	6359895	0.0052
19	5370	288	5.36%	0.001348649	6406410	0.0052
20	5407	284	5.25%	0.001176898	6515435	0.0053
21	5412	317	5.86%	0.001349843	6575580	0.0054
22	5446	326	5.99%	0.001762041	6660458	0.0054
23	5460	305	5.59%	0.001356943	6743100	0.0055
24	5480	312	5.69%	0.001648942	6811640	0.0056
25	5506	291	5.29%	0.001224195	6893512	0.0056
26	5516	298	5.40%	0.001628023	6955676	0.0057
27	5535	294	5.31%	0.001296545	7029450	0.0057
28	5562	298	5.36%	0.001461594	7136046	0.0058
29	5585	293	5.25%	0.001096341	7215820	0.0059
30	5601	318	5.68%	0.001566223	7309305	0.006
31	5618	336	5.98%	0.001773766	7387670	0.006

The marginal PDs for each group is straighforward

For Joint PD, we need the ratio of all possible default pair and the possible pairs of

JPD **0.001393**

Realized Default correlation **0.09384**

True Default Correlation 0.09384089

Default implied asset correlation

ρ 0.01960

JDP **0.001519**

Asset correlation with Market is given in Basel paper

Correlation (R) = 0.12 × (1 – EXP (-50 × PD)) / (1 - EXP(-50))
+ 0.24 × [1 - (1 - EXP(-50 × PD))/(1 - EXP(-50))]

Group 1 0.153894
Group 2 0.127673

Inter group asset correlation 0.0196

24.7 Credit Portfolio VAR

PEAKS2TAILS

PEAKS2TAILS
http://peaks2tails.com

Modeling Credit Portfolio VaR with Cholesky

Obligor	Exposures	PD	k
A1	₹ 50.00	5%	-1.6449
A2	₹ 100.00	4%	-1.7507
A3	₹ 75.00	3%	-1.8808

$\rho^A = AA^T$

1	0.5	0.2
0.5	1	-0.3
0.2	-0.3	1

A

1	0	0
0.4999	0.86617535	0
0.19985	-0.46116039	0.86419041

ρ^D

1	19.42%	7.32%
19.42%	1	-3.31%
7.32%	-3.31%	1

#	z1	z2	z3	z1'	z2'	z3'	D1	D2	D3	L
1	1.91264	-0.945241	1.22725	1.91264	0.137385	1.87873	0	0	0	₹ -
2	0.52046	-0.4721	1.57913	0.52046	-0.148743	1.68639	0	0	0	₹ -
3	0.96135	0.935071	-1.0672	0.96135	1.290514	-1.1613	0	0	0	₹ -
4	-0.9253	0.673762	-0.8655	-0.9253	0.121041	-1.2436	0	0	0	₹ -
5	1.67888	-2.279414	-0.9085	1.67888	-1.135099	0.6016	0	0	0	₹ -
6	0.8969	-0.648558	0.56927	0.8969	-0.113406	0.97029	0	0	0	₹ -
7	-0.4776	1.487514	-0.4674	-0.4776	1.049681	-1.1853	0	0	0	₹ -
8	1.81739	-1.243297	-0.1802	1.81739	-0.168401	0.78082	0	0	0	₹ -
9	0.18391	0.165364	-1.517	0.18391	0.235171	-1.3505	0	0	0	₹ -
10	-0.5491	0.325202	-0.8769	-0.5491	0.007178	-1.0175	0	0	0	₹ -
11	-1.85	-0.986833	-0.1684	-1.85	-1.779592	-0.0602	1	1	0	₹ 150.00
12	0.19171	-1.177284	0.45223	0.19171	-0.923898	0.97205	0	0	0	₹ -
13	-0.7638	0.886596	-0.1091	-0.7638	0.386117	-0.6558	0	0	0	₹ -
14	-0.6667	1.542606	-1.266	-0.6667	1.002906	-1.9387	0	0	1	₹ 75.00
15	-0.4339	-1.249377	-1.5772	-0.4339	-1.29908	-0.8736	0	0	0	₹ -
16	-0.407	0.891458	-0.3993	-0.407	0.568704	-0.8375	0	0	0	₹ -
17	-1.3345	-1.710483	-0.5429	-1.3345	-2.148677	0.05298	0	1	0	₹ 100.00

Confidence **99%**

VaR ₹ 150.00

EL $ 8.47

Theoretical EL ₹ 8.75

UL $ 141.25

Loss Distribution

Loss		Frequency	Prob	Cum Prob
₹	-	4490	0.898	0.898
$	50.00	181	0.0362	0.9342
$	75.00	108	0.0216	0.9558
$	100.00	150	0.03	0.9858
$	125.00	19	0.0038	0.9896
$	150.00	52	0.0104	1
$	175.00	0	0	1
$	225.00	0	0	1

END TO END PROJECTS IN PYTHON ON:

1. Scorecard Building
2. Transition Matrices
3. Stress Testing

TRAINING

SOLUTIONING

CONSULTING

9780564549

PEAKS2TAILS

Frequently asked questions

PREQUISITE

Basic knowledge of statistics and excel

CERTIFICATE

On successful completion of assignment and exam

FEES

40,000 For 1 Year Access
48,000 For Lifetime Access

DURATION

125 Hours

DATES

Online

TIMING

Flexible Timing